

MSE, 기초 공업수학

[연습문제 정답 이용 안내]

- 본 연습문제 정답의 저작권은 **마인속과 한빛아카데미(주)**에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

Chapter 01 연습문제 정답

[1.1 도함수]

1. $f'(x) = 20(x + x^3)^{19} (1 + 3x^2)$

2. $\frac{-t^2 - 4t - 2}{(t^2 - 2)^2}$

3. $g'(x) = 6x^2(x^3 + 1)(x^2 - 1)^4 + 8x(x^3 + 1)^2(x^2 - 1)^3$

4. $\frac{4t^5 - 5t^4 + 1}{(t - 1)^2}$

5. $\frac{d}{dt} \{f(x^2 + 1)\} = 2xf'(x^2 + 1) \frac{dx}{dt}$

6. $-\frac{2}{(x-1)^2} f' \left(\frac{x+1}{x-1} \right) \frac{dx}{dt}$

7. $\frac{d}{dt} \left\{ \frac{f(x)+1}{f(x)-1} \right\} = \frac{-2f'(x)}{(f(x)-1)^2} \frac{dx}{dt}$

8. $\frac{dy}{dx} = -\frac{2x+y^2}{y(2x+9y)}$

9. $\frac{dy}{dx} = \frac{1}{2y-2}$

10. $4(2x+1)(3(2x+1)^2+2)$

11. $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$

[1.2 여러 함수의 미분법]

12. $-3x^2 \sin(x^3 - 1)$

13. $y' = 2 \sec^2 x \tan x$

14. $\frac{dy}{dx} = \frac{1}{2y + \sec^2 y}$

15. $\frac{dy}{dx} = \frac{\sin x + y^2}{\cos y - 2xy}$

16. $-\frac{2}{x\sqrt{x^4 - 1}}$

17. $\frac{dy}{dx} = -\frac{\cos x}{\sqrt{1 - \sin^2 x}}$

18. $\frac{x}{(x^2 - 1)\sqrt{x^2 - 2}}$

19. $\frac{dy}{dx} = \frac{1}{2(1+x)\sqrt{x}}$

20. $-\tan x$

21. $\frac{dy}{dx} = \frac{\sin^3 x + \cos^2 x}{\cos x \sin^2 x - \cos^3 x \sin x}$

22. $2x \ln x + x$

23. $\frac{dy}{dx} = \frac{1}{2x}$

24. $3^x \ln 3 \sin x + 3^x \cos x$

25. $\frac{dy}{dx} = -\frac{3e^{3x}}{2\sqrt{1 - e^{3x}}}$

26. $e^x \left(\ln x + \frac{1}{x} \right)$

27. $\frac{dy}{dx} = x^{\cos x} \left(-\sin x \ln x + \frac{1}{x} \cos x \right)$

28. $6x^2 \sinh(2x^3)$

29. $\frac{dy}{dx} = -3x^2 \operatorname{sech}^2(1-x^3)$

30. $\frac{6x}{\sqrt{1+9x^4}}$

31. $\frac{dy}{dx} = 2 \frac{\cosh^{-1} x}{\sqrt{x^2-1}}$

[1.3 편도함수]

32. $\frac{9z^2}{x^2-2y+3z^3}$

33. $\frac{\partial f}{\partial z}(1, 2, 3) = -\frac{6}{169}$

34. $4x \sin(x-2y) + 2x^2 \cos(x-2y)$

35. $\frac{\partial z}{\partial x} = -\frac{2x+2yz}{2xy+2z}$

Chapter 02 연습문제 정답

[2.1 부정적분과 정적분]

1. $\int x^{\frac{1}{3}} dx = \frac{3}{4}x^{\frac{4}{3}} + C$

2. $\frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} + x + C$

3. $\int x(1 + \sqrt{x}) dx = \frac{1}{2}x^2 + \frac{2}{5}x^{\frac{5}{2}} + C$

4. $2\sqrt{x} + C$

5. 36

6. $\frac{13}{12}$

7. 13

8. $\frac{38}{45}$

9. $\frac{16}{5} - \frac{6}{5}2^{\frac{5}{3}}$

10. $\frac{41}{35}$

[2.2 여러 함수의 적분법]

11. $\int \tan 2x dx = -\frac{1}{2} \ln |\cos 2x| + C$

12. $-\frac{1}{2} \cos t^2 + C$

13. $\int \tan^2 x dx = \tan x - x + C$

$$14. \frac{1}{2} \tan^2 x + \ln |\cos x| + C$$

$$15. \int \sin^3 \theta \, d\theta = -\cos \theta + \frac{1}{3} \cos^3 \theta + C$$

$$16. \tan \theta - \sec \theta + C$$

$$17. \int \frac{\cos^2 x}{1 - \sin x} \, dx = x - \cos x + C$$

$$18. \frac{1}{2} \sin^4 x + C$$

$$19. \int \frac{e^x}{1 - e^x} \, dx = -\ln |1 - e^x| + C$$

$$20. \ln |1 - \cos x| + C$$

$$21. \int \frac{1}{t(1 - \ln t)} \, dt = -\ln |1 - \ln t| + C$$

$$22. \ln(e^x + 1) + C$$

$$23. \int \frac{e^{2x} + e^{-2x}}{3} \, dx = \frac{1}{6} (e^{2x} - e^{-2x}) + C$$

$$24. \frac{1}{2} (\ln(\ln x))^2 + C$$

$$25. \int x^2 e^{x^3} \, dx = \frac{1}{3} e^{x^3} + C$$

$$26. \frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} + 2x + C$$

$$27. \int \frac{1}{x \sqrt{9 - 4x^2}} \, dx = -\frac{1}{3} \ln \left| \frac{3}{2x} \left(1 + \sqrt{1 - \frac{4x^2}{9}} \right) \right| + C$$

$$28. \frac{1}{3} \tan^{-1} \left(\frac{x+1}{3} \right) + C$$

$$29. \int \frac{1}{\sqrt{4 - x^2}} \, dx = \sin^{-1} \frac{x}{2} + C$$

$$30. \frac{1}{6} \sec^{-1} \frac{2}{3} x + C$$

$$31. \int \frac{1}{t\sqrt{t+1}} dt = \ln |\sqrt{t+1}-1| - \ln |\sqrt{t+1}+1| + C$$

$$32. \frac{1}{2} \sec^{-1} \frac{x-1}{2} + C$$

$$33. \int x\sqrt{x+1} dx = 2\left(\frac{1}{5}\sqrt{(x+1)^5} - \frac{1}{3}\sqrt{(x+1)^3}\right) + C$$

$$34. \sqrt{x^2-1} - \tan^{-1} \sqrt{x^2-1} + C$$

[2.3 부분분수에 의한 유리함수의 적분법]

$$35. -\frac{1}{2}\ln|x| + \frac{1}{3}\ln|x+1| + \frac{1}{6}\ln|x-2| + C$$

$$36. \frac{2}{9}\ln\left|\frac{x-1}{x+2}\right| - \frac{1}{3}\frac{1}{x+2} + C$$

$$37. \ln|x-1| - \frac{2}{x-1} - \frac{1}{2}\frac{1}{(x-1)^2} + C$$

$$38. \ln|x| - \frac{1}{2}\ln(x^2-x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + C$$

$$39. 4\ln|x| - 2\ln(x^2+1) + \frac{2}{x^2+1} + C$$

[2.4 부분적분법]

$$40. \int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) + C$$

$$41. x \tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$$

$$42. \sin x - \frac{1}{3}\sin^3 x + C$$

$$43. x \sin^{-1} x + \sqrt{1-x^2} + C$$

44. $x(\ln x)^2 - 2x \ln x + 2x + C$

45. $(x^2 - x)\ln(x - 1) - \frac{1}{2}x^2 + C$

46. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$

47. $\frac{x}{1-x}\ln x + \ln|x-1| + C$

Chapter 03 연습문제 정답

[3.1 미분방정식의 소개]

