CHAPTER 1 BASIC ARITHMETIC

EXERCISE 1, Page 4

1. Evaluate 67 kg – 82 kg + 34 kg without using a calculator

67 kg – 82 kg + 34 kg = 67 kg + 34 kg – 82 kg = 101 kg – 82 kg = **19 kg**

2. Evaluate 851 mm – 372 mm without using a calculator

$$\begin{array}{r} 851 \\ -\ \underline{372} \\ \underline{479} \end{array}$$

Hence, 851 mm – 372 mm = **479 mm**

3. Evaluate 124 - 273 + 481 – 398 without using a calculator

$$\begin{array}{r} 124 \\ +\ \underline{481} \\ \underline{605} \end{array} \qquad \begin{array}{r} 273 \\ \underline{398} \\ \underline{671} \end{array}$$

Hence, 124 - 273 + 481 – 398 = 605 – 671 = **-** (671 – 605) = **- 66**

4. Evaluate £927 - £114 + £182 - £183 - £247 without using a calculator

$$\begin{array}{r} 927 \\ \underline{182} \\ \underline{1109} \end{array} \qquad \begin{array}{r} 114 \\ 183 \\ \underline{247} \\ \underline{544} \end{array} \qquad \begin{array}{r} 1109 \\ \underline{544} \\ \underline{565} \end{array}$$

Hence, £927 - £114 + £182 - £183 - £247 = £1109 - £544 = **£565**

5. Evaluate 647 – 872 without using a calculator

647 – 872 = **-** (872 – 647)

$$\begin{array}{r} 872 \\ \underline{647} \\ \underline{225} \end{array}$$

Hence, 647 – 872 = **- 225**

6. Evaluate 2417 - 487 + 2424 - 1778 – 4712 without using a calculator

2417 - 487 + 2424 - 1778 – 4712 = 2417 + 2424 - 487 - 1778 – 4712

$$\begin{array}{r} 2417 \\ \underline{2424} \\ \underline{4841} \end{array} \qquad \begin{array}{r} 487 \\ 1778 \\ \underline{4712} \\ \underline{6977} \end{array} \qquad \begin{array}{r} 6977 \\ -\ \underline{4841} \\ \underline{2136} \end{array}$$

Hence, 2417 + 2424 - 487 - 1778 – 4712 = 4841 – 6977

= - (6977 – 4841) = **- 2136**

7. Evaluate £2715 - £18250 + £11471 - £1509 + £113274 without using a calculator

£2715 - £18250 + £11471 - £1509 + £113274 = £2715 + £11471 + £113274 - £18250 - £1509

$$\begin{array}{r} 2715 \\ 11471 \\ +\ \underline{113274} \\ \underline{127460} \end{array} \qquad \begin{array}{r} 18250 \\ +\ \underline{1509} \\ \underline{19759} \end{array}$$

Hence, £2715 + £11471 + £113274 - £18250 - £1509 = £127460 – £19759 = **£107,701**

8. Evaluate 47 + (- 74) – (- 23) without using a calculator

47 + (- 74) – (- 23) = 47 – 74 + 23 = 47 + 23 – 74

= 70 – 74 = **- 4**

9. Evaluate 813 - (- 674) without using a calculator

813 - (- 674) = 813 + 674

$$\begin{array}{r} 813 \\ +\ \underline{674} \\ \underline{1487} \end{array}$$

Hence, 813 + 674 = **1487**

10. Evaluate - 23148 – 47724 without using a calculator

- 23148 – 47724 = - (23148 + 47724)

$$\begin{array}{r} 23148 \\ +\ \underline{47724} \\ \underline{70872} \end{array}$$

Hence, - 23148 – 47724 = **- 70872**

11. Evaluate $53774 - $38441 without using a calculator

$$\begin{array}{r} 53774 \\ -\ \underline{38441} \\ \underline{15333} \end{array}$$

Hence, $53774 - $38441 = **$15,333**

EXERCISE 2, Page 6

1. Evaluate without using a calculator: (a) 78×6 (b) 124×7

(a)
$$\begin{array}{r} 78 \\ \times\ \underline{\quad 6} \\ \underline{468} \\ {\scriptstyle 4\ \ } \end{array}$$

Hence, $78 \times 6 =$ **468**

(b)
$$\begin{array}{r} 124 \\ \times\ \underline{\quad 7} \\ \underline{868} \\ {\scriptstyle 1\,2\ } \end{array}$$

Hence, $124 \times 7 =$ **868**

2. Evaluate without using a calculator: (a) £261 × 7 (b) £462 × 9

(a)
$$\begin{array}{r} 261 \\ \times\ \underline{\quad 7} \\ \underline{1827} \\ {\scriptstyle 4\ \ } \end{array}$$

Hence, £261 × 7 = **£1827**

(b)
$$\begin{array}{r} 462 \\ \times\ \underline{\quad 9} \\ \underline{4158} \\ {\scriptstyle 5\,1\ \ } \end{array}$$

Hence, £462 × 9 = **£4158**

3. Evaluate without using a calculator: (a) 783 kg × 11 (b) 73 kg × 8

(a)
$$\begin{array}{r} 783 \\ \times\ \underline{\ \ 11} \\ 783 \\ \underline{7830} \\ \underline{8613} \\ {\scriptstyle 1\,1\ \ \ \ } \end{array}$$

Hence, 783 kg × 11 = **8613 kg**

(b)

$$\begin{array}{r} 73 \\ \times \underline{\quad 8} \\ \underline{584} \\ {\scriptstyle 2} \end{array}$$

Hence, 73 kg × 8 = **584 kg**

4. Evaluate without using a calculator: (a) 27 mm × 13 (b) 77 mm × 12

(a)

$$\begin{array}{r} 27 \\ \times \underline{\ 13} \\ 81 \\ \underline{270} \\ \underline{351} \\ {\scriptstyle 1} \end{array}$$

Hence, 27 mm × 13 = **351 mm**

(b)

$$\begin{array}{r} 77 \\ \times \underline{\ 12} \\ 154 \\ \underline{770} \\ \underline{924} \\ {\scriptstyle 1} \end{array}$$

Hence, 77 mm × 12 = **924 mm**

5. Evaluate without using a calculator: (a) 288 m ÷ 6 (b) 979 m ÷ 11

(a)

$$6\overline{)288}\quad = 48$$

Hence, 288 m ÷ 6 = **48 mm**

(b)

$$11\overline{)979}\quad = 89$$

Hence, 979 m ÷ 11 = **89 mm**

6. Evaluate without using a calculator: (a) $\frac{1813}{7}$ (b) $\frac{896}{16}$

(a) $\begin{array}{r} 259 \\ 7\overline{)1813} \end{array}$

Hence, $\frac{1813}{7} = 1813 \div 7 = \mathbf{259}$

(b) $\begin{array}{r} 56 \\ 16\overline{)896} \end{array}$

Hence, $\frac{896}{16} = 896 \div 16 = \mathbf{56}$

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7. Evaluate without using a calculator: (a) $\frac{88737}{11}$ (b) $46858 \div 14$

(a)

$$\begin{array}{r} 8067 \\ 11\overline{)88737} \\ \underline{88} \\ 73 \\ \underline{66} \\ 77 \\ \underline{77} \\ .\,. \end{array}$$

Hence, $\frac{88737}{11} = 88737 \div 11 = \mathbf{8067}$

(b)

$$\begin{array}{r} 3347 \\ 14\overline{)46858} \\ \underline{42} \\ 48 \\ \underline{42} \\ 65 \\ \underline{56} \\ 98 \\ \underline{98} \\ .\,. \end{array}$$

Hence, $46858 \div 14 = \mathbf{3347}$

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8. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws. (1 kg = 1000 g)

Mass of 1200 screws = $1200 \times 15 = 18000$ g = $18000 \div 1000$ = **18 kg**

9. Holes are drilled 36 mm apart in a metal plate. If a row of 26 holes is drilled, determine the distance, in centimetres, between the centres of the first and last holes.

The number of spaces if 26 holes are drilled = 25

Hence, distance between the centres of the first and last holes = 36 × 25 = 900 mm

Thus, distance in centimetres = 900 ÷ 10 = **90 cm**

10. A builder needs to clear a site of bricks and top soil. The total weight to be removed is 696 tonnes. Trucks can carry a maximum load of 24 tonnes. Determine the number of truck loads needed to clear the site.

Number of truck loads = 696 ÷ 24

$$24\overline{)696}\quad = 29$$

Hence, **number of truck loads needed = 29**

EXERCISE 3, Page 7

1. Find (a) the HCF and (b) the LCM of the following numbers: 8, 12

(a) $8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

Hence, **HCF** $= 2 \times 2 =$ **4** i.e. 4 is the highest number that will divide into **both** 8 and 12

(b) **LCM** $= 2 \times 2 \times 2 \times 3 =$ **24** i.e. 24 is the lowest number that **both** 8 and 12 will divide into.

2. Find (a) the HCF and (b) the LCM of the following numbers: 60, 72

(a) $60 = 2 \times 2 \times 3 \times 5$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

Hence, **HCF** $= 2 \times 2 \times 3 =$ **12** i.e. 12 is the highest number that will divide into **both** 60 and 72

(b) **LCM** $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 =$ **360** i.e. 360 is the lowest number that **both** 60 and 72 will divide into.

3. Find (a) the HCF and (b) the LCM of the following numbers: 50, 70

(a) $50 = 2 \times 5 \times 5$

$70 = 2 \times 5 \times 7$

Hence, **HCF** $= 2 \times 5 =$ **10** i.e. 10 is the highest number that will divide into **both** 50 and 70

(b) **LCM** $= 2 \times 5 \times 5 \times 7 =$ **350** i.e. 350 is the lowest number that **both** 50 and 79 will divide into

4. Find (a) the HCF and (b) the LCM of the following numbers: 270, 900

(a) $270 = 2 \times 3 \times 3 \times 3 \times 5$

$900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$

Hence, **HCF** $= 2 \times 3 \times 3 \times 5 =$ **90** i.e. 90 is the highest number that will divide into **both** 270 and 900

(b) **LCM** = $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$ = **2700** i.e. 2700 is the lowest number that **both** 270 and 900 will divide into

5. Find (a) the HCF and (b) the LCM of the following numbers: 6, 10, 14

(a) $6 = 2 \times 3$

$10 = 2 \times 5$

$14 = 2 \times 7$

Hence, **HCF** = 2 i.e. is the highest number that will divide into 6, 10 and 14

(b) **LCM** = $2 \times 3 \times 5 \times 7$ = **210** i.e. 210 is the lowest number that 6, 10 and 14 will divide into

6. Find (a) the HCF and (b) the LCM of the following numbers: 12, 30, 45

(a) $12 = 2 \times 2 \times 3$

$30 = 2 \times 3 \times 5$

$45 = 3 \times 3 \times 5$

Hence, **HCF** = **3** i.e. is the highest number that will divide into 12, 30 and 45

(b) **LCM** = $2 \times 2 \times 3 \times 3 \times 5$ = **180** i.e. 180 is the lowest number that 12, 30 and 45 will divide into

7. Find (a) the HCF and (b) the LCM of the following numbers: 10, 15, 70, 105

(a) $10 = 2 \times 5$

$15 = 3 \times 5$

$70 = 2 \times 5 \times 7$

$105 = 3 \times 5 \times 7$

Hence, **HCF** = **5** i.e. is the highest number that will divide into 10, 15, 70 and 105

(b) **LCM** = $2 \times 3 \times 5 \times 7$ = **210** i.e. 210 is the lowest number that 10, 15, 70 and 105 will divide into

8. Find (a) the HCF and (b) the LCM of the following numbers: 90, 105, 300

(a) $90 = 2 \times 3 \times 3 \times 5$

$105 = 3 \times 5 \times 7$

$300 = 2 \times 2 \times 3 \times 5 \times 5$

Hence, **HCF** = 3×5 = **15** i.e. 15 is the highest number that will divide into 90, 105 and 300

(b) **LCM** = $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$ = **6300** i.e. 6300 is the lowest number that 90, 105 and 300 will divide into

EXERCISE 4, Page 8

1. Evaluate: $14 + 3 \times 15$

$14 + 3 \times 15 = 14 + 45$ (M)

$= \mathbf{59}$ (A)

2. Evaluate: $17 - 12 \div 4$

$17 - 12 \div 4 = 17 - 3$ (D)

$= \mathbf{14}$ (S)

3. Evaluate: $86 + 24 \div (14 - 2)$

$86 + 24 \div (14 - 2) = 86 + 24 \div 12$ (B)

$= 86 + 2$ (D)

$= \mathbf{88}$ (A)

4. Evaluate: $7(23 - 18) \div (12 - 5)$

$7(23 - 18) \div (12 - 5) = 7 \times 5 \div 7$ (B)

$= \mathbf{5}$ (D/M)