1. 2계 1차
2. 2계 2차
3. 1계 1차
4. 2계 1차
5. 3계 1차
6. 3계 1차
7. 선형
8. 비선형
9. 선형
10. 비선형
11. 비선형
12. 선형
13. $c_1 = 1, c_2 = 1$
14. $c_1 = 0, c_2 = 1$
15. $c_1 = 0, c_2 = -1$
16. $c_1 = 0, c_2 = 2$
17. 존재하지 않는다.
18. $x^3 + (y+1)^3 = C \quad (C = 3C')$

[3.2 변수분리형 미분방정식]

19. $(x+1)^2 + (y+1)^2 + 2\ln|(x+1)(y-1)| = C$

20. $y = C \frac{x}{x-1}$

21. $yx^2 = ce^y$

22. $\frac{1}{x^2} - \frac{1}{y^2} = -2c' = c$

23. $\sqrt{2}$

24. $\sqrt{2}$

25. $\frac{\pi}{4}$

26. 999.99

27. $10 \times e^{-5}$

28. 동차 2차

29. 동차 0차

30. 동차 $\frac{3}{2}$ 차

31. 동차 아님

32. 동차 3차

33. 동차 0차

34. 동차 -1차

35. 동차 -2차

36. $\ln|x| + \frac{\sqrt{41}+1}{2\sqrt{41}} \ln|\frac{2y}{x} + 7 - \sqrt{41}| + \frac{\sqrt{41}-1}{2\sqrt{41}} \ln|\frac{2y}{x} + 7 + \sqrt{41}| = c$

$$37. x^3 - 2y^3 = cx \quad (c : \text{임의 상수})$$

$$38. c \sin^3 \frac{y}{x}$$

$$39. y = c\left(\frac{y}{x} - 1\right) \quad (c : \text{임의 상수})$$

$$40. x \ln |\sqrt{x^2 + y^2} - x| = cx - \sqrt{x^2 + y^2}$$

$$41. x^2 + 4xy + 3y^2 = c \quad (c : \text{임의 상수})$$

$$42. \ln |x| = -e^{\frac{y}{x}} + 1$$

$$43. 1 + \cos \frac{x}{y} = \frac{\pi}{2} y \sin \frac{x}{y}$$

$$44. y \ln \left(\frac{x}{y} \right) = 1$$

$$45. x^2 \left(1 + 2\sqrt{\frac{y}{x}} \right) = e^{2\sqrt{\frac{y}{x}}}$$

$$46. x + 2y + \ln |x + y| = c$$

$$47. \frac{5}{2}(x - y) - \ln |15x + 10y - 1| = c$$

$$48. \frac{1 + \sqrt{3}(2x + 3y)}{1 - \sqrt{3}(2x + 3y)} = ce^{4\sqrt{3}x} \quad (c \text{는 임의 상수})$$

$$49. 4(x + 2\sin y) + 9\ln |4x - 8\sin y + 3| = c$$

$$50. 1 - \cos(x + y) = (x + c)\sin(x + y)$$

$$51. 2\sqrt{2x + y + 1} - \ln(2\sqrt{2x + y + 1} + 1) = 2y + c$$

$$52. e^{x+y+2} = \frac{3ce^{3x}}{1 - ce^{3x}}$$

[3.3 완전 미분방정식]

53. 일반해 : $ye^{x^2} + x^2 = c$

54. 일반해 : $e^{3x}y + x^2 = c$

55. 일반해 : $\frac{1}{2}x^4 + 3xy + \frac{1}{3}y^3 - y = c$ 또는 $3x^4 + 18xy + 2y^3 - 6y = c$

56. 일반해 : $e^{xy^2} + \frac{2}{3}x^3 - y^3 = c$

57. 일반해 : $-\rho + \rho e^{2\theta} = c$

58. 일반해 : $x^4y^3 + \ln x + \ln y = c$

59. 일반해 : $\frac{1}{3}(x^2 + y^2)^{\frac{3}{2}} - xy = c$

60. 일반해 : $r \csc \theta + \ln \cos \theta = c$

61. 일반해 : $xy^2 - \ln x + \ln(x+y) + x + y^2 = c$

62. 일반해 : $e^{x^2y} + e^{xy^2} + 2x + y^2 = c$

63. 일반해 : $\frac{1}{4}x^4 + \frac{1}{2}x^2y^2 + \frac{1}{3}x^3 + \frac{1}{2}x^2 = c$ 또는 $3x^4 + 6x^2y^2 + 4x^3 + 6x^2 = c$

64. 일반해 : $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$

65. 일반해 : $e^{x^2}(\frac{1}{2}y^4 + x^2y^2 + 2xy) = c$

66. 일반해 : $\frac{y}{x} + \frac{1}{x} + x = c$ or $y + 1 + x^2 = cx$

67. 일반해 : $x^2y - x^3 = c$

68. 일반해 : $\ln x + \frac{y^2}{x} = c$

69. 일반해 : $e^x \sin y + y^2 = c$

70. 일반해 : $x^2 e^x \sin y = c$

[3.4 선형 미분방정식]

71. $y \cos^2 x = \int \sin 2x \, dx = -\frac{1}{2} \cos 2x + c$

72. $2yx^3 = 2ce^{\frac{1}{x^2}} + 1$

73. $y = \sin 2x \left(x - \frac{1}{2} \ln |\csc 2x + \cot 2x| + c \right)$

74. $r = \sin^2 \theta (2 \ln |\sin \theta| + c)$

75. $y = c(1 + e^x)$

76. $y = \frac{1}{1+x^3} \left(\frac{1}{3} x^3 + c \right)$

77. $i = -\frac{1}{2} \cos 2t + \frac{1}{4} \frac{\sin 2t}{t} + \frac{c}{t}$

78. $y = \frac{1}{x} e^x \left(-\frac{1}{2} \cos 2x + c \right)$

79. $x = \frac{4}{7} y^3 + c y^{-4}$

80. $(y^2 - 2y + 2)e^y + c$

81. $y = \begin{cases} \frac{1}{x^2} & 0 < x \leq 2 \\ \frac{1}{3}x - \frac{5}{3} \frac{1}{x^2} & x > 2 \end{cases}$

82. $y = \begin{cases} \frac{1}{3} + \frac{5}{3} e^{-x^3} & 0 \leq x < 1 \\ \left(\frac{1}{3} e + \frac{5}{3} \right) e^{-x^3} & x \geq 1 \end{cases}$

$$83. \ y = \begin{cases} 2 + e^{-x^2} & 0 \leq x < 1 \\ 4x^2 \ln x + (e^{-1} - 2)x^2 & x \geq 1 \end{cases}$$

$$84. \ y^3(e^{3x^2} + c) = 2e^{3x^2} \quad (c = 2c_1)$$

$$85. \ y^2(-2x + ce^{2x}) = 1$$

$$86. \ e^x = (-e^x \sin x + c)y$$

$$87. \ y\left(-\frac{1}{4}x - \frac{1}{2}x \ln x + \frac{c}{x}\right) = 1$$

Chapter 04 연습문제 정답

[4.1 1계 선형모형]

1. $x = 6.727171x_0$

2. $x = 3 \times 10^5 e^{0.099021t}$

3. $y = e^{\frac{1}{5}(t - \cos t + 1)} y_0$

4. $T = 290 + 80e^{-0.028768t}$

5. $241920(J)$

6. $A = A_0 e^{-0.000433t}$

7. $0.574(g)$

8. $x(3600) = 20e^{-3 \times 10^{-4} \times 3600} \doteq 6.79(kg)$

9. $18.08(kg)$

10. 소금 양 $484(kg)$

소금의 농도 : $\frac{484}{500}(kg/l) = 0.968$

11. 0.06%

12. $i = 55 + ce^{-4t}$

13. $i(t) = 2(\sin t + 2\cos t) + ce^{-2t}$

14. 2.217164

15. $v = \frac{18}{49}(-\frac{1}{2}\cos 2t + \frac{49}{20}\sin 2t) + ce^{-\frac{49}{5}t}$

16. $512.69(s)$

17. $t = 38.88$

18. $\frac{10^6}{3^9} (3125 \ln \frac{625}{463} - 810) \approx 6482$ (초)

19. 5.7개월

20. $\frac{1}{30}x_1 - \frac{2}{100+t}x_2 + q_2$

21. $\frac{dx_1}{dt} = (1.5 + \frac{1.5}{20-1.5t}x_2) - (\frac{x_1}{30} \cdot 3)$
 $\frac{dx_2}{dt} = (3 + \frac{x_1}{30} \cdot 3) - (\frac{x_2}{20-1.5t} \cdot 5.5)$

22. $\begin{cases} -4i_3' + 6i_2' + 4i_2 = 0 \\ i_2' + i_3' + 4i_3 = 12 \end{cases}$

23. $10i_2 + 10i_3 + 4i_3' = 4$
 $10i_2' + 10i_3' + 100i_2 = 0$

[4.2 비선형모형]

24. (a) $c_1 = \frac{y_0}{1000 - y_0}, y = \frac{1000y_0}{(1000 - y_0)e^{-1000\alpha t} + y_0}$
 (b) 0명

25. $P(t) = \frac{2 \times 10^6 [-3(2 \times 10^4 - 100)]^t}{(2 \times 10^4 - 100)(300 - 2 \times 10^4)^t + 100[-3(2 \times 10^4 - 100)]^t}$

26. (a) 10^5 (명) (b) 35개월

27. (a) $x = \frac{bce^{\alpha(a-b)t} - a}{ce^{\alpha(a-b)t} - 1} \Rightarrow \begin{cases} \textcircled{1} a > b : t \rightarrow \infty \text{ 일 때 } x \rightarrow b \\ \textcircled{2} a < b : t \rightarrow \infty \text{ 일 때 } x \rightarrow a \end{cases}$
 (b) $x = a - \frac{1}{\alpha t + c_1}$ 그리고 $t \rightarrow \infty : x \rightarrow a$

28. (a) 27.41 (b) $\frac{200}{3}$ (c) A의 양은 $\frac{100}{3}$ (g), B의 양은 0 (g)

[4.3 해의 존재성과 유일성]

29. 생략

30. 판단할 수 없다.

31. $y < -3$ 이거나 $y > 3$

32. x 는 모든 실수이고 $y \neq 4$ 인 모든 실수이다.

33. $y \neq x$ 인 모든 점

34. ① $y(0) = k \neq 0$ 이거나 $y(1) = k \neq 0$ 이면
해가 존재하지 않음
② $y(0) = 0$ 이거나 $y(1) = 0$ 이면
무수히 많은 해가 존재
③ 임의의 y_0 에 대해, $y(x_0) = y_0$, $x_0 \neq 0$
이고 $x_0 \neq 4$ 이면 유일한 해 존재

Chapter 05 연습문제 정답

[5.1 선형미분방정식의 기본이론]

1. $2x + x^2$
2. $(4x + 4)e^x$
3. e^{2x}
4. $a \cos ax + a \sin ax$
5. $2D^2 - D - 6$
6. $x^2 D^2 + 2xD - 2$
7. $D^2 - 4$
8. $x^2 D^2 + D - x^2 - x$
9. $(D+3)(D-3)$
10. $(2D+1)(D+1)$
11. $D(D+2)$
12. $D(D-1)(D-2)$
13. $(D-2)(D-6)$
14. $(D^2 + 2 + \sqrt{5}D)(D^2 + 2 - \sqrt{5}D)$
15. $D^3(1+3x+2x^2) = 0$
16. $(D^2 - 2D)y = 0$
17. 생략
18. $\cos x \neq 0$ 인 구간 I 에서 연속

[5.2 선형미분방정식의 성질]

19. 일차독립

20. 일차독립

21. 일차종속

22. 일차독립

23. 일차독립

24. 일차독립

25. 일차독립

26. 일반해 $y = c_1 e^{3x} + c_2 e^{4x}$

27. 일반해 $y = c_1 x^4 + c_2 x^{-\frac{1}{2}}$

28. 일반해 $y = c_1 \cos(2\ln x) + c_2 \sin(2\ln x)$

29. 일반해 $y = c_1 e^x \cos 2x + c_2 e^x \sin 2x = e^x (c_1 \cos 2x + c_2 \sin 2x)$

30. $= e^x ((-x+1)e^{-x} - e^{-x}) = -x$

31. x^3

32. xe^x

33. $\frac{2}{3}x^{\frac{1}{2}}$

34. $\frac{1}{2} \sin x^2$

35. $\frac{\sin x}{x}$

[5.3 상수계수를 갖는 선형 제차 미분방정식]

36. $y = c_1 e^{-x} + c_2 e^{\frac{1}{2}x}$

37. $y = c_1 + c_2 e^{\frac{1}{2}x} + c_3 x e^{\frac{1}{2}x}$

38. $y = c_1 e^{-x} + e^{\frac{1}{2}x} (c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x)$

39. $y = c_1 \cos 3x + c_2 \sin 3x + c_3 e^{\sqrt{2}x} + c_4 e^{-\sqrt{2}x}$

40. $y = c_1 + c_2 e^{\sqrt{3}x} + c_3 e^{-\sqrt{3}x} + c_4 \cos \sqrt{3}x + c_5 \sin \sqrt{3}x$

41. $y = c_1 \cos x + c_2 \sin x + c_3 x \cos x + c_4 x \sin x$

42. $y = c_1 + c_2 x + e^{\frac{1}{2}x} (c_3 \cos \frac{\sqrt{3}}{2}x + c_4 \sin \frac{\sqrt{3}}{2}x)$