5. Evaluate: $63 - 8(14 \div 2) + 26$

$63 - 8(14 \div 2) + 26 = 63 - 8 \times 7 + 26$ (B)

$= 63 - 56 + 26$ (M)

$= 89 - 56 = \mathbf{33}$ (D/M)

6. Evaluate: $\frac{40}{5}$ - 42 ÷ 6 + (3 × 7)

$$\frac{40}{5} - 42 \div 6 + (3 \times 7) = \frac{40}{5} - 42 \div 6 + 21 \qquad \text{(B)}$$

$$= 8 - 7 + 21 \qquad \text{(D)}$$

$$= 29 - 7 = \mathbf{22} \qquad \text{(A/S)}$$

7. Evaluate: $\frac{(50-14)}{3}$ + 7(16 - 7) - 7

$$\frac{(50-14)}{3} + 7(16 - 7) - 7 = \frac{36}{3} + 7 \times 9 - 7 \qquad \text{(B)}$$

$$= 12 + 63 - 7 \qquad \text{(D/M)}$$

$$= \mathbf{68} \qquad \text{(A/D)}$$

8. Evaluate: $\frac{(7-3)(1-6)}{4(11-6) \div (3-8)}$

$$\frac{(7-3)(1-6)}{4(11-6) \div (3-8)} = \frac{4 \times -5}{4 \times 5 \div -5} \qquad \text{(B)}$$

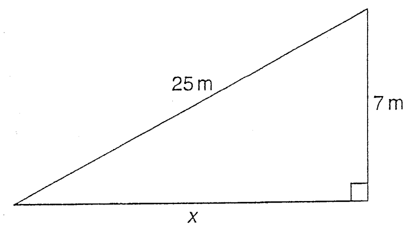
$$= \frac{4 \times -5}{4 \times -1} \qquad \text{(D)}$$

$$= \frac{-20}{-4} = \mathbf{5} \qquad \text{(D/M)}$$

CHAPTER 10 TRIGONOMETRY

EXERCISE 39, Page 87

1. Find the length of side x in the diagram below.

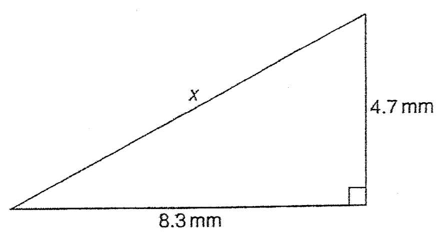


By Pythagoras, $25^2 = x^2 + 7^2$

from which, $x^2 = 25^2 - 7^2$

and $\mathbf{x} = \sqrt{25^2 - 7^2} = \mathbf{24\ m}$

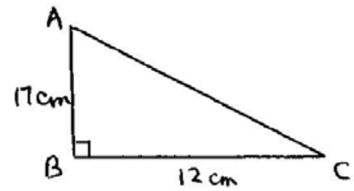
2. Find the length of side x in the diagram below, correct to 3 significant figures.



By Pythagoras, $x^2 = 8.3^2 + 4.7^2$

from which, $\mathbf{x} = \sqrt{8.3^2 + 4.7^2} = \mathbf{9.54\ mm}$

3. In a triangle ABC, AB = 17 cm, BC = 12 cm and ∠ABC = 90°. Determine the length of AC, correct to 2 decimal places.

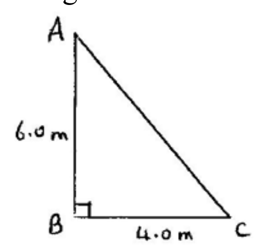


By Pythagoras, $AC^2 = 17^2 + 12^2$

from which, $\mathbf{AC} = \sqrt{17^2 + 12^2} = \mathbf{20.81\ mm}$

4. A tent peg is 4.0 m away from a 6.0 m high tent. What length of rope, correct to the nearest centimetre, runs from the top of the tent to the peg?

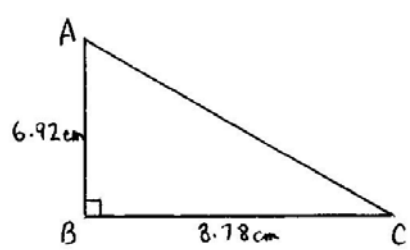
In the side view shown below, AB is the height of the tent and C is the tent peg.



By Pythagoras, $AC^2 = 6.0^2 + 4.0^2$

from which, length of rope, $\mathbf{AC} = \sqrt{6.0^2 + 4.0^2} = \mathbf{7.21\ m}$

5. In a triangle ABC, $\angle B$ is a right angle, AB = 6.92 cm and BC = 8.78 cm. Find the length of the hypotenuse.



By Pythagoras, $AC^2 = 6.92^2 + 8.78^2$

from which, $\mathbf{AC} = \sqrt{6.92^2 + 8.78^2} = \mathbf{11.18\ cm}$

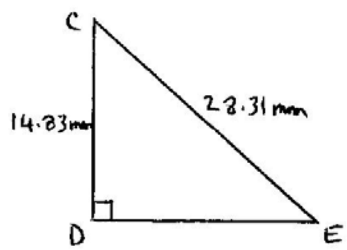
6. In a triangle CDE, D = 90°, CD = 14.83 mm and CE = 28.31 mm. Determine the length of DE.

Triangle CDE is shown below.

By Pythagoras, $28.31^2 = DE^2 + 14.83^2$

from which, $DE^2 = 28.31^2 - 14.83^2$

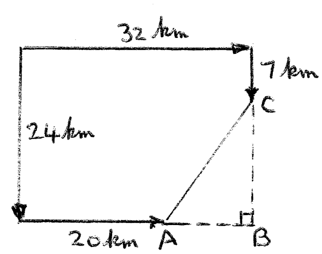
and **DE** = $\sqrt{28.31^2 - 14.83^2}$ = **24.11 mm**



7. A man cycles 24 km due south and then 20 km due east. Another man, starting at the same time as the first man, cycles 32 km due east and then 7 km due south. Find the distance between the two men.

With reference to the diagram below, AB = 32 – 20 = 12 km

and BC = 24 – 7 = 17 km



Hence, **distance between the two men, AC** = $\sqrt{(12^2 + 17^2)}$ = **20.81 km** by Pythagoras.

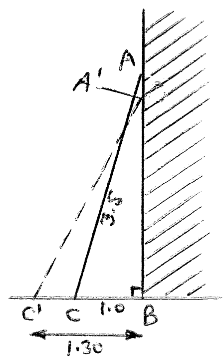
8. A ladder 3.5 m long is placed against a perpendicular wall with its foot 1.0 m from the wall. How far up the wall (to the nearest centimetre) does the ladder reach? If the foot of the ladder is now moved 30 cm further away from the wall, how far does the top of the ladder fall?

Distance up the wall, AB = $\sqrt{(3.5^2 - 1.0^2)}$ = **3.35 m** by Pythagoras.

$$A'B = \sqrt{\left[(A'C')^2 - (BC')^2\right]} = \sqrt{(3.5^2 - 1.30^2)} = 3.25\,\text{m}$$

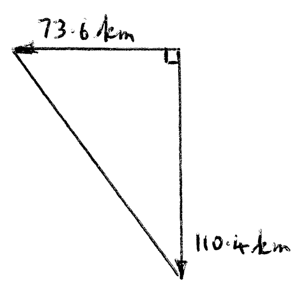
Hence, the amount the top of the ladder has moved down the wall,

given by AA′ = 3.35 – 3.25 = **0.10 m** or **10 cm**



9. Two ships leave a port at the same time. One travels due west at 18.4 knots and the other due south at 27.6 knots. If 1 knot = 1 nautical mile per hour, calculate how far apart the two ships are after 4 hours.

After 4 hours, the ship travelling west travels 4 × 18.4 = 73.6 km, and the ship travelling south travels 4 × 27.6 = 110.4 km, as shown in the diagram below.

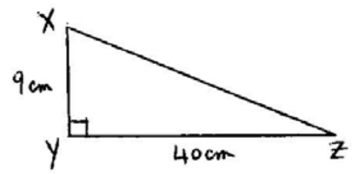


Hence, **distance apart after 4 hours** = $\sqrt{(73.6^2 + 110.4^2)}$ = **132.7 km** by Pythagoras.

EXERCISE 40, Page 89

1. Sketch a triangle XYZ such that ∠Y = 90°, XY = 9 cm and YZ = 40 cm. Determine sin Z, cos Z, tan X and cos X.

Triangle XYZ is shown sketched below.



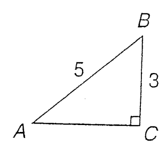
By Pythagoras, $XZ^2 = 9^2 + 40^2$

from which, $\mathbf{XZ} = \sqrt{9^2 - 40^2} = \mathbf{41\ cm}$

$\sin Z = \frac{XY}{XZ} = \mathbf{\frac{9}{41}}$ $\cos Z = \frac{YZ}{XZ} = \mathbf{\frac{40}{41}}$

$\tan X = \frac{YZ}{XY} = \mathbf{\frac{40}{9}}$ $\cos X = \frac{XY}{XZ} = \mathbf{\frac{9}{41}}$

2. In triangle ABC shown below, find sin A, cos A, tan A, sin B, cos B and tan B



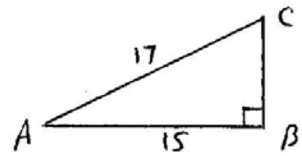
By Pythagoras' theorem, $AC = \sqrt{5^2 - 3^2} = 4$

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB} = \mathbf{\frac{3}{5}}$ $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB} = \mathbf{\frac{4}{5}}$ $\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} = \mathbf{\frac{3}{4}}$

$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB} = \mathbf{\frac{4}{5}}$ $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB} = \mathbf{\frac{3}{5}}$ $\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC} = \mathbf{\frac{4}{3}}$

3. If $\cos A = \frac{15}{17}$ find sin A and tan A, in fraction form.

Triangle ABC is shown below with $\cos A = \frac{15}{17}$



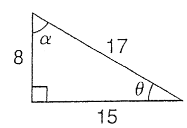
By Pythagoras, $17^2 = 15^2 + BC^2$

from which, $BC^2 = 17^2 - 15^2$

and $\mathbf{BC} = \sqrt{17^2 - 15^2} = \mathbf{8}$

Hence, $\mathbf{\sin A} = \frac{BC}{AC} = \mathbf{\frac{8}{17}}$ and $\mathbf{\tan A} = \frac{BC}{AB} = \mathbf{\frac{8}{15}}$

4. For the right-angled triangle shown below, find: (a) $\sin\alpha$ (b) $\cos\theta$ (c) $\tan\theta$



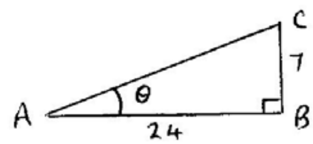
(a) $\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \mathbf{\frac{15}{17}}$

(b) $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \mathbf{\frac{15}{17}}$

(c) $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \mathbf{\frac{8}{15}}$

5. If $\tan\theta = \frac{7}{24}$, find $\sin\theta$ and $\cos\theta$ in fraction form.

Triangle ABC is shown below with $\tan\theta = \frac{7}{24}$



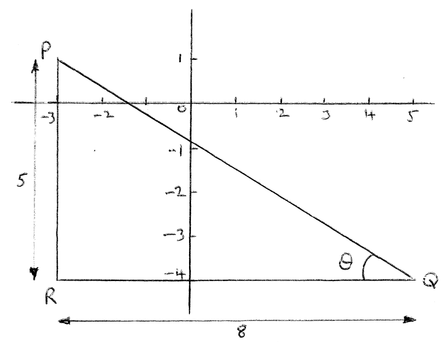
By Pythagoras, $AC^2 = 24^2 + 7^2$

and $\mathbf{AC} = \sqrt{24^2 + 7^2} = \mathbf{25}$

Hence, $\mathbf{sin}\ \theta = \frac{BC}{AC} = \mathbf{\frac{7}{25}}$ and $\mathbf{cos}\ \theta = \frac{AB}{AC} = \mathbf{\frac{24}{25}}$

6. Point P lies at co-ordinate (- 3, 1) and point Q at (5, - 4). Determine the distance PQ.

From the diagram below, $\mathbf{PQ} = \sqrt{(5^2 + 8^2)} = \mathbf{9.434}$ by Pythagoras



EXERCISE 41, Page 91

1. Determine, correct to 4 decimal places, 3 sin 66° 41′

Using a calculator, 3 sin 66° 41′ = **2.7550**, correct to 4 decimal places

2. Determine, correct to 3 decimal places, 5 cos 14° 15′

Using a calculator, 5 cos 14° 15′= **4.846**, correct to 3 decimal places

3. Determine, correct to 4 significant figures, 7 tan 79° 9′

Using a calculator, 7 tan 79° 9′ ′= **36.52**, correct to 4 significant figures

4. Determine (a) cos 1.681 (b) tan 3.672

Using a calculator,

(a) cos 1.681 = cos(1.681 rad) = **- 0.1010**, correct to 4 significant figures

(b) tan 3.672 = tan(3.672 rad) = **0.5865**, correct to 4 significant figures

5. Find the acute angle $\sin^{-1} 0.6734$ in degrees, correct to 2 decimal places

Using a calculator, $\sin^{-1} 0.6734 =$ **42.33º**, correct to 2 decimal places

6. Find the acute angle $\cos^{-1} 0.9648$ in degrees, correct to 2 decimal places

Using a calculator, $\cos^{-1} 0.9648$ = **15.25º**, correct to 2 decimal places

7. Find the acute angle $\tan^{-1} 3.4385$ in degrees, correct to 2 decimal places

Using a calculator, $\tan^{-1} 3.4385$ = **73.78º**, correct to 2 decimal places

8. Find the acute angle $\sin^{-1} 0.1381$ in degrees and minutes

Using a calculator, $\sin^{-1} 0.1381$ = **7.94º = 7º56′**

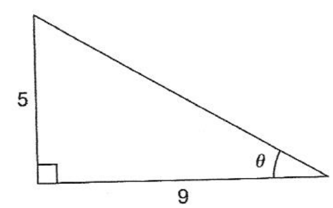
9. Find the acute angle $\cos^{-1} 0.8539$ in degrees and minutes

Using a calculator, $\cos^{-1} 0.8539$ = **31.36º = 31º22′**

10. Find the acute angle $\tan^{-1} 0.8971$ in degrees and minutes

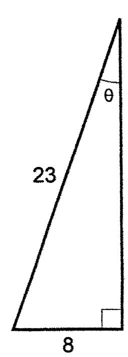
Using a calculator, $\tan^{-1} 0.8971$ = **41.90º = 41º54′**

11. In the triangle shown below, determine angle θ, correct to 2 decimal places.



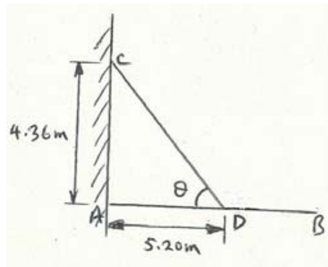
From trigonometric ratios, $\tan\theta = \frac{5}{9}$ from which, $\theta = \tan^{-1}\left(\frac{5}{9}\right)$ = **29.05°**

12. In the triangle shown, determine angle θ in degrees and minutes.



From trigonometric ratios, $\sin\theta = \frac{8}{23}$ from which, $\theta = \sin^{-1}\left(\frac{8}{23}\right) = 20.35° = \mathbf{20°21'}$

13. For the supported beam AB shown in the diagram, determine (a) the angle the supporting stay CD makes with the beam, i.e. θ, correct to the nearest degree, (b) the length of the stay, CD, correct to the nearest centimetre**.**



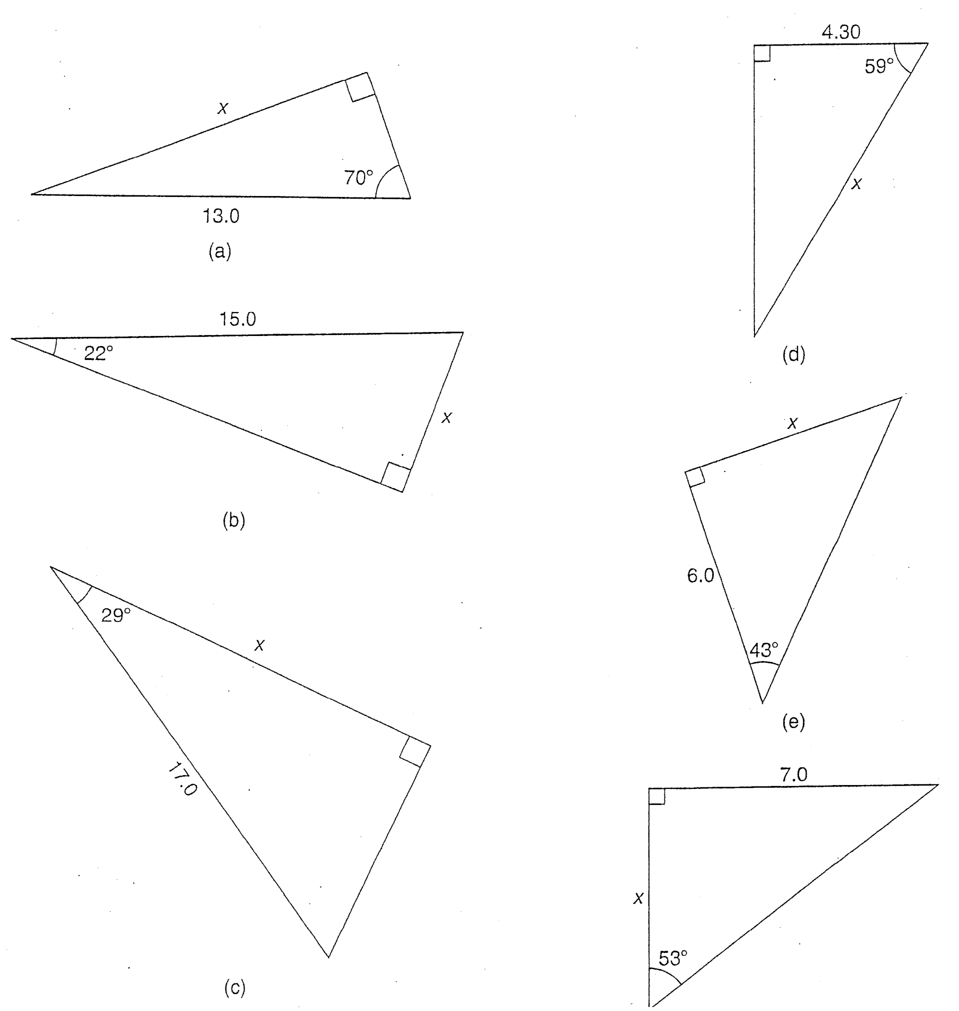
(a) $\tan\theta = \frac{AC}{AD} = \frac{4.36}{5.20}$ hence **angle θ** $= \tan^{-1}\left(\frac{4.36}{5.20}\right)$ **=** $39.98^{\circ} =$ **40º** correct to nearest degree

(b) By Pythagoras, $CD^2 = 4.36^2 + 5.20^2$

from which, **CD** $= \sqrt{4.36^2 + 5.20^2} =$ **6.79 m**

EXERCISE 42, Page 93

1. Calculate the dimensions shown as x in (a) to (f) below, each correct to 4 significant figures.



(a) $\text{Sin } 70° = \dfrac{x}{13.0}$ from which, $x = 13.0 \sin 70° =$ **12.22**, correct to 4 significant figures.

(b) $\text{Sin } 22° = \dfrac{x}{15.0}$ from which, $x = 15.0 \sin 22° =$ **5.619**, correct to 4 significant figures.

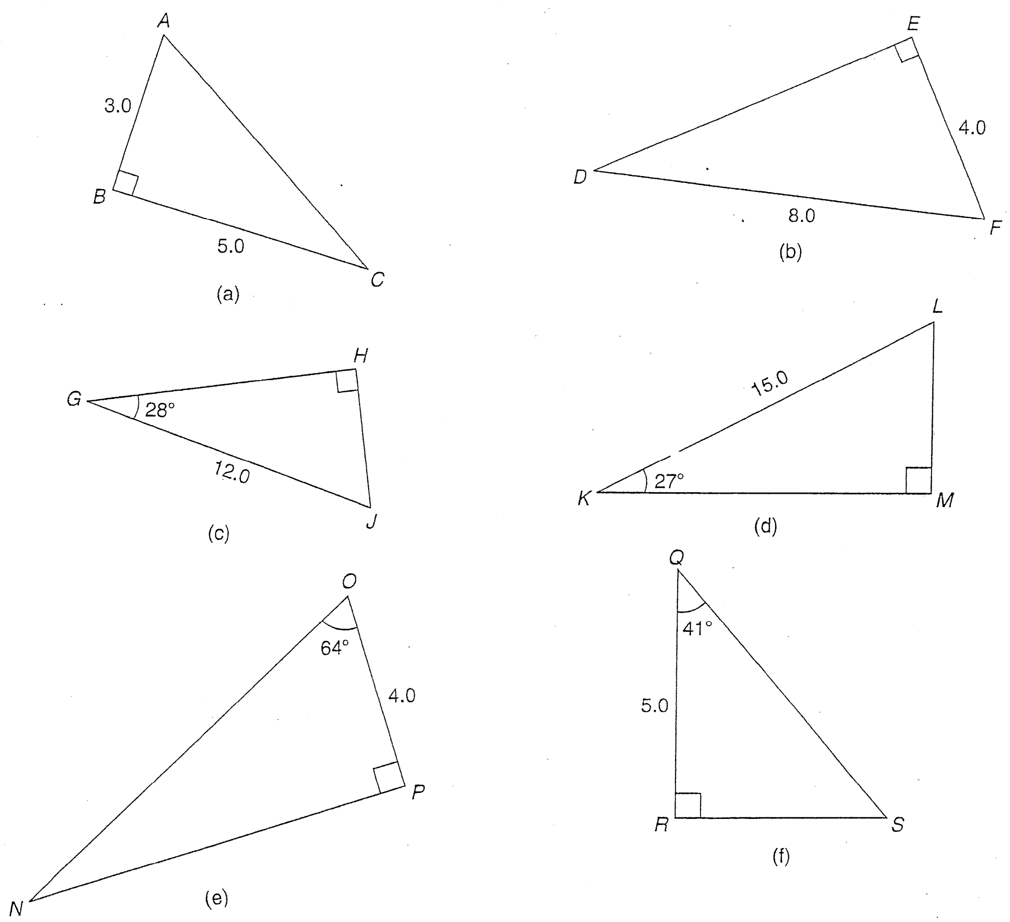
(c) $\text{Cos } 29° = \dfrac{x}{17.0}$ from which, $x = 17.0 \cos 29° =$ **14.87**, correct to 4 significant figures.

(d) $\text{Cos } 59° = \dfrac{4.30}{x}$ from which, $x = \dfrac{4.30}{\cos 59°} =$ **8.349**, correct to 4 significant figures.

(e) $\tan 43° = \dfrac{x}{6.0}$ from which, $x = 6.0 \tan 43° = \mathbf{5.595}$, correct to 4 significant figures.

(f) $\tan 53° = \dfrac{7.0}{x}$ from which, $x = \dfrac{7.0}{\tan 53°} = \mathbf{5.275}$, correct to 4 significant figures.

2. Find the unknown sides and angles in the right-angled triangles shown below. The dimensions shown are in centimetres.



(a) By Pythagoras, $AC = \sqrt{3.0^2 + 5.0^2} = \mathbf{5.831\ cm}$

$\tan C = \dfrac{3.0}{5.0}$ and $\angle \mathbf{C} = \tan^{-1}\left(\dfrac{3.0}{5.0}\right) = \mathbf{30.96°}$

Hence, $\angle A = 180° - 90° - 30.96° = \mathbf{59.04°}$

(b) By Pythagoras, $DE = \sqrt{8.0^2 - 4.0^2} = \mathbf{6.928\ cm}$

$\text{Sin D} = \frac{4.0}{8.0}$ and $\angle \mathbf{D} = \sin^{-1}\left(\frac{4.0}{8.0}\right) = \mathbf{30^o}$

Hence, $\angle F = 180^o - 90^o - 30^o = \mathbf{60^o}$

(c) $\angle J = 180^o - 90^o - 28^o = \mathbf{62^o}$

$\sin 28^o = \frac{HJ}{12.0}$ from which, $HJ = 12.0 \sin 28^o = \mathbf{5.634\ cm}$

By Pythagoras, $GH = \sqrt{12.0^2 - 5.634^2} = \mathbf{10.60\ cm}$

(d) $\angle L = 180^o - 90^o - 27^o = \mathbf{63^o}$

$\sin 27^o = \frac{LM}{15.0}$ from which, $LM = 15.0 \sin 27^o = \mathbf{6.810\ cm}$

By Pythagoras, $KM = \sqrt{15.0^2 - 6.810^2} = \mathbf{13.37\ cm}$

(e) $\angle N = 180^o - 90^o - 64^o = \mathbf{26^o}$

$\cos 64^o = \frac{4.0}{ON}$ from which, $ON = \frac{4.0}{\cos 64°} = \mathbf{9.125\ cm}$

By Pythagoras, $NP = \sqrt{9.125^2 - 4.0^2} = \mathbf{8.201\ cm}$

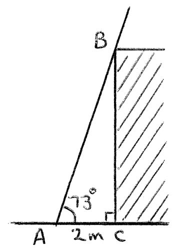
(f) $\angle S = 180^o - 90^o - 41^o = \mathbf{49^o}$

$\cos 41^o = \frac{5.0}{QS}$ from which, $QS = \frac{5.0}{\cos 41°} = \mathbf{6.625\ cm}$

By Pythagoras, $RS = \sqrt{6.625^2 - 5.0^2} = \mathbf{4.346\ cm}$

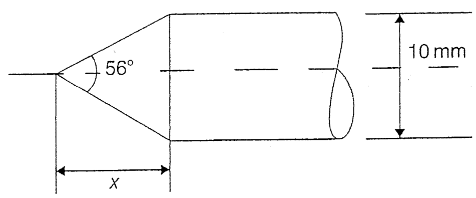
3. A ladder rests against the top of the perpendicular wall of a building and makes an angle of 73° with the ground. If the foot of the ladder is 2 m from the wall, calculate the height of the building.