43. $y = \cos 2x - \frac{1}{2} \sin 2x$

44. $y = \frac{3}{4} e^x + \frac{1}{4} e^{5x}$

45. $y = \frac{23}{18} - \frac{5}{18} e^{6x} + \frac{2}{3} x e^{6x}$

46. $y = x + x^2 + x^3$

47. $y = (1-x)e^{-5x}$

48. $y = \cos 3x + \sin 3x$

49. $y''' - 3y'' + 4y' - 2y = 0$

50. $y'' + 4y = 0$

51. $y''' - 4y'' + y' + 6y = 0$

52. $y'' - 9y = 0$

$$53. y''' - y'' = 0$$

$$54. y'' + 4y = 0$$

[5.4 미정계수법]

$$55. y = c_1 e^{-3x} + c_2 e^x + \frac{1}{5} e^{2x}$$

$$56. y = c_1 e^{-3x} + c_2 e^x - \frac{1}{4} x e^{-3x}$$

$$57. y = e^x (c_1 \cos 2x + c_2 \sin 2x) + \frac{2}{17} \cos 2x - \frac{8}{17} \sin 2x$$

$$58. y = e^x (c_1 \cos 2x + c_2 \sin 2x) - \frac{3}{4} x e^x \cos 2x$$

$$59. y = \frac{3}{4} e^{-2x} + \frac{3}{4} e^{2x} - \frac{1}{2}$$

$$60. y = -2 + 4e^{-x} + x e^{-x} + 5x - \left(\frac{1}{6} x^3 + \frac{1}{2} x^2\right) e^{-x}$$

$$61. y = e^{-2x} \left(-\frac{1}{32} \cos 2x + \frac{1}{32} \sin 2x\right) + \frac{1}{8} x^2 - \frac{1}{8} x + \frac{1}{32}$$

$$62. y = 4 - 3x + x^2 - 4e^{-x} - \frac{1}{6} x^3 + \frac{1}{24} x^4 - x e^{-x}$$

$$63. y_P = Ax \cos 2x + Bx \sin 2x + Cx^2 \cos 2x + Ex^2 \sin 2x$$

[5.5 매개변수 변화법]

$$64. y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{4} e^{2x} \int \frac{1}{x} e^{-4x} dx - \frac{1}{4} e^{-2x} \ln |x|$$

$$65. y = c_1 \cos x + c_2 \sin x - \cos x \ln |\sec x + \tan x|$$

$$66. y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{12} e^{2x} - \frac{1}{6} e^x + e^{-x} - (e^{-x} + e^{-2x}) \ln(1 + e^x) + \frac{1}{2}$$

$$67. y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x} \cos e^x$$

$$68. y = e^{-x} (c_1 \cos x + c_2 \sin x) + e^{-x} (\cos x \cdot \ln |\cos x| + x \sin x)$$

$$69. y = c_1 e^{5x} + c_2 x e^{5x} + x e^{5x} (-1 + \ln |x|) = c_1 e^{5x} + c_2 x e^{5x} + x e^{5x} \ln |x|$$

$$70. y = e^{-x} (c_1 \cos 3x + c_2 \sin 3x) - \frac{1}{9} (e^{-x} \cos 3x) \ln |\sec 3x + \tan 3x|$$

$$71. y = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x} + \frac{1}{12} (1-x^2)^{\frac{3}{2}} e^{-\frac{1}{2}x} + \frac{1}{8} x e^{-\frac{1}{2}x} [\sin^{-1} x + x \sqrt{1-x^2}]$$

$$72. y = c_1 x + c_2 x \ln x + \frac{1}{4x}$$

$$73. y = c_1 x^{-\frac{1}{2}} + c_2 x - \frac{8}{9} x + \frac{2}{5} x^2 + \frac{4}{3} x \ln |x|$$

[5.6 코시-오일러 방정식]

$$74. y = c_1 x^{-3} + c_2 x^2 - \frac{1}{125} x^2 (5 \ln x - 1) + \frac{1}{10} x^2 (\ln x)^2$$

$$75. y = c_1 x^2 + c_2 x^2 \ln x + \frac{1}{4} \ln x + \frac{1}{4}$$

$$76. y = c_1 x^{-2} + c_2 x^2 - \frac{1}{2} \cos(\ln x) + \frac{3}{20} \sin(\ln x)$$

$$77. y = c_1 + c_2 x^{-3} + \frac{1}{28} x^4$$

$$78. y = c_1 x + c_2 x \ln x + x^2$$

$$79. y = c_1 x + c_2 x^2 - x e^x + x^2 \int \frac{1}{x} e^x dx$$

$$80. y_c = c_1 x + c_2 x \ln x + c_3 x (\ln x)^2$$

$$81. y = c_1 x^{-2} + c_2 x^2 - \frac{1}{4x} + \frac{1}{4x^2} \ln |x+1| + \frac{1}{4} x^2 \ln |x| + \frac{x}{4} - \frac{1}{8} - \frac{1}{4} x^2 \ln |x+1|$$

82. $a \frac{d^2 y}{dt^2} + (b - a) \frac{dy}{dt} + cy = 0$

83. $y = c_1 x + c_2 x^2 + (\ln x - 1)x^2 + 2$

Chapter 06 연습문제 정답

[6.1 고계 선형모형]

1. $x = 5\sin(2t + \phi)$

$$\text{단 } \sin \phi = \frac{3}{5}, \quad \cos \phi = \frac{4}{5}, \quad \tan \phi = \frac{3}{4}$$

$$\text{즉, } \phi = 36.8698^\circ \quad (0.6435 \text{ rad})$$

2. $x = 3\sin(t + \phi)$

$$\text{단, } \sin \phi = -\frac{2}{3}, \quad \cos \phi = \frac{\sqrt{5}}{3}, \quad \tan \phi = -\frac{2}{\sqrt{5}}$$

$$\text{즉, } \phi = -41.8104^\circ \quad (-0.72973 \text{ rad})$$

3. $x = 2\sqrt{2} \sin(3t + \frac{3}{4}\pi)$

$$\text{단 } \sin \phi = \frac{1}{\sqrt{2}}, \quad \cos \phi = \frac{-1}{\sqrt{2}}$$

$$\text{즉, } \phi = \frac{3}{4}\pi \text{ (rad)} = 135^\circ$$

4. $x = \sqrt{5} \sin(\pi t + \phi)$

$$\text{단 } \sin \phi = \frac{-1}{\sqrt{5}}, \quad \cos \phi = -\frac{2}{\sqrt{5}}, \quad \tan \phi = \frac{1}{2}$$

$$\text{즉, } \phi = 26.5623^\circ \quad (0.4636 \text{ rad})$$

5. $x = 3\cos \frac{7}{\sqrt{30}}t$

6. $x = \frac{16}{7} \sin(\frac{7}{\sqrt{15}}t + \phi)$

$$\text{단, } \sin \phi = -\frac{7}{8}, \quad \cos \phi = \frac{\sqrt{15}}{8}, \quad \tan \phi = -\frac{7}{\sqrt{15}}$$

$$\text{즉, } \phi = -61.045^\circ \quad (-1.0654 \text{ rad})$$

7. $x = \frac{5}{7} \sin 14t$

8. $x = e^{-10t} (2\cos 4\sqrt{6}t + \frac{5}{\sqrt{6}} \sin 4\sqrt{6}t)$

[6.2 고계 연립 선형 미분방정식의 모형]

9. $x_1 = c_1 \cos 2t + c_2 \sin 2t$

$$x_2 = -c_1 \sin 2t + c_2 \cos 2t$$

10. $x_1 = e^{\frac{5}{2}t} (c_1 \cos \frac{\sqrt{7}}{2}t + c_2 \sin \frac{\sqrt{7}}{2}t)$

$$x_2 = e^{\frac{5}{2}t} (-(\frac{3}{4}c_1 + \frac{\sqrt{7}}{4}c_2) \cos \frac{\sqrt{7}}{2}t + (-\frac{3}{4}c_2 + \frac{\sqrt{7}}{4}c_1) \sin \frac{\sqrt{7}}{2}t)$$

11. $x_1 = -\frac{13}{2}e^{2t} - \frac{3-2\sqrt{2}}{1-\sqrt{2}}c_2e^{(1-\sqrt{2})t} - \frac{3+2\sqrt{2}}{1+\sqrt{2}}c_3e^{(1+\sqrt{2})t}$

$$x_2 = c_1 + c_2e^{(1-\sqrt{2})t} + c_3e^{(1+\sqrt{2})t} + \frac{7}{2}e^{2t}$$

12. $x = 2c_1e^t + 2c_2(t-1)e^t - c_3e^{-2t}$

$$y = c_1e^t + c_2te^t + c_3e^{-2t}$$

$$z = (c_1 - 2c_2)e^t + c_2(t-1)e^t + c_3e^{-2t}$$

13. $q_1' - q_2' - q_3^3 = 0$

$$q_2' - q_4' - q_5^4 = 0$$

$$10(q_3' + q_4' + q_5') + 5q_3'' = 6$$

$$10q_3' + 40q_4' + 40q_5'' + 10q_5 = 6$$

$$10q_3' + 55q_4' + 40q_5 + \frac{5}{2}q_4 = 6$$

14. $\frac{3}{2}c_1e^{-\frac{1}{10}t} - c_2e^{-\frac{3}{5}t}, c_1e^{-\frac{1}{10}t} + c_2e^{-\frac{3}{5}t}$

15. $(D+0.1)x_1 - 1.5x_2 = 1.5$

$$0.1x_1 - (D + \frac{3}{20+t})x_2 = 3$$

16. $86.4e^{-2.16} \doteq 9.9641(kg)$

Chapter 07 연습문제 정답

[7.1 라플라스 변환]

1. $e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$

2. $F(s) = \frac{s}{s^2 + 1}(e^{-\pi s} + 1)$

3. $-\frac{k}{s}(e^{-sa} - 1)$

4. $F(s) = \frac{1}{s}(e^{-3s} - 2e^{-s} + 1)$

5. $\frac{s}{s^2 + \pi^2}$

6. $\frac{w \cos \theta + s \sin \theta}{s^2 + w^2}$

7. $L\{\sin^2 x\} = \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2 + 4}\right)$

8. $e^{-b}\frac{1}{s-a}$

9. $\frac{1}{s^2 - 2s + 2}$

10. $\frac{s+1}{s^2 + 2s + 2}$

11. 생략

12. $\frac{1}{s^{\alpha+1}}\Gamma(\alpha+1), \quad \alpha > -1$

13. $\frac{\sqrt{\pi}}{\sqrt{s}}$

$$14. \frac{1}{2} \frac{\sqrt{\pi}}{s^{\frac{3}{2}}}$$

$$15. \frac{3\sqrt{\pi}}{4 s^{\frac{5}{2}}}$$

$$16. \frac{1}{5!} t^5$$

$$17. \frac{1}{2} e^{\frac{\sqrt{2}}{2} t}$$

$$18. 2\cos 2t$$

$$19. \frac{1}{2} \sin 2t$$

$$20. \cos \frac{1}{2} t$$

$$21. \frac{1}{2} \cos \frac{1}{2} t$$

$$22. \cosh 5t - \frac{6}{5} \sinh 5t$$

$$23. \cos 6t - \frac{6}{5} \sin 5t$$

$$24. -e^{-t} + e^{4t}$$

$$25. \frac{1}{5} e^{4t} + \frac{1}{5} e^{-t}$$

$$26. 1 + 3\cos 3t - \sin 3t$$

$$27. \frac{2}{13} e^{2t} - \frac{2}{13} \cos 3t + \frac{3}{13} \sin 3t$$

$$28. y(t) = \frac{4}{3} \cos t + \sin t - \frac{1}{3} \cos 2t$$

$$29. y(t) = 2e^t - e^{2t}$$

$$30. y(t) = \cos t + \frac{2}{3} \sin t - \frac{1}{3} \sin 2t$$

$$31. y(t) = \frac{1}{2}(e^t - e^{-t})$$

[7.2 라플라스 변환의 성질]

$$32. \frac{1}{(s-2)^2}$$

$$33. \frac{2}{(s+2)^3}$$

$$34. \frac{2}{(s-2)^3} + 2\frac{1}{(s-2)^2} + \frac{1}{s-2}$$

$$35. \frac{2}{(s-2)^2 + 4}$$

$$36. \frac{s-2}{(s-2)^2 + 4}$$

$$37. \frac{1}{2}t^2e^{-t}$$

$$38. \frac{1}{6}t^3e^t$$

$$39. (1+3t)e^{-2t}$$

$$40. (2\cos \sqrt{3}t - \frac{2}{\sqrt{3}} \sin \sqrt{3}t)e^t$$

$$41. (7\cos t + 6\sin t)e^{-t}$$

$$42. (t - t^2 + \frac{2}{3}t^3)e^t$$

$$43. (1+3t)e^{-2t}$$

44. $(2\cos \sqrt{3} t - \frac{2}{\sqrt{3}} \sin \sqrt{3} t)e^t$

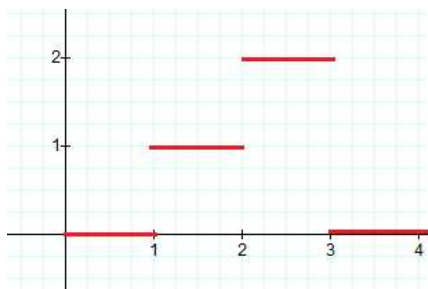
45. $y(t) = (\frac{7}{5} \cos t + \frac{6}{5} \sin t)e^{-t} - \frac{2}{5} \cos t + \frac{1}{5} \sin t$

46. $y(t) = (-\cos t + \sin t)e^{-t} + e^{-t}$

47. $y(t) = -\frac{3}{4}e^t - \frac{1}{2}t^2e^t + \frac{1}{12}e^{-t} + \frac{2}{3}e^{2t}$

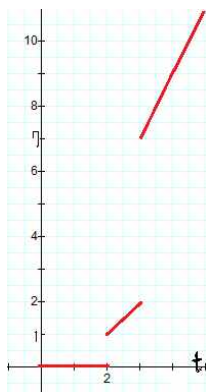
48. $y(t) = (t - t^2 + \frac{2}{3}t^3)e^t$

49.



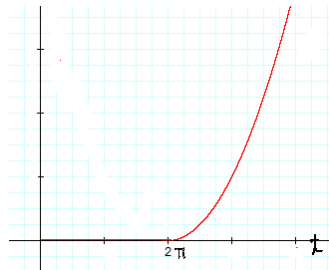
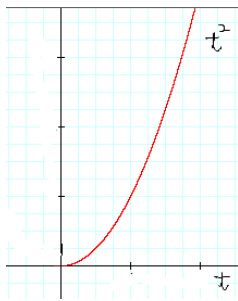
$$\begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t < 2 \\ 2, & 2 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

50.



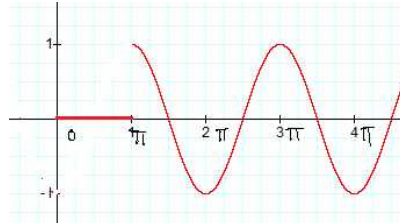
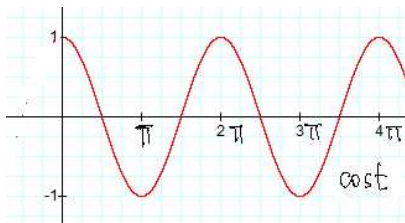
$$\begin{cases} 0 & 0 \leq t < 2 \\ t-1 & 2 \leq t < 3 \\ 2t+1 & t \geq 3 \end{cases}$$

51.



$$\begin{cases} 0 & 0 \leq t < 2\pi \\ f(t-2\pi) = (t-2\pi)^2 & t \geq 2\pi \end{cases}$$

52.



$$\begin{cases} 0 & 0 \leq t \leq \pi \\ f(t-\pi) = \cos(t-\pi) & t \geq \pi \\ = \cos(\pi-t) & \\ = -\cos t & \end{cases}$$