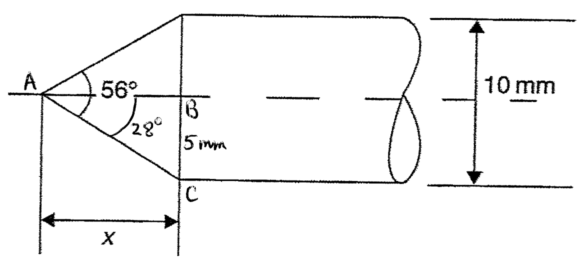
The ladder is shown in the diagram below, where BC is the height of the building.



Tan 73° = $\dfrac{BC}{2}$ from which, **height of building, BC** = 2 tan 73° = **6.54 m**

4. Determine the length x in the diagram below.



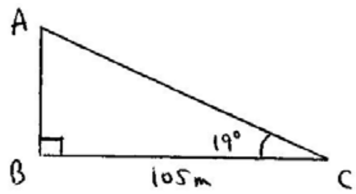


From triangle ABC in the sketch above,

$$\tan 28° = \frac{BC}{AB} = \frac{5}{x} \quad \text{from which,} \quad \mathbf{x} = \frac{5}{\tan 28°} = \mathbf{9.40\ mm}$$

5. A vertical tower stands on level ground. At a point 105 m from the foot of the tower the angle of elevation of the top is 19°. Find the height of the tower.

A side view is shown below where AB is the tower.

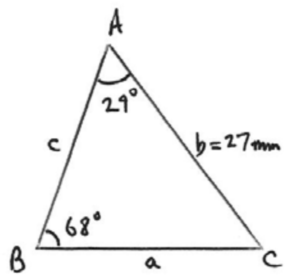


Tan 19° = $\frac{AB}{105}$ from which, **height of tower, AB** = 105 tan 19° = **36.15 m**

EXERCISE 43, Page 98

1. Use the sine rule to solve triangle ABC and find its area given: A = 29°, B = 68°, b = 27 mm

Triangle ABC is shown below.



Since the angles in a triangle add up to 180°, then: **C** = 180° - 29° - 68° = **83°**

Applying the sine rule: $\frac{27}{\sin 68°} = \frac{a}{\sin 29°} = \frac{c}{\sin 83°}$

Using $\frac{27}{\sin 68°} = \frac{a}{\sin 29°}$

and transposing gives: $\mathbf{a} = \frac{27 \sin 29°}{\sin 68°}$ = **14.1 mm = BC**

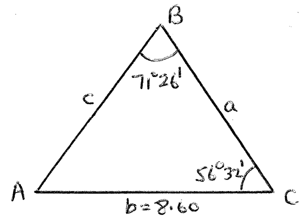
Using $\frac{27}{\sin 68°} = \frac{c}{\sin 83°}$

and transposing gives: $\mathbf{c} = \frac{27 \sin 83°}{\sin 68°}$ = **28.9 mm = AB**

Area of triangle XYZ = $\frac{1}{2}$ ab sin C = $\frac{1}{2}$(14.1)(27) sin 83° = **189 mm²**

2. Use the sine rule to solve triangle ABC and find its area given: B = 71°26', C = 56°32', b = 8.60 cm

Triangle ABC is shown below.



$\angle A = 180° - 71°26' - 56°32' = \mathbf{52°2'}$

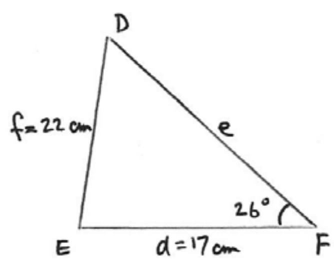
From the sine rule, $\dfrac{8.60}{\sin 71°26'} = \dfrac{c}{\sin 56°32'}$ from which, $\mathbf{c} = \dfrac{8.60 \sin 56°32'}{\sin 71°26'} = \mathbf{7.568\ cm}$

Also from the sine rule, $\dfrac{a}{\sin 52°2'} = \dfrac{8.60}{\sin 71°26'}$ from which, $\mathbf{a} = \dfrac{8.60 \sin 52°2'}{\sin 71°26'} = \mathbf{7.152\ cm}$

Area = $\dfrac{1}{2} a c \sin B = \dfrac{1}{2}(7.152)(7.568) \sin 71°26' = \mathbf{25.65\ cm^2}$

3. Use the sine rule to solve the triangle DEF and find its area given: d = 17 cm, f = 22 cm, F = 26°

Triangle DEF is shown below.



Applying the sine rule: $\dfrac{22}{\sin 26°} = \dfrac{17}{\sin D}$

from which, $\sin D = \dfrac{17 \sin 26°}{22} = 0.338741$

Hence, $D = \sin^{-1} 0.338741 = 19.80°$ or $160.20°$

Since F = 26°, C cannot be 160.20°, since 26° + 160.20° is greater than 180°.

Thus only **D = 19.80° or 19°48′** is valid. Angle **E** = 180° - 26° - 19°48′ = **134°12′**.

Applying the sine rule: $\dfrac{e}{\sin 134°12'} = \dfrac{22}{\sin 26°}$

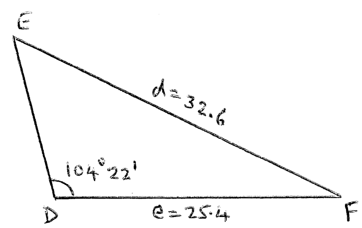
from which, $\mathbf{e} = \dfrac{22\sin 134°12'}{\sin 26°} = \mathbf{36.0\ cm}$

Hence, **D = 19°48′, E = 134°12′** and **DF = 36.0 mm**

Area of triangle ABC = $\frac{1}{2}de\sin F = \frac{1}{2}(17)(36.0)\sin 26° = \mathbf{134\ cm^2}$

4. Use the sine rule to solve the triangle DEF and find its area given: d = 32.6 mm, e = 25.4 mm, D = 104°22′

Triangle DEF is shown below.



From the sine rule, $\dfrac{32.6}{\sin 104°22'} = \dfrac{25.4}{\sin E}$ from which, $\sin E = \dfrac{25.4\sin 104°22'}{32.6} = 0.75477555$

and $\mathbf{E} = \sin^{-1} 0.75477555 = \mathbf{49.0°}$ or $\mathbf{49°0'}$

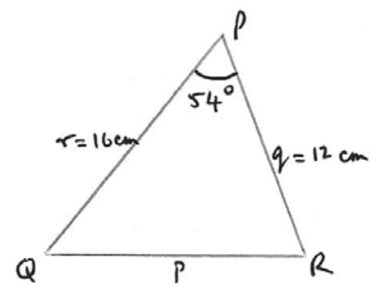
Hence, $\angle\mathbf{F} = 180° - 104°22' - 49°0' = \mathbf{26°38'}$

From the sine rule, $\dfrac{32.6}{\sin 104°22'} = \dfrac{f}{\sin 26°38'}$ from which, $\mathbf{f} = \dfrac{32.6\sin 26°38'}{\sin 104°22'} = \mathbf{15.09\ mm}$

Area = $\frac{1}{2}de\sin F = \frac{1}{2}(32.6)(25.4)\sin 26°38' = \mathbf{185.6\ mm^2}$

5. Use the cosine and sine rules to solve triangle PQR and find its area given: q = 12 cm, r = 16 cm, P = 54°

Triangle PQR is shown below.



By the cosine rule, $p^2 = 12^2 + 16^2 - 2(12)(16)\cos 54°$

$= 144 + 256 - 225.71 = 174.29$

and $\mathbf{p} = \sqrt{174.29} = \mathbf{13.2\ cm}$

From the sine rule, $\dfrac{13.2}{\sin 54°} = \dfrac{12}{\sin Q}$ from which, $\sin Q = \dfrac{12\sin 54°}{13.2} = 0.735470$

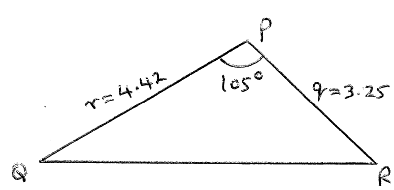
and $\mathbf{Q} = \sin^{-1} 0.735470 = \mathbf{47.35°}$

$\angle\mathbf{Q} = 180° - 54° - 47.35° = \mathbf{78.65°}$

Area = $\dfrac{1}{2}(12)(16)\sin 54° = \mathbf{77.7\ cm^2}$

6. Use the cosine and sine rules to solve triangle PQR and find its area given: $q = 3.25$ m, $r = 4.42$ m, $P = 105°$

Triangle PQR is shown below.



By the cosine rule, $p^2 = 4.42^2 + 3.25^2 - 2(4.42)(3.25)\cos 105°$

$= 19.5364 + 10.5625 - (-7.4359) = 37.5348$

and $\mathbf{p} = \sqrt{37.5348} = \mathbf{6.127\ m}$

From the sine rule, $\dfrac{6.127}{\sin 105°} = \dfrac{4.42}{\sin R}$ from which, $\sin R = \dfrac{4.42\sin 105°}{6.127} = 0.696816$

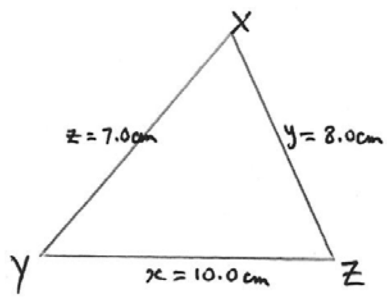
and $\mathbf{R} = \sin^{-1} 0.696816 = \mathbf{44.17°}$

$\angle\mathbf{Q} = 180° - 105° - 44.17° = \mathbf{30.83°}$

Area = $\frac{1}{2}(4.42)(3.25)\sin 105° = \mathbf{6.938\,m^2}$

7. Use the cosine and sine rules to solve triangle XYZ and find its area given: x = 10.0 cm, y = 8.0 cm, z = 7.0 cm

Triangle XYZ is shown below.



By the cosine rule, $10.0^2 = 7.0^2 + 8.0^2 - 2(7.0)(8.0)\cos X$

from which, $\cos X = \dfrac{7.0^2 + 8.0^2 - 10.0^2}{2(7.0)(8.0)} = 0.1160714$

and $\angle\mathbf{X} = \cos^{-1}(0.1160714) = \mathbf{83.33°}$

From the sine rule, $\dfrac{10.0}{\sin 83.33°} = \dfrac{8.0}{\sin Y}$ from which, $\sin Y = \dfrac{8.0\sin 83.33°}{10.0} = 0.7945853$

and $\angle Y = \sin^{-1} 0.7945853 = \mathbf{52.62°}$

Hence, $\angle\mathbf{Z} = 180° - 83.33° - 52.62° = \mathbf{44.05°}$

Area = $\frac{1}{2}(7.0)(8.0)\sin 83.33° = \mathbf{27.8\,cm^2}$

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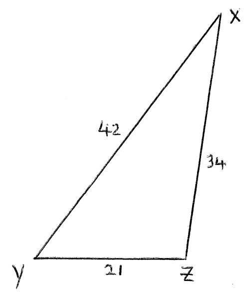
8. Use the cosine and sine rules to solve triangle XYZ and find its area given: x = 21 mm, y = 34 mm, z = 42 mm

Triangle XYZ is shown below.



By the cosine rule, $21^2 = 42^2 + 34^2 - 2(42)(34)\cos X$

from which, $\cos X = \dfrac{42^2 + 34^2 - 21^2}{2(42)(34)} = 0.867997$

and $\angle\mathbf{X} = \cos^{-1}(0.867997) = \mathbf{29.77°}$

From the sine rule, $\dfrac{21}{\sin 29.77°} = \dfrac{34}{\sin Y}$ from which, $\sin Y = \dfrac{34\sin 29.77°}{21} = 0.8038888$

and $\angle Y = \sin^{-1} 0.8038888 = \mathbf{53.50°}$

Hence, $\angle\mathbf{Z}$ = 180° - 29.77° - 53.50° = **96.73°**

Area = $\dfrac{1}{2}(21)(34)\sin 96.73° = \mathbf{355\,mm^2}$

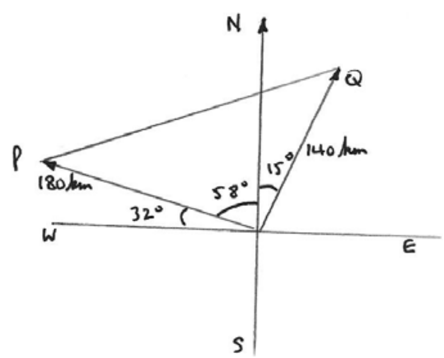
EXERCISE 44, Page 99

1. A ship P sails at a steady speed of 45 km/h in a direction of W 32° N (i.e. a bearing of 302°) from a port. At the same time another ship Q leaves the port at a steady speed of 35 km/h in a direction N 15° E (i.e. a bearing of 015°). Determine their distance apart after 4 hours.