53. $\frac{2}{s^3} e^{-s}$

54. $(\frac{2}{s^3} - \frac{2}{s}) e^{-2s}$

55. $\frac{1}{s^2} e^{-\pi s} - \frac{1}{s^2} e^{-3\pi s} - \frac{2\pi}{s} e^{-3\pi s}$

56. $\frac{1}{s} (e^{-s} + e^{-2s} - 2e^{-3s})$

57. $(\frac{1}{s^2} + \frac{1}{s}) e^{-2s} + (\frac{1}{s^2} + \frac{5}{s}) e^{-3s}$

58. $t^2 e^{-2t}$

59. $\frac{1}{\sqrt{3}} \sin \sqrt{3}(t-2) e^{-(t-2)} u(t-2)$

60. $e^{t-1} \cos \sqrt{2}(t-1) \cdot u(t-1)$

61. $\frac{1}{2} \sinh 2t$

$$62. e^{3(t-2)} \cosh \sqrt{6}(t-2)u(t-2)$$

$$63. e^{(t-1)}u(t-1) + e^{(t-2)}u(t-2) + e^{(t-3)}u(t-3) + e^{(t-4)}u(t-4)$$

$$64. 1 - \cos t - (1 + \cos t)u(t - \pi)$$

$$65. (\cos t - \cos \sqrt{2}t) + (\cos t + \cos \sqrt{2}(t - \pi))u(t - \pi)$$

$$66. e^{2t} - e^t + \left(\frac{1}{2} - e^{t-3} + \frac{1}{2}e^{2(t-3)}\right)u(t-3)$$

$$67. \cos t + 2t - 2\sin t - 2[2t + 3\sin t - \pi - 1 + (\pi - 1)\cos t]u(t - \pi)$$

$$68. \frac{1}{17}[\cos t + 4\sin t - (\cos t + \frac{9}{2}\sin t)e^{-\frac{1}{2}t}] \\ - \frac{1}{17}[\cos t + 4\sin t - (\cos t + \frac{9}{2}\sin t)e^{-\frac{1}{2}(t-\pi)}]u(t - \pi)$$

$$69. [-1 + \frac{1}{2}\cos t(t-1) + \frac{1}{4}e^{-(t-1)} + \frac{1}{4}e^{(t-1)}]u(t-1) \\ + [-1 + \frac{1}{2}\cos t(t-2) + \frac{1}{4}e^{-(t-2)} + \frac{1}{4}e^{(t-2)}]u(t-2)$$

$$70. i(t) = -\frac{1}{10} + t + \frac{1}{10}e^{-10t} - \left(\frac{19}{10} + (t-2) - \frac{19}{10}e^{-10(t-2)}\right)u(t-2)$$

[7.3 합성곱과 주기함수의 라플라스 변환]

$$71. \frac{4s}{(s^2 + 4)^2}$$

$$72. \frac{-2(5s^2 - 1)}{(s^2 + 1)^3}$$

$$73. \frac{s^2 - 2s}{(s^2 + 2s + 2)^2}$$

$$74. \frac{2s + 2}{(s^2 + 2s + 2)^2}$$

$$75. -\frac{1}{2}t \sin t - \frac{1}{2}(\sin t - t \cos t)$$

$$76. y(t) = 2\cos 2t + \sin 2t + \frac{1}{4}t \sin 2t$$

$$77. \frac{2}{s^3} \frac{1}{s^2 + 1}$$

$$78. \frac{1}{s+1} \frac{s}{s^2 + 4}$$

$$79. \frac{1}{s^2(s-1)}$$

$$80. \frac{s}{(s^2 + 1)(s^2 + 1)}$$

$$81. t + e^{-t} - 1$$

$$82. \frac{1}{2}(-\cos t + \sin t + e^t)$$

$$83. \frac{1}{2}te^t - \frac{1}{2}e^t + \frac{1}{2}\cos t$$

$$84. \frac{1}{2}(t^2 - 2t - 2e^{-t} + 2)$$

$$85. \frac{2}{5}e^{-t}\cos t + \frac{1}{5}e^{-t}\sin t - \frac{2}{5}\cos t + \frac{1}{5}\sin t$$

$$86. -e^{-2t} + 12te^{-2t} + te^{-2t} * g(t)$$

$$87. -1 + t + \frac{2}{3}e^t + \frac{4}{3}e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2}t$$

$$88. 2e^{-t}$$

$$89. 1 - \cos t + \sin t$$

$$90. -1 + e^{-t} + 2t^2e^{-t}$$

$$91. \frac{1}{3}e^{-t} - \frac{1}{3}e^{\frac{1}{2}t} \cos t \frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{\frac{1}{2}t} \sin \frac{\sqrt{3}}{2}t$$

$$92. (1 - e^{-2(t-3)})u(t-3)$$

$$93. 100\left(\frac{1}{2} - t - \frac{1}{2} \cos \sqrt{2}t + \frac{1}{\sqrt{2}} \sin \sqrt{2}t\right) + 100\left(\frac{1}{2} + (t-1) - \frac{1}{2} \cos \sqrt{2}(t-1) - \frac{1}{\sqrt{2}} \sin \sqrt{2}(t-1)\right)u(t-1)\Big\}$$

$$94. \frac{1 - se^{-s} - e^{-s}}{s^2(1 - e^{-s})}$$

$$95. \frac{1 + e^{-\pi s}}{(1 - e^{-\pi s})(1 + s^2)}$$

$$96. \frac{1}{e^{-s} + 1} \left(\frac{e^{-s}}{s} + \frac{e^{-s} - 1}{s^2} \right)$$

[7.4 라플라스 변환의 성질]

$$97. \begin{cases} 0 & t < \pi \\ -\frac{1}{3} \sin 3t & t \geq \pi \end{cases}$$

$$98. \begin{cases} 0 & t < \pi \\ -\sin t & \pi \leq t < 2\pi \\ -2\sin t & t \geq 2\pi \end{cases}$$

$$99. \begin{cases} \cosh t & t < 1 \\ \cosh t + 10\sinh(t-1) & t \geq 1 \end{cases}$$

$$100. \begin{cases} e^t - 1 & t < 2 \\ e^t + e^{t-2} - 2 & t \geq 2 \end{cases}$$

$$101. -\frac{2}{5} \cos t + \frac{1}{5} \sin t + \left(\frac{2}{5} \cos t + \frac{1}{5} \sin t\right)e^{-t} - e^{-(t-\pi)} \sin t u(t-\pi)$$

$$102. \frac{2}{\sqrt{31}} \sin \frac{\sqrt{31}}{4} \left(t - \frac{\pi}{2}\right) e^{-\frac{1}{4}(t - \frac{\pi}{2})} u\left(t - \frac{\pi}{2}\right)$$

$$103. \quad x_1 = c_1 \cos 2t + c_2 \sin 2t$$

$$x_2 = -c_1 \sin 2t + c_2 \cos 2t$$

$$104. \quad x_1 = (c_1 \cos \frac{\sqrt{7}}{2}t + c_2 \sin \frac{\sqrt{7}}{2}t)e^{\frac{5}{2}t}$$

$$x_2 = (-\frac{3}{4}c_1 + \frac{\sqrt{7}}{4}c_2) \cos \frac{\sqrt{7}}{2}t + (\frac{\sqrt{7}}{4}c_1 - \frac{3}{4}c_2) \sin \frac{\sqrt{7}}{2}t)e^{\frac{5}{2}t}$$

$$105. \quad x_1 = -\frac{1}{2} - \frac{13}{2}e^{2t} + 7(\cosh \sqrt{2}t)e^t + \frac{10}{\sqrt{2}}(\sinh \sqrt{2}t)e^t$$

$$x_2 = -\frac{1}{2} + \frac{7}{2}e^{2t} - 3(\cosh \sqrt{2}t)e^t - \frac{4}{\sqrt{2}}(\sinh \sqrt{2}t)e^t$$

$$106. \quad x = -\frac{1}{3}e^{-2t} + \frac{4}{3}e^t, \quad y = \frac{1}{3}e^{-2t} + \frac{2}{3}e^t$$

$$107. \quad y_1 = 1 - \cos t + \sin t + \sin(t-2)u(t-2) + (1 - \cos(t-2))u(t-2)$$

$$y_2 = (\cos t + \sin t - 1) + (\cos(t-2) + \sin(t-2) - 1)u(t-2)$$

$$108. \quad y_1 = -2\cos t + \frac{7}{2}\sin t - \frac{1}{2}t\cos t + e^t + e^{-t}$$

$$y_2 = e^t + e^{-t} + 2\cos t - \frac{7}{2}\sin t + \frac{1}{2}t\cos t$$

$$109. \quad y = -\frac{1}{2} + \frac{1}{2}e^t - \frac{1}{2}te^t$$

$$x = \frac{13}{2} + 5t + \frac{3}{4}t^2 - \frac{13}{2}e^t + \frac{5}{2}te^t$$

Chapter 08 연습문제 정답

[8.1 벡터]

1. $\overrightarrow{AB} = \langle -1, 5, -11 \rangle$, $|\overrightarrow{AB}| = \sqrt{147}$

2. $3\mathbf{a} - 2\mathbf{b} = \langle -7, 8, 7 \rangle$, $|3\mathbf{a} - 2\mathbf{b}| = \sqrt{49 + 64 + 49} = \sqrt{162}$

$3\mathbf{a} - 2\mathbf{b}$ 의 단위 벡터 $= \frac{3\mathbf{a} - 2\mathbf{b}}{|3\mathbf{a} - 2\mathbf{b}|} = \langle -\frac{7}{\sqrt{162}}, \frac{8}{\sqrt{162}}, \frac{7}{\sqrt{162}} \rangle$

3. $B = (3, -6, 4)$

4. $A = (-1, -2, -1)$

5. ②

6. $t = 1$

7. $\mathbf{b} = \langle \frac{2}{\sqrt{14}}, -\frac{4}{\sqrt{14}}, -\frac{6}{\sqrt{14}} \rangle$

8. $\mathbf{a} = -\frac{5}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$

9. $k_1 = -\frac{5}{31}$, $k_2 = \frac{81}{31}$, $k_3 = \frac{27}{31}$

10. $\overrightarrow{PM} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b}$

11. $k = \pm \frac{2}{|\vec{a}|}$

12. $k = \pm \sqrt{6}$

13. $k = -1$

[8.2 내적]

14. 6

15. 27

16. -9

17. $c = -9$

18. $\mathbf{c} = \langle -\frac{1}{8}, 1, -\frac{7}{8} \rangle$

19. $\cos\alpha = \frac{2}{\sqrt{56}}, \cos\beta = -\frac{6}{\sqrt{56}}, \cos\gamma = \frac{4}{\sqrt{56}}$

$$\begin{cases} \alpha = \cos^{-1}\left(\frac{2}{\sqrt{56}}\right) \approx 74.49864^\circ \\ \beta = \cos^{-1}\left(-\frac{6}{\sqrt{56}}\right) \approx 143.30077^\circ \\ \gamma = \cos^{-1}\left(\frac{4}{\sqrt{56}}\right) \approx 57.68847^\circ \end{cases}$$

20. $\cos A = \frac{50}{\sqrt{66}\sqrt{40}} = \frac{5\sqrt{165}}{66}$

$$\cos B = \frac{16}{\sqrt{66}\sqrt{6}} = \frac{8\sqrt{11}}{33}$$

$$\cos C = \frac{-10}{\sqrt{40}\sqrt{6}} = -\frac{\sqrt{15}}{6}$$

21. $\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{9}{29} \langle 2, 3, -4 \rangle$, 크기 : $\frac{9}{\sqrt{29}}$

22. $3920\frac{\sqrt{3}}{3} (J)$

23. $\text{proj}_{\mathbf{a}^\perp}\mathbf{b} = \frac{1}{29} \langle 11, 2, 7 \rangle$

[8.3 외적]

24. $\sqrt{22}$

25. $\langle 30, -12, -6 \rangle$

26. $\langle -2k, -7k, 5k \rangle$, ($k \neq 0$)

27. 16

28. 3

$$29. c \neq \frac{5}{2}$$

$$30. \sqrt{281}$$

$$31. \frac{3}{2}\sqrt{13}$$

[8.4 3차원 공간에서의 직선과 평면]

$$32. \langle 1, 2, -2 \rangle + t \langle 4, -1, -2 \rangle$$

$$33. \langle x, y, z \rangle = \langle 1, -2, 3 \rangle + t \langle 5, 3, -2 \rangle$$

$$34. \frac{x-3}{1} = \frac{y-2}{3} = \frac{z+1}{5}$$

$$35. \langle x, y, z \rangle = \langle 2, 1, -1 \rangle + t \langle 1, 2, 3 \rangle \quad \text{또는} \quad \frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{3}$$

$$36. \left(\frac{13}{3}, 2, -\frac{5}{3} \right)$$

$$37. 12 \langle 1, 0, 0 \rangle \quad \text{또는} \quad 12 \langle -1, 0, 0 \rangle$$

$$38. x + z = 5$$

$$39. 4x - y + z - 5 = 0$$

$$40. 2x + 3y - 4z - 13 = 0$$

$$41. 39x + 22y + 36z - 112 = 0$$

$$42. x = 1$$

$$43. \begin{cases} x = \frac{13}{5} + \frac{7}{5}t \\ y = \frac{6}{5} + \frac{9}{5}t \\ z = t \end{cases}$$

$$44. \left(-\frac{2}{5}, -\frac{4}{5}, -\frac{21}{5} \right)$$

[8.5 벡터공간]

45. 생략

46. V 는 벡터공간이 아니다.

47. 생략

48. \mathbb{R}^3 는 주어진 합과 스칼라 곱의 연산에 대해서는 벡터공간을 이루지 못한다.

49. V 는 \mathbb{R}^4 의 부분공간이다.

50. V 는 \mathbb{R}^4 의 부분공간이다.

51. A 는 P_3 의 부분공간이다.

52. B 는 \mathbb{R}^2 의 부분공간이 아니다.

53. A 는 \mathbb{R}^2 의 부분공간이 아니다.

54. B 는 \mathbb{R}^2 의 부분공간이다.

55. 일차독립

56. 일차독립

57. 일차독립

58. 일차종속

59. $\langle x, y, z \rangle = (x-y) \langle 1, 0, 0 \rangle + (y-z) \langle 1, 1, 0 \rangle + z \langle 1, 1, 1 \rangle \in \mathbb{R}^3$

60. $\langle 1, 2 \rangle = \frac{3}{2} \langle 1, 1 \rangle + (-\frac{1}{2}) \langle 1, -1 \rangle$

61. 49번-3차원, 50번-2차원, 51번-2차원

Chapter 09 연습문제 정답

[9.1 행렬과 연산]

1. $x = \pm 3, y = \pm 12$ (복호동순)