After 4 hours, ship P has travelled $4 \times 45 = 180$ km.

After 4 hours, ship Q has travelled $4 \times 35 = 140$ km.

The directions of travel are shown in the diagram below. After 4 hours their distance apart is given by PQ

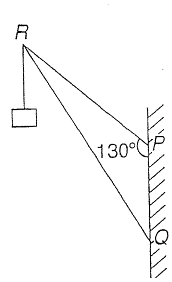


By the cosine rule, $(PQ)^2 = 180^2 + 140^2 - 2(180)(140)\cos(58+15)°$

$= 32400 + 19600 - 14735.534 = 37264.466$

and **PQ** $= \sqrt{37264.466}$ = **193 km**

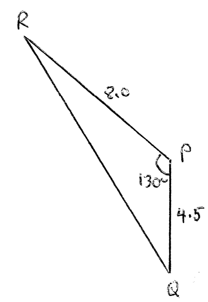
2. A jib crane is shown below. If the tie rod PR is 8.0 long and PQ is 4.5 m long determine (a) the length of jib RQ, and (b) the angle between the jib and the tie rod.



(a) Using the cosine rule on triangle PQR shown below gives:

$$RQ^2 = 8.0^2 + 4.5^2 - 2(8.0)(4.5)\cos 130° = 130.53$$

and **jib, RQ** = $\sqrt{130.53}$ = 11.43 m = **11.4 m**, correct to 3 significant figures

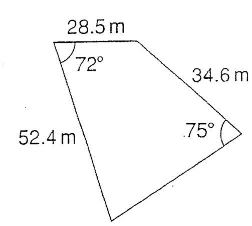


(b) From the sine rule, $\dfrac{4.5}{\sin R} = \dfrac{11.43}{\sin 130°}$ from which, $\sin R = \dfrac{4.5\sin 130°}{11.43} = 0.3015923$

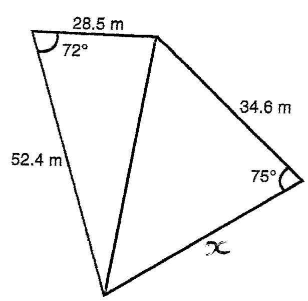
and **the angle between the jib and the tie rod,** $\angle\mathbf{R} = \sin^{-1} 0.3015923 = \mathbf{17.55°}$

3. A building site is in the form of a quadrilateral as shown below, and its area is 1510 m^2.

Determine the length of the perimeter of the site.



The quadrilateral is split into two triangles as shown in the diagram below.



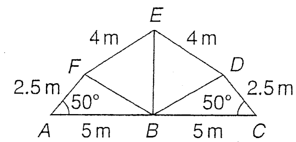
$$\text{Area} = 1510 = \frac{1}{2}(52.4)(28.5)\sin 72° + \frac{1}{2}(34.6)(x)\sin 75°$$

i.e. $1510 = 710.15 + 16.71\,x$

from which, $x = \dfrac{1510 - 710.15}{16.71} = 47.87$ m

Hence, **perimeter of quadrilateral** = 52.4 + 28.5 + 34.6 + 47.9 = **163.4 m**

4. Determine the length of members BF and EB in the roof truss shown below.



Using the cosine rule on triangle ABF gives: $BF^2 = 2.5^2 + 5^2 - 2(2.5)(5)\cos 50° = 15.18$

from which, $\mathbf{BF} = \sqrt{15.18} = \mathbf{3.9\ m}$

Using the sine rule on triangle ABF gives:

$$\frac{3.9}{\sin 50°} = \frac{2.5}{\sin B} \quad \text{from which,} \quad \sin B = \frac{2.5 \sin 50°}{3.9} = 0.491054$$

and $\angle ABF = \sin^{-1} 0.491054 = 29.41°$

Assuming $\angle ABE = 90°$, then $\angle FBE = 90° - 29.41° = 60.59°$

Using the sine rule on triangle BEF gives:

$$\frac{4}{\sin 60.59°} = \frac{3.9}{\sin E} \quad \text{from which,} \quad \sin E = \frac{3.9 \sin 60.59°}{4} = 0.8493499$$

and $\angle E = \sin^{-1} 0.8493499 = 58.14°$

Thus, $\angle EFB = 180° - 58.14° - 60.59° = 61.27°$

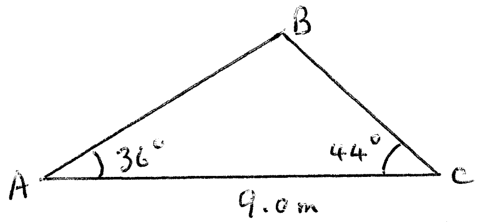
Using the sine rule on triangle BEF again gives:

$$\frac{BE}{\sin 61.27°} = \frac{4}{\sin 60.59} \quad \text{from which,} \quad \mathbf{BE} = \frac{4 \sin 61.27°}{\sin 60.59°} = \mathbf{4.0\ m}$$

5. A laboratory 9.0 m wide has a span roof which slopes at 36° on one side and 44° on the other. Determine the lengths of the roof slopes.

A cross-sectional view is shown below.

Angle ABC = 180° - 36° - 44° = 100°

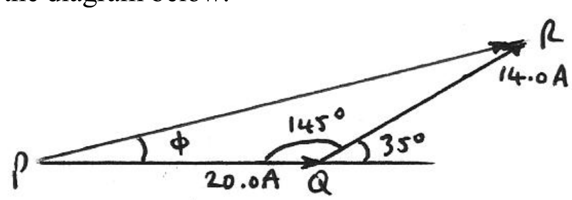


Using the sine rule, $\frac{AB}{\sin 44°} = \frac{9.0}{\sin 100°}$ from which, $\mathbf{AB} = \frac{9.0 \sin 44°}{\sin 100°} = \mathbf{6.35\ m}$

and $\frac{BC}{\sin 36°} = \frac{9.0}{\sin 100°}$ from which, $\mathbf{BC} = \frac{9.0 \sin 36°}{\sin 100°} = \mathbf{5.37\ m}$

6. PQ and QR are the phasors representing the alternating currents in two branches of a circuit. Phasor PQ is 20.0 A and is horizontal. Phasor QR (which is joined to the end of PQ to form triangle PQR) is 14.0 A and is at an angle of 35° to the horizontal. Determine the resultant phasor PR and the angle it makes with phasor PQ.

The phasors are shown in the diagram below.



By the cosine rule, $(PR)^2 = 20.0^2 + 14.0^2 - 2(20.0)(14.0)\cos 145°$

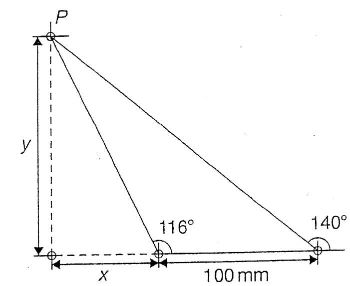
$= 400 + 196 - (-458.7251) = 1054.7251$

and **resultant phasor, PR** = $\sqrt{1054.7251}$ = **32.48 A**

Using the sine rule, $\frac{32.48}{\sin 145°} = \frac{14.0}{\sin \phi}$ from which, $\sin \phi = \frac{14.0 \sin 145°}{32.48} = 0.247231$

and $\phi = \sin^{-1} 0.247231 = \mathbf{14.31°}$

7. Calculate, correct to 3 significant figures, the co-ordinates x and y to locate the hole centre at P shown below.



$\tan(180° - 116°) = \dfrac{y}{x}$ i.e. $y = x \tan 64°$ (1)

$\tan(180° - 140°) = \dfrac{y}{x+100}$ i.e. $y = (x + 100) \tan 40°$ (2)

Equating (1) and (2) gives: $x \tan 64° = (x + 100) \tan 40°$

i.e. $x \tan 64° - x \tan 40° = 100 \tan 40°$

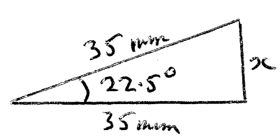
and $x(\tan 64° - \tan 40°) = 100 \tan 40°$

from which, $\mathbf{x} = \dfrac{100 \tan 40°}{(\tan 64° - \tan 40°)}$ = 69.278 mm = **69.3 mm**, correct to 3 significant figures.

Substituting in (1) gives: $\mathbf{y} = x \tan 64° = 66.278 \tan 64°$ = **142 mm**

8. 16 holes are equally spaced on a pitch circle of 70 mm diameter. Determine the length of the chord joining the centres of two adjacent holes.

If 16 holes are equally spaced around a circle of diameter 70 mm, i.e. radius 35 mm, then the holes are spaced $\dfrac{360°}{16} = 22.5°$ apart.



Length x in the diagram is the chord joining the centres of two adjacent holes.

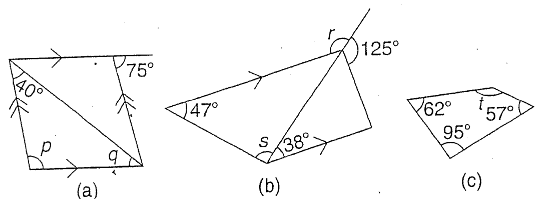
Using the cosine rule, $x^2 = 35^2 + 35^2 - 2(35)(35)\cos 22.5° = 186.495$

from which, $\mathbf{x} = \sqrt{186.495} = \mathbf{13.66\ mm}$

CHAPTER 11 AREAS OF PLANE FIGURES

EXERCISE 45, Page 106

1. Find the angles p and q in diagram (a) below.



p = 180° - 75° = **105°** (interior opposite angles of a parallelogram are equal)

q = 180° - 105° - 40° = **35°**

2. Find the angles r and s in diagram (b) above.

r = 180° - 38° = **142°** (the 38° angle is the alternate angle between parallel lines)

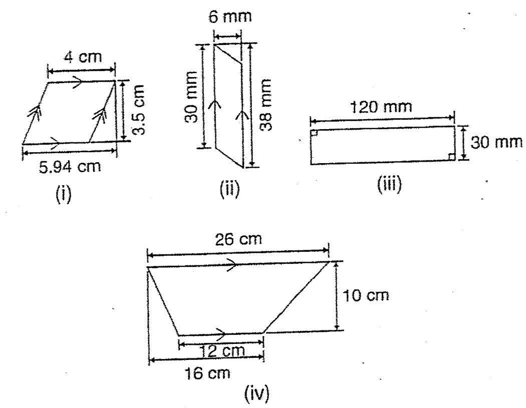
s = 180° - 47° - 38° = **95°**

3. Find the angle t in diagram (c) above.

t = 360° - 62° - 95° - 57° = **146°**

EXERCISE 46, Page 110

1. Name the types of quadrilateral shown in diagrams (i) to (iv) below, and determine for each (a) the area, and (b) the perimeter.



(i) **Rhombus**

(a) Area = 4 × 3.5 = **14 cm^2**

(b) Perimeter = 4 + 4 + 4 + 4 = **16 cm**

(ii) **Parallelogram**

(a) Area = 30 × 6 = **180 mm^2**

(b) Perimeter = 30 + 30 + $2\left[\sqrt{(6^2+8^2)}\right]$ = **80 mm**

(iii) **Rectangle**

(a) Area = 120 × 30 = **3600 mm^2**

(b) Perimeter = (2 × 120) + (2 × 30) = **300 mm**

(iv) **Trapezium**

(a) Area = $\frac{1}{2}(26+12)\times 10$ = **190 cm^2**

(b) Perimeter = 26 + 12 + $\left[\sqrt{(10^2+10^2)}\right]+\left[\sqrt{(4^2+10^2)}\right]$

= 26 + 12 + 14.14 + 10.77 = **62.91 cm**

2. A rectangular plate is 85 mm long and 42 mm wide. Find its area in square centimetres.

85 mm = 8.5 cm and 42 mm = 4.2 cm

Area of plate = $8.5 \times 4.2 = \mathbf{35.7\,cm^2}$

3. A rectangular field has an area of 1.2 hectares and a length of 150 m. If 1 hectare = 10000 m^2 find (a) its width, and (b) the length of a diagonal.

Area of field = 1.2 ha = $1.2 \times 10000\,m^2 = 12000\,m^2$

(a) Area = length × width from which, **width** = $\dfrac{\text{area}}{\text{length}} = \dfrac{12000}{150} = \mathbf{80\ m}$

(b) By Pythagoras, **length of diagonal** = $\sqrt{(150^2 + 80^2)} = \mathbf{170\ m}$

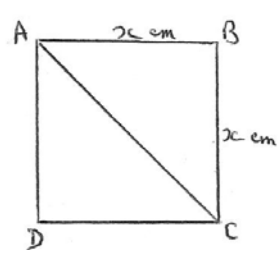
4. Find the area of a triangle whose base is 8.5 cm and perpendicular height 6.4 cm.

Area of triangle = $\dfrac{1}{2} \times$ base × perpendicular height

$= \dfrac{1}{2} \times 8.5 \times 6.4 = \mathbf{27.2\,cm^2}$

5. A square has an area of $162\,cm^2$. Determine the length of a diagonal.

A square ABCD is shown below of side x cm. The diagonal is given by length AC



Area of square = $x^2 = 162$

By Pythagoras, $(AC)^2 = x^2 + x^2 = 2x^2 = 2 \times 162$

from which, **diagonal, AC** = $\sqrt{2x^2} = \sqrt{[2\times162]}$ = **18 cm**

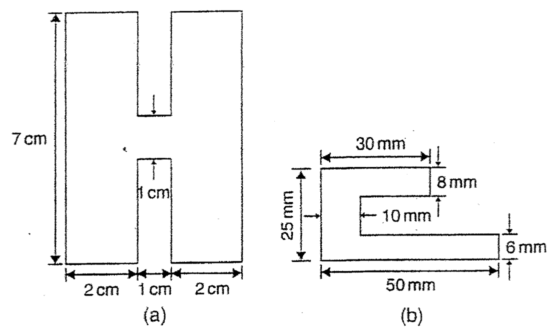
6. A rectangular picture has an area of $0.96\,m^2$. If one of the sides has a length of 800 mm, calculate, in millimetres, the length of the other side.

Area = $0.96\,m^2 = 0.96\times10^6\,mm^2$

and area = length × breadth, i.e. $0.96\times10^6\,mm^2$ = 800 mm × breadth

from which, **breadth** = $\dfrac{0.96\times10^6}{800}$ = **1200 mm**

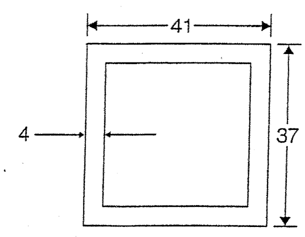
7. Determine the area of each of the angle iron sections shown in below.



(a) Area = 2 (7 × 2) + (1 × 1) = 28 + 1 = **29 cm^2**

(b) Area = (30 × 8) + 10(25 – 8 – 6) + (6 × 50) = 240 + 110 + 300 = **650 mm^2**

8. The diagram below shows a 4 m wide path around the outside of a 41 m by 37 m garden. Calculate the area of the path.



Area of garden = $41 \times 37\,\text{m}^2$

Area of garden, neglecting the path = $(41 - 8) \times (37 - 8) = 33 \times 29\,\text{m}^2$

Hence, **area of path** = $(41 \times 37) - (33 \times 29)$

$= 1517 - 957 = \mathbf{560\,m^2}$

9. The area of a trapezium is 13.5 cm^2 and the perpendicular distance between its parallel sides is 3 cm. If the length of one of the parallel sides is 5.6 cm, find the length of the other parallel side.

Area of a trapezium = $\frac{1}{2} \times$ (sum of parallel sides) $\times$ (perpendicular distance between the parallel sides)

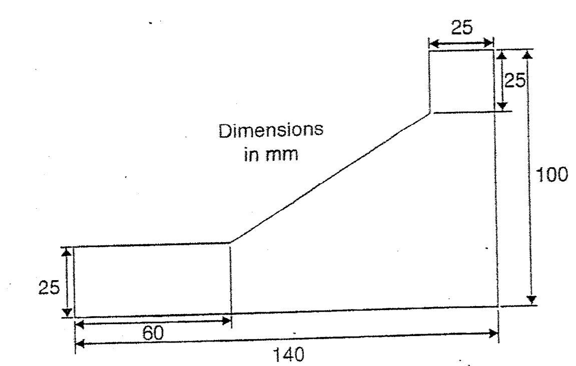
i.e. $13.5 = \frac{1}{2} \times (5.6 + x) \times (3)$ where x is the unknown parallel side

i.e. $27 = 3(5.6 + x)$

i.e. $9 = 5.6 + x$

from which, $\mathbf{x} = 9 - 5.6 = \mathbf{3.4\ cm}$

10. Calculate the area of the steel plate shown below.



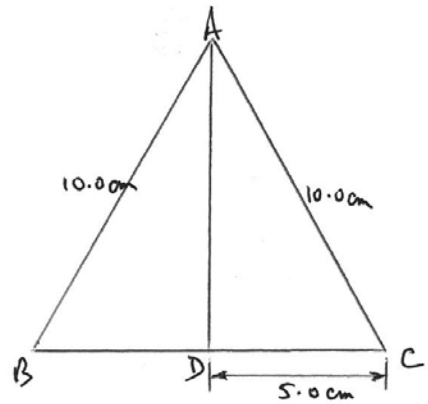
Area of steel plate = $(25 \times 60) + (140 - 60)(25) + (25)^2 + (50 \times 25) + \left(\frac{1}{2} \times 55 \times 50\right)$

$= 1500 + 2000 + 625 + 1250 + 1375$

$= \mathbf{6750\,mm^2}$

11. Determine the area of an equilateral triangle of side 10.0 cm.

An equilateral triangle ABC is shown below.



Perpendicular height, AD = $\sqrt{10.0^2 - 5.0^2}$ by Pythagoras

= 8.6603 cm

Hence, **area of triangle** = $\frac{1}{2}$ × base × perpendicular height

= $\frac{1}{2}$ × 10.0 × 8.6603

= $\mathbf{43.30\ cm^2}$

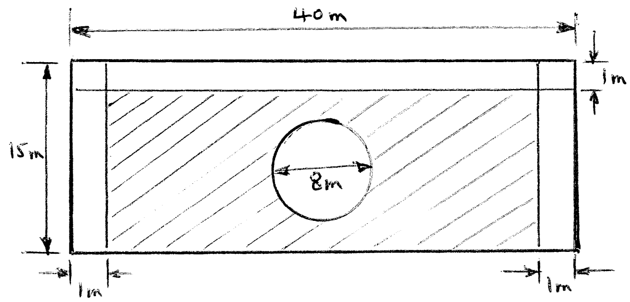
12. If paving slabs are produced in 250 mm by 250 mm squares, determine the number of slabs required to cover an area of $2\,\text{m}^2$.

Number of slabs = $\frac{2\times 10^6\,\text{mm}^2}{250\times 250}$ = **32**

EXERCISE 47, Page 111

1. A rectangular garden measures 40 m by 15 m. A 1 m flower border is made round the two shorter sides and one long side. A circular swimming pool of diameter 8 m is constructed in the middle of the garden. Find, correct to the nearest square metre, the area remaining.

A sketch of a plan of the garden is shown below.



Shaded area = (40 × 15) – [(15 × 1) + (38 × 1) + (15 × 1) + $\pi\left(4^2\right)$]

$= 600 – [15 + 38 + 15 + 16\pi]$

$= 600 – 118.27 = 481.73\ \text{m}^2$ = $\mathbf{482\ m^2}$, correct to the nearest square metre.

2. Determine the area of circles having (a) a radius of 4 cm (b) a diameter of 30 mm (c) a circumference of 200 mm.

(a) **Area** = $\pi r^2 = \pi(4)^2 = \mathbf{50.27\ cm^2}$

(b) **Area** = $\pi r^2 = \pi\left(\dfrac{d}{2}\right)^2 = \dfrac{\pi d^2}{4} = \dfrac{\pi(30)^2}{4} = \mathbf{706.9\ mm^2}$

(c) Circumference = $2\pi r$ = 200 mm, from which, radius, $r = \dfrac{200}{2\pi} = \dfrac{100}{\pi}$ mm

Hence, **area** = $\pi r^2 = \pi\left(\dfrac{100}{\pi}\right)^2 = \mathbf{3183\ mm^2}$

3. An annulus has an outside diameter of 60 mm and an inside diameter of 20 mm. Determine its area.

Area of annulus $= \frac{\pi d_2^{\,2}}{4} - \frac{\pi d_1^{\,2}}{4} = \frac{\pi(60)^2}{4} - \frac{\pi(20)^2}{4} = \frac{\pi}{4}\left(60^2 - 20^2\right) = \mathbf{2513\,mm^2}$

4. If the area of a circle is 320 mm^2, find (a) its diameter, and (b) its circumference.

(a) Area of circle, $A = \frac{\pi d^2}{4}$ i.e. $320 = \frac{\pi d^2}{4}$

from which, **diameter, d** $= \sqrt{\left(\frac{4\times 320}{\pi}\right)}$ = 20.185 mm = **20.19 mm** correct to 2 decimal places.

(b) **Circumference of circle** = $2\pi r = \pi d = \pi \times 20.185$ = **63.41 mm**

5. Calculate the areas of the following sectors of circles:

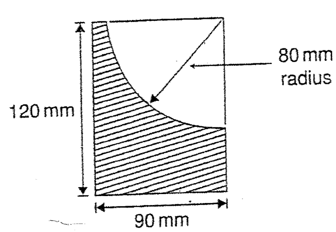
(a) radius 9 cm, angle subtended at centre 75°

(b) diameter 35 mm, angle subtended at centre 48°37'

(a) Area of sector $= \left(\frac{\theta}{360}\right)\pi r^2 = \left(\frac{75}{360}\right)\left(\pi\times 9^2\right) = \mathbf{53.01\,cm^2}$

(b) Area of sector $= \left(\frac{\theta}{360}\right)\pi r^2 = \left(\frac{48\frac{37}{60}}{360}\right)\left(\pi\times\left(\frac{35}{2}\right)^2\right) = \mathbf{129.9\,mm^2}$

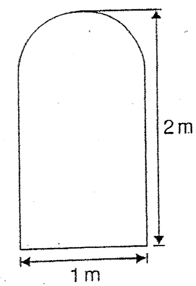
6. Determine the shaded area of the template shown below.



Area of template = shaded area = (120 × 90) - $\frac{1}{4}\pi(80)^2$ = 10800 – 5026.55 = $\mathbf{5773\,mm^2}$

7. An archway consists of a rectangular opening topped by a semi-circular arch as shown below.

Determine the area of the opening if the width is 1 m and the greatest height is 2 m.



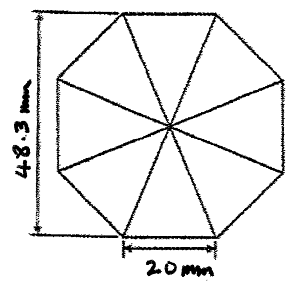
The semicircle has a diameter of 1 m, i.e. a radius of 0.5 m. Hence, the archway shown is made up of a rectangle of sides 1 m by 1.5 m and a semicircle of radius 0.5 m.

Thus, **area of opening** = (1.5 × 1) + $\frac{1}{2}\left[\pi(0.5)^2\right]$ = 1.5 + 0.393 = $\mathbf{1.89\,m^2}$

EXERCISE 48, Page 114

1. Calculate the area of a regular octagon if each side is 20 mm and the width across the flats is 48.3 mm.

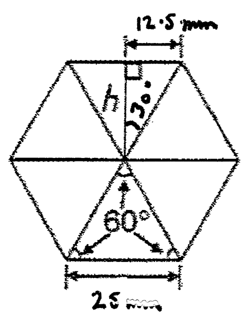
The octagon is shown sketched below and is comprised of 8 triangles of base length 20 mm and perpendicular height 48.3/2



Area of octagon $= 8\left[\frac{1}{2} \times 20 \times \frac{48.3}{2}\right] = \mathbf{1932\,mm^2}$

2. Determine the area of a regular hexagon which has sides 25 mm.

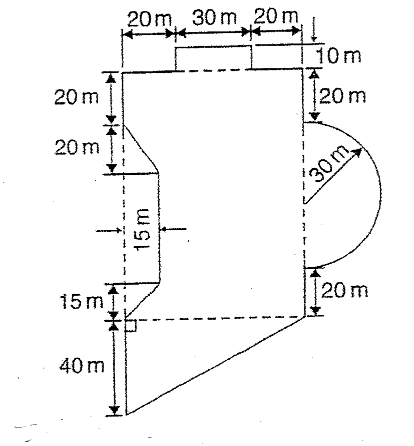
The hexagon is shown sketched below and is comprised of 6 triangles of base length 25 mm and perpendicular height h as shown.



Tan 30° = $\frac{12.5}{h}$ from which, $h = \frac{12.5}{\tan 30°} = 21.65$ mm

Hence, **area of hexagon** $= 6\left[\frac{1}{2} \times 25 \times 21.65\right] = \mathbf{1624\,mm^2}$

3. A plot of land is in the shape shown below. Determine (a) its area in hectares (1 ha = 10^4 m^2), and (b) the length of fencing required, to the nearest metre, to completely enclose the plot of land.



(a) Area of land = $(30 \times 10) + \frac{1}{2}\pi(30)^2 + \frac{1}{2}(70)(40) + \left[(70 \times 100) - \frac{1}{2}(80+45)(15)\right]$

$= 300 + 450\pi + 1400 + [7000 - 937.5]$

$= 9176\,\text{m}^2 = \frac{9176}{10^4}\,\text{ha} =$ **0.918 ha**

(b) Perimeter $= 20 + 10 + 30 + 10 + 20 + 20 + \frac{1}{2}(2\pi \times 30) + 20 + \sqrt{(70^2 + 40^2)} + 40$

$+ \sqrt{(15^2 + 15^2)} + 45 + \sqrt{(20^2 + 15^2)} + 20$

$= 110 + 30\pi + 20 + 80.62 + 40 + 21.21 + 45 + 25 + 20$

= **456 m**, to the nearest metre.

EXERCISE 49, Page 115

1. The area of a park on a map is 500 mm^2. If the scale of the map is 1 to 40000 determine the true area of the park in hectares (1 hectare = 10^4 m^2)

Area of park = $500\times10^{-6}\times(40000)^2$ m^2 = $\dfrac{500\times10^{-6}\times(40000)^2}{10^4}$ ha = **80 ha**

2. A model of a boiler is made having an overall height of 75 mm corresponding to an overall height of the actual boiler of 6 m. If the area of metal required for the model is 12500 mm^2, determine, in square metres, the area of metal required for the actual boiler.

The scale is $\dfrac{6000}{75}$: 1 i.e. 80 : 1

Area of metal required for actual boiler = 12500×10^{-6} m^2 $\times(80)^2$ = $\mathbf{80\,m^2}$

3. The scale of an Ordnance Survey map is 1:2500. A circular sports field has a diameter of 8 cm on the map. Calculate its area in hectares, giving your answer correct to 3 significant figures. (1 hectare = 10^4 m^2)

Area of sports field on map = $\dfrac{\pi d^2}{4}=\dfrac{\pi(8)^2}{4}\times10^{-4}\,\text{m}^2$

True area of sports field = $\dfrac{\pi(8)^2}{4}\times10^{-4}\times(2500)^2\,\text{m}^2=\dfrac{\dfrac{\pi(8)^2}{4}\times10^{-4}\times(2500)^2}{10^4}$ ha

= **3.14 ha**

CHAPTER 12 THE CIRCLE

EXERCISE 50, Page 118

1. Calculate the length of the circumference of a circle of radius 7.2 cm

Circumference, c = $2\pi r = 2\pi(7.2) =$ **45.24 cm**

2. If the diameter of a circle is 82.6 mm, calculate the circumference of the circle

Circumference, c = $2\pi r = \pi d = \pi(82.6) =$ **259.5 mm**

3. Determine the radius of a circle whose circumference is 16.52 cm

Circumference = $2\pi r$ i.e. 16.52 cm = $2\pi r$

from which, **radius, r** = $\dfrac{16.52}{2\pi}$ = **2.629 cm**

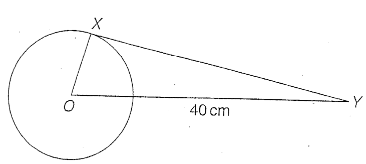
4. Find the diameter of a circle whose perimeter is 149.8 cm

If perimeter, or circumference, $c = \pi d$,

then $149.8 = \pi d$

and **diameter, d** = $\dfrac{149.8}{\pi}$ = **47.68 cm**

5. A crank mechanism is shown below, where XY is a tangent to the circle at point X. If the circle radius 0X is 10 cm and length 0Y is 40 cm, determine the length of the connecting rod XY.



If XY is a tangent to the circle, then $\angle 0XY = 90°$

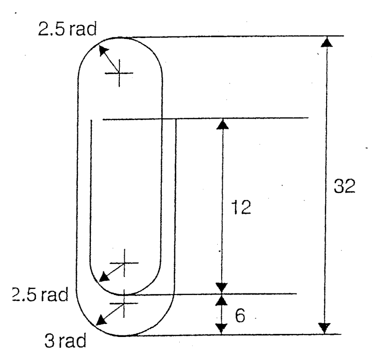
Thus, by Pythagoras, $0Y^2 = 0X^2 + XY^2$

from which, $\mathbf{XY} = \sqrt{(0Y^2 - 0X^2)} = \sqrt{40^2 - 10^2} = \sqrt{1500} = \mathbf{38.73\ cm}$

6. If the circumference of the earth is 40 000 km at the equator, calculate its diameter.

Circumference, $c = 2\pi r = \pi d$ from which, **diameter, d** $= \dfrac{c}{\pi} = \dfrac{40000}{\pi} = 12732$ km = **12,730 km**, correct to 4 significant figures.

7. Calculate the length of wire in the paper clip shown below. The dimensions are in millimetres.



$$\text{Length of wire} = (12 - 2.5) + \frac{1}{2}(2\pi \times 2.5) + (32 - 2.5 - 6 - 2.5) + \frac{1}{2}(2\pi \times 2.5) + (32 - 2.5 - 3) + \frac{1}{2}(2\pi \times 3) + 3 + 12$$

$= 9.5 + 2.5\pi + 21 + 2.5\pi + 26.5 + 3\pi + 3 + 12$

$= 72 + 8\pi$

$=$ **97.13 mm**

EXERCISE 51, Page 119

1. Convert to radians in terms of π: (a) 30° (b) 75° (c) 225°

(a) $30° = 30 \times \frac{\pi}{180}\text{rad} = \mathbf{\frac{\pi}{6}\ rad}$

(b) $75° = 75 \times \frac{\pi}{180}\text{rad} = \mathbf{\frac{5\pi}{12}\ rad}$

(c) $225° = 225 \times \frac{\pi}{180}\text{rad} = \frac{45\pi}{36}\text{rad} = \frac{15\pi}{12}\text{rad} = \mathbf{\frac{5\pi}{4}\ rad}$

2. Convert to radians, correct to 3 decimal places: (a) 48° (b) 84°51' (c) 232°15'

(a) $48° = 48 \times \frac{\pi}{180}\text{rad} = \mathbf{0.838\ rad}$

(b) $84°51' = \left(84\frac{51}{60}\right) \times \frac{\pi}{180} = 84.85 \times \frac{\pi}{180}\text{rad} = \mathbf{1.481\ rad}$

(c) $232°15' = 232.25 \times \frac{\pi}{180}\text{rad} = \mathbf{4.054\ rad}$

3. Convert to degrees: (a) $\frac{7\pi}{6}$ rad (b) $\frac{4\pi}{9}$ rad (c) $\frac{7\pi}{12}$ rad

(a) $\frac{7\pi}{6}\text{rad} = \frac{7\pi}{6} \times \frac{180°}{\pi} = 7 \times 30 = \mathbf{210°}$

(b) $\frac{4\pi}{9}\text{rad} = \frac{4\pi}{9} \times \frac{180°}{\pi} = 4 \times 20 = \mathbf{80°}$

(c) $\frac{7\pi}{12}\text{rad} = \frac{7\pi}{12} \times \frac{180°}{\pi} = 7 \times 15 = \mathbf{105°}$

4. Convert to degrees and minutes: (a) 0.0125 rad (b) 2.69 rad (c) 7.241 rad

(a) 0.0125 rad = $0.0125 \times \frac{180°}{\pi}$ = **0.716°** or **0°43′**

(b) 2.69 rad = $2.69 \times \frac{180°}{\pi}$ = **154.126°** or **154°8′**

(c) 7.241 rad = $7.241 \times \frac{180°}{\pi}$ = **414.879°** or **414°53′**

5. A car engine speed is 1000 rev/min. Convert this speed into rad/s.

1000 rev/min = $\frac{1000\,\text{rev/min} \times 2\pi\ \text{rad/rev}}{60\,\text{s/min}}$ = **104.7 rad/s**

EXERCISE 52, Page 121

1. Calculate the area of a circle of radius 6.0 cm, correct to the nearest square centimetre.

Area = $\pi r^2 = \pi(6.0)^2$ = $\mathbf{113\,cm^2}$, correct to the nearest square centimetre.

2. The diameter of a circle is 55.0 mm. Determine its area, correct to the nearest square millimetre.

Area of circle = $\pi r^2 = \pi\dfrac{d^2}{4} = \pi\left(\dfrac{55.0^2}{4}\right)$ = $\mathbf{2376\,mm^2}$

3. The perimeter of a circle is 150 mm. Find its area, correct to the nearest square millimetre.

Perimeter = circumference = $2\pi r$ i.e. 150 mm = $2\pi r$ from which, radius, r = $\dfrac{150}{2\pi} = \dfrac{75}{\pi}$ mm

Area = $\pi r^2 = \pi\left(\dfrac{75}{\pi}\right)^2$ = $\mathbf{1790\,mm^2}$, correct to the nearest square millimetre.

4. Find the area of the sector, correct to the nearest square millimetre, of a circle having a radius of 35 mm, with angle subtended at centre of 75°.

Area of sector = $\dfrac{\theta}{360}(\pi r^2) = \dfrac{75}{360}(\pi\times 35^2)$ = $\mathbf{802\,mm^2}$

5. An annulus has an outside diameter of 49.0 mm and an inside diameter of 15.0 mm. Find its area correct to 4 significant figures.

Area of annulus = $\pi r_1^2 - \pi r_2^2 = \pi\dfrac{d_1^2}{4} - \pi\dfrac{d_2^2}{4} = \dfrac{\pi}{4}(d_1^2 - d_2^2) = \dfrac{\pi}{4}(49.0^2 - 15.0^2)$ = $\mathbf{1709\,mm^2}$

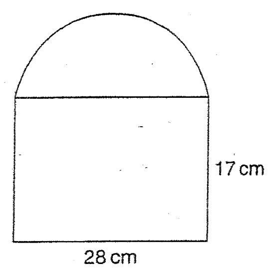
6. Find the area, correct to the nearest square metre, of a 2 m wide path surrounding a circular plot of land 200 m in diameter.

Area of path $= \frac{\pi}{4}\left(d_1^2 - d_2^2\right) = \frac{\pi}{4}\left(204^2 - 200^2\right)$ = $\mathbf{1269\,m^2}$

7. A rectangular park measures 50 m by 40 m. A 3 m flower bed is made round the two longer sides and one short side. A circular fish pond of diameter 8.0 m is constructed in the centre of the park. It is planned to grass the remaining area. Find, correct to the nearest square metre, the area of grass.

Area of grass $= (50 \times 40) - 2(50 \times 3) - (34 \times 3) - \pi\frac{8.0^2}{4} = 2000 - 300 - 102 - 16\pi =$ $\mathbf{1548\,m^2}$

8. With reference to the shape below, determine (a) the perimeter, and (b) the area.



(a) **Perimeter** $= 17 + 28 + 17 + \frac{1}{2}(2\pi r) = 17 + 28 + 17 + (\pi \times 14)$

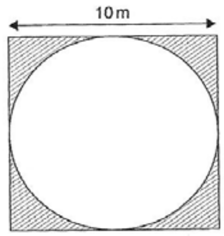
$= \mathbf{106.0\ cm}$

(b) **Area** = area of rectangle + area of semicircle

$$= (17 \times 28) + \left(\frac{1}{2}\pi r^2\right) = 476 + \left(\frac{1}{2}\pi(14)^2\right)$$

$$= 476 + 98\pi = \mathbf{783.9\,cm^2}$$

9. Find the area of the shaded portion of the shape below.



Shaded portion = area of square – area of circle

$$= 10^2 - \pi r^2 = 100 - \pi(5)^2 = 100 - 25\pi = \mathbf{21.46\,m^2}$$

10. Find the length of an arc of a circle of radius 8.32 cm when the angle subtended at the centre is 2.14 radians. Calculate also the area of the minor sector formed.

Length of arc, s = rθ = (8.32)(2.14) = **17.80 cm**

Area of minor sector $= \dfrac{2.14}{2\pi}\left(\pi r^2\right) = \dfrac{2.14}{2\pi}\left(\pi(8.32)^2\right)$ $= \mathbf{74.07\,cm^2}$

11. If the angle subtended at the centre of a circle of diameter 82 mm is 1.46 rad, find the lengths of the (a) minor arc, and (b) major arc.

If diameter d = 82 mm, radius $r = \dfrac{82}{2} = 41$ mm

(a) Minor arc length, s = rθ = (41)(1.46) = **59.86 mm**

(b) Major arc length = circumference – minor arc

$$= 2\pi(41) - 59.86 = 257.61 - 59.86 = \mathbf{197.8\ mm}$$

12. A pendulum of length 1.5 m swings through an angle of 10° in a single swing. Find, in centimetres, the length of the arc traced by the pendulum bob.

Arc length of pendulum bob, s = $r\theta = (1.5)\left(10\times\frac{\pi}{180}\right)$ = 0.262 m or **26.2 cm**

13. Determine the angle of lap, in degrees and minutes, if 180 mm of a belt drive are in contact with a pulley of diameter 250 mm.

Arc length, s = 180 mm, radius, r = $\frac{250}{2}$ = 125 mm

Since s = rθ, **the angle of lap, θ** = $\frac{s}{r}=\frac{180}{125}$ = 1.44 rad = $1.44\times\frac{180}{\pi}$ = **82.5°**

14. Determine the number of complete revolutions a motorcycle wheel will make in travelling 2 km, if the wheel's diameter is 85.1 cm.

If wheel diameter = 85.1 cm, then circumference, c = πd = π(85.1) cm = 267.35 cm = 2.6735 m

Hence, number of revolutions of wheel in travelling 2000 m = $\frac{2000}{2.6735}$ = 748.08

Thus, **number of complete revolutions = 748**

15. The floodlights at a sports ground spread its illumination over an angle of 40° to a distance of 48 m. Determine (a) the angle in radians, and (b) the maximum area that is floodlit.

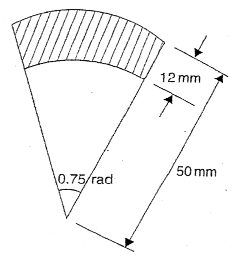
(a) In radians, 40° = $40\times\frac{\pi}{180}$ rad = 0.69813 rad = **0.698 rad**, correct to 3 decimal places

(b) **Maximum area floodlit** = area of sector = $\frac{1}{2}r^2\theta=\frac{1}{2}(48)^2(0.69813)=\mathbf{804.2\,m^2}$

16. Find the area swept out in 50 minutes by the minute hand of a large floral clock, if the hand is 2 m long.

Area swept out = $\frac{50}{60}\times\pi r^2=\frac{50}{60}\times\pi(2)^2=\mathbf{10.47\,m^2}$

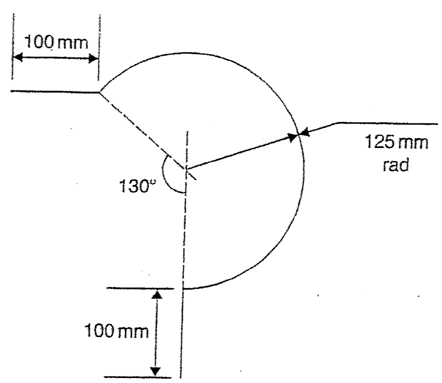
17. Determine (a) the shaded area shown below, (b) the percentage of the whole sector that the area of the shaded area represents.



(a) Shaded area = $\frac{1}{2}(50)^2(0.75) - \frac{1}{2}(38)^2(0.75) = \frac{1}{2}(0.75)\left[50^2 - 38^2\right] = \mathbf{396\ mm^2}$

(b) Percentage of whole sector = $\dfrac{396}{\frac{1}{2}(50)^2(0.75)} \times 100\% = \mathbf{42.24\%}$

18. Determine the length of steel strip required to make the clip shown below.

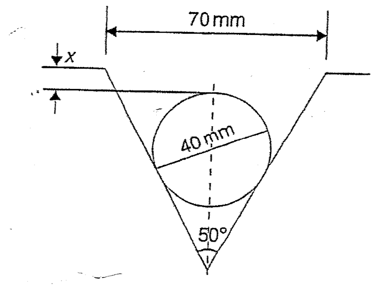


Angle of sector = 360° - 130° = 230° = $230 \times \frac{\pi}{180}$ rad = 4.01426 rad

Thus, arc length, s = rθ = (125)(4.01426) = 501.783 mm

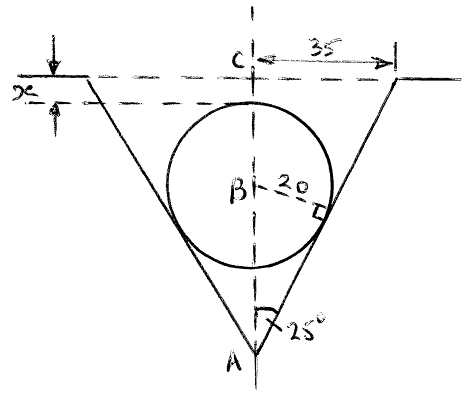
Length of steel strip in clip = 100 + 501.783 + 100 = **701.8 mm**

19. A 50° tapered hole is checked with a 40 mm diameter ball as shown below. Determine the length shown as x.



From the sketch below, $\tan 25° = \frac{35}{AC}$ from which, $AC = \frac{35}{\tan 25°} = 75.06$ mm

and $\sin 25° = \frac{20}{AB}$ from which, $AB = \frac{20}{\sin 25°} = 47.32$ mm



i.e. $AC = 75.06 = x + 20 + AB$

$= x + 20 + 47.32$

and $x = 75.06 - 20 - 47.32$

i.e. $\mathbf{x = 7.74\ mm}$

CHAPTER 13 VOLUMES OF COMMON SOLIDS

EXERCISE 53, Page 127

1. Change a volume of $1{,}200{,}000\,\text{cm}^3$ to cubic metres.

$1{,}200{,}000\,\text{cm}^3 = 1{,}200{,}000 \times 10^{-6}\,\text{m}^3 = \mathbf{1.2\ m^3}$

2. Change a volume of $5000\,\text{mm}^3$ to cubic centimetres.

$5000\,\text{mm}^3 = 5000 \times 10^{-3}\,\text{cm}^3 = \mathbf{5\ cm^3}$

3. A metal cube has a surface area of $24\,\text{cm}^2$. Determine its volume.

A cube has 6 identical faces. Hence each face has an area of $24 \div 6 = 4\,\text{cm}^2$

If each side of each face has length x cm then $x^2 = 4$ from which, x = 2 cm.

Hence, **volume of cube** = $x^3 = 2^3 = \mathbf{8\,cm^3}$

4. A rectangular block of wood has dimensions of 40 mm by 12 mm by 8 mm. Determine (a) its volume, in cubic millimetres, and (b) its total surface area in square millimetres.

(a) Volume = 40 × 12 × 8 = $\mathbf{3840\,mm^3}$

(b) Total surface area = 2(40 × 12 + 40 × 8 + 12 × 8) = 2(480 + 320 + 96)

= 2(896) = $\mathbf{1792\ mm^2}$

5. Determine the capacity, in litres, of a fish tank measuring 90 cm by 60 cm by 1.8 m, given 1 litre = $1000\,\text{cm}^3$.

Volume of tank = $90 \times 60 \times 180\,\text{cm}^3 = 972{,}000\,\text{cm}^3$

Capacity , in litres = $\dfrac{972000\,\text{cm}^3}{1000\,\text{cm}^3\,/\,\text{litre}}$ = **972 litres**

6. A rectangular block of metal has dimensions of 40 mm by 25 mm by 15 mm. Determine its Volume in cm^3. Find also its mass if the metal has a density of 9 g/cm^3.

Volume = length × breadth × width = 40 × 25 × 15 = 15000 mm^3

$$= 15000 \times 10^{-3}\ \text{cm}^3 = \mathbf{15\,cm^3}$$

Mass = density × volume = $9\,\text{g}/\text{cm}^3 \times 15\,\text{cm}^3$ = **135 g**

7. Determine the maximum capacity, in litres, of a fish tank measuring 50 cm by 40 cm by 2.5 m (1 litre = 1000 cm^3)

Volume = $50 \times 40 \times 250\,\text{cm}^3$

Tank capacity = $\dfrac{50\times 40\times 250\,\text{cm}^3}{1000\,\text{cm}^3\,/\,\text{litre}}$ = **500 litre**

8. Determine how many cubic metres of concrete are required for a 120 m long path, 150 mm wide and 80 mm deep.

Volume of concrete = 120 × 0.15 × 0.08 = $\mathbf{1.44\,m^3}$

9. A cylinder has a diameter 30 mm and height 50 mm. calculate (a) its volume in cubic centimetres, correct to 1 decimal place, and (b) the total surface area in square centimetres, correct to 1 decimal place.

(a) **Volume** = $\pi r^2 h = \pi\left(\dfrac{3}{2}\right)^2(5) = \mathbf{35.3\,cm^3}$, correct to 1 decimal place.

(b) **Total surface area** = $2\pi rh + 2\pi r^2 = 2\pi(1.5)(5) = 2\pi(1.5)^2$

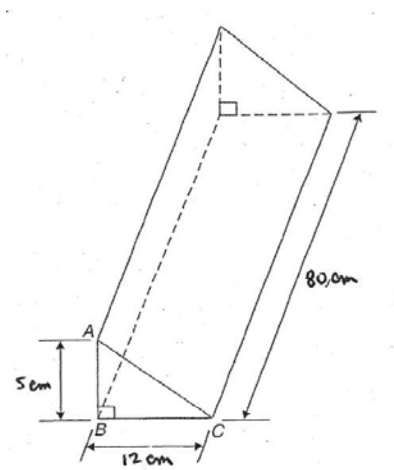
$$=15\pi + 4.5\,\pi = 19.5\pi = \mathbf{61.3\,cm^2}$$

10. Find (a) the volume, and (b) the total surface area of a right-angled triangular prism of length 80 cm and whose triangular end has a base of 12 cm and perpendicular height 5 cm.

(a) Volume = Ah = $\frac{1}{2} \times \text{base} \times \text{perpendicular height} \times \text{height of prism}$

$$= \frac{1}{2} \times 12 \times 5 \times 80 = \mathbf{2400\,cm^3}$$

(b) Total surface area = area of each end + area of three sides



In triangle ABC, $AC^2 = AB^2 + BC^2$

from which, $AC = \sqrt{AB^2 + BC^2} = \sqrt{5^2 + 12^2} = 13$ cm

Hence, total surface area = $2\left(\frac{1}{2}bh\right) + (AC \times 80) + (BC \times 80) + (AB \times 80)$

$$= (12 \times 5) + (13 \times 80) + (12 \times 80) + (5 \times 80)$$

$$= 60 + 1040 + 960 + 400$$

i.e. **total surface area = 2460 cm²**

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11. A steel ingot whose volume is 2 m^2 is rolled out into a plate which is 30 mm thick and 1.80 m wide. Calculate the length of the plate in metres.

Volume = length × width × thickness

i.e. $2 = \text{length} \times 1.80 \times 0.030$

from which, **length** = $\dfrac{2}{1.80 \times 0.30}$ = **37.04 m**

12. Calculate the volume of a metal tube whose outside diameter is 8 cm and whose inside diameter is 6 cm, if the length of the tube is 4 m.

Volume of tube = area of an end × length of tube

$$= \left(\pi\left(R^2 - r^2\right)\right)(\text{length})$$

$$= \left(\pi\left(4^2 - 3^2\right)\right)(400)\,\text{cm}^3 = 2800\pi = \mathbf{8796\,cm^3}$$

13. The volume of a cylinder is 400 cm^3. If its radius is 5.20 cm, find its height. Determine also its curved surface area.

Volume of cylinder = $\pi r^2 h$

i.e. $400 = \pi(5.20)^2 h$

from which, **height, h** = $\dfrac{400}{\pi(5.20)^2}$ = **4.709 cm**

Curved surface area = $2\pi rh = 2\pi(5.20)(4.709)$ = $\mathbf{153.9\,cm^2}$

14. A cylinder is cast from a rectangular piece of alloy 5 cm by 7 cm by 12 cm. If the length of the cylinder is to be 60 cm, find its diameter.

Volume of rectangular piece of alloy = 5 × 7 × 12 = 420 cm^3

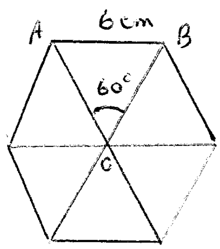
Volume of cylinder = $\pi r^2 h$

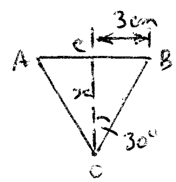
Hence, $420 = \pi r^2 (60)$ from which, $r^2 = \dfrac{420}{\pi(60)} = \dfrac{7}{\pi}$ and radius, $r = \sqrt{\dfrac{7}{\pi}} = 1.4927$ cm

and **diameter of cylinder, d** = 2r = 2 × 1.4927 = **2.99 cm**

15. Find the volume and the total surface area of a regular hexagonal bar of metal of length 3 m if each side of the hexagon is 6 cm.

A hexagon is shown