2. $-\frac{1}{3}A = \begin{pmatrix} -\frac{1}{3} & -\frac{2}{3} \\ -1 & -\frac{4}{3} \end{pmatrix}$

$$A + 2B = \begin{pmatrix} 11 & 14 \\ 17 & 20 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$$

$$A - B^T = \begin{pmatrix} -4 & -5 \\ -3 & -4 \end{pmatrix}$$

3. $AB = \begin{pmatrix} 14 & -10 \\ 28 & -14 \end{pmatrix}, BA = \begin{pmatrix} 18 & 24 \\ -17 & -18 \end{pmatrix}$

$$\therefore AB \neq BA$$

4. $B^T A^T = \begin{pmatrix} 5 & 2 \\ 10 & -5 \end{pmatrix} \begin{pmatrix} 3 & 7 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 23 & 39 \\ 10 & 60 \end{pmatrix}$

$$(AB)^T = B^T A^T$$

5. 생략

6. $A = \begin{pmatrix} a & b \\ 2a & 2b \end{pmatrix}, B = \begin{pmatrix} -b & b \\ a & -a \end{pmatrix}$

7. 생략

[9.2 연립 선형방정식과 가우스 소거법]

8. $x_1 = \frac{27}{8}, x_2 = \frac{9}{8}, x_3 = \frac{15}{8}$

9. $x_1 = -\frac{3}{5}t + 1$

$$x_2 = \frac{1}{5}t$$

$$x_3 = t$$

10. $x_1 = 1, x_2 = t, x_3 = 4 - 2t, x_4 = t$

11. $x_1 = x_2 = x_3 = 0$

12. $x_1 = t, x_2 = -2t, x_3 = t$

[9.3 행렬의 계수]

13. $\text{rank } A = 3$

14. $\text{rank } B = 3$

15. $\text{rank } C = 2$

16. 일차종속

17. 일차독립

18. 자명한 해를 갖는다.

19. $a = 2$

20. $a - b - c = 0$

[9.4 행렬식]

21. $x - 2y + z$

22. 0

23. 0

24. 34

25. $x = 1$

26. $x = 2, -2 \pm \sqrt{-3}$

27. k

28. $-k$

29. $6k$

30. $-k$

31. 24

32. 24

33. -6

34. 0

[9.5 역행렬]

35. $A^{-1} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

36. $A^{-1} = \begin{pmatrix} 1 & -\frac{5}{2} & \frac{3}{2} \\ 2 & -8 & 5 \\ -1 & 5 & -3 \end{pmatrix}$

37. $A^{-1} = -\frac{1}{3} \begin{pmatrix} -12 & -3 & 3 \\ 31 & 7 & -9 \\ -20 & -5 & 6 \end{pmatrix}$

38. $A^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

39. $A^{-1} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}$

40. $x_1 = \frac{61}{53}, x_2 = -\frac{51}{53}, x_3 = \frac{13}{53}$

41. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 25 \\ -15 \end{pmatrix}$

42. $\lambda = 4, -1$

43. $A = \frac{1}{26} \begin{pmatrix} 8 & -1 \\ 3 & 11 \end{pmatrix}$

Chapter 10 연습문제 정답

[10.1 크래머의 법칙]

1. $x_1 = \frac{61}{53}, x_2 = -\frac{51}{53}, x_3 = \frac{13}{53}$

2. $x_1 = 7, x_2 = 25, x_3 = -15$

3. $k \neq \frac{6}{5}$ 인 경우 $x_1 = \frac{3k-2}{-5k+6}, x_2 = \frac{3k-6}{-5k+6}$ (유일해)

$k = \frac{6}{5}$ 인 경우 해를 갖지 않는다.

[10.2 고윳값과 고유벡터]

4. $\lambda = 1 \pm \sqrt{6}$

$$K_1 = \begin{pmatrix} 2 \\ \sqrt{6} \end{pmatrix}, K_2 = \begin{pmatrix} -2 \\ \sqrt{6} \end{pmatrix}$$

5. $\lambda = 4, -4$

$$K_1 = \begin{pmatrix} 8 \\ 3 \end{pmatrix}, K_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

6. $\lambda = 0$ (중근)

$$K = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

7. $\lambda_1 = 1, \lambda_2 = -1, \lambda_3 = 2$

$$K_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, K_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, K_3 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

8. $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$

$$K_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, K_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, K_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

9. $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$

$$K_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, K_2 = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}, K_3 = \begin{pmatrix} -5 \\ -7 \\ 3 \end{pmatrix}$$

[10.3 직교행렬]

$$10. \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$11. \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$12. \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$13. \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

[10.4 대각화]

$$14. \begin{pmatrix} 1 & -1 + 3^5 \\ 0 & 3^5 \end{pmatrix}$$

$$15. \begin{pmatrix} 5^6 & 2 \times 5^6 \\ 2 \times 5^6 & 4 \times 5^6 \end{pmatrix}$$

$$16. \begin{pmatrix} \frac{5}{2} - 2^{11} + \frac{5}{2} \times 3^9 & -2 + 2^{10} & -\frac{1}{2} - 2^{10} + \frac{5}{2} \times 3^9 \\ \frac{5}{2} - 3 \times 2^{10} + \frac{7}{2} \times 3^9 & -2 + 3 \times 2^9 & -\frac{1}{2} - 3 \times 2^9 + \frac{7}{2} \times 3^9 \\ -\frac{5}{2} + 2^{11} - \frac{1}{2} \times 3^{10} & 2 - 2^{10} & \frac{1}{2} + 2^{10} - \frac{1}{2} \times 3^{10} \end{pmatrix}$$

$$17. \begin{pmatrix} 1 & 2 & -3 + 3 \cdot 2^{11} \\ 0 & -1 & 0 \\ 0 & 0 & 2^{11} \end{pmatrix}$$

18. E^{13} 은 대각화를 이용한 방법으로 계산할 수 없다.

$$19. \begin{pmatrix} \frac{1}{3}5^{15} - \frac{2}{3} & -\frac{1}{3}5^{15} - \frac{1}{3} & \frac{1}{3}5^{15} + \frac{1}{3} \\ -\frac{1}{3}5^{15} - \frac{1}{3} & \frac{1}{3}5^{15} - \frac{2}{3} & -\frac{1}{3}5^{15} - \frac{1}{3} \\ \frac{1}{3}5^{15} + \frac{1}{3} & -\frac{1}{3}5^{15} - \frac{1}{3} & \frac{1}{3}5^{15} - \frac{2}{3} \end{pmatrix}$$

$$20. \begin{pmatrix} 4 \times 3^{15} + 16 \times 6^{15} + 9^{16} & 4 \times 3^{15} - 8 \times 6^{15} - 2 \times 9^{16} & 2 \times 3^{15} - 16 \times 6^{15} + 2 \times 9^{16} \\ 4 \times 3^{15} - 8 \times 6^{15} - 2 \times 9^{16} & 4 \times 3^{15} + 4 \times 6^{15} + 4 \times 9^{16} & 2 \times 3^{15} + 8 \times 6^{15} - 4 \times 9^{16} \\ 2 \times 3^{15} - 16 \times 6^{15} + 2 \times 9^{16} & 2 \times 3^{15} + 8 \times 6^{15} - 4 \times 9^{16} & 3^{15} + 16 \times 6^{16} + 4 \times 9^{16} \end{pmatrix}$$

$$21. \frac{3}{10}x'^2 - \frac{1}{5}y'^2 = 1$$

$$22. -x'^2 + y'^2 = 5$$

$$23. \frac{x'^2}{\left(\sqrt{\frac{31}{3}}\right)^2} + \frac{\left(y' + \frac{1}{3}\right)^2}{\left(\frac{\sqrt{31}}{3}\right)^2} = 1$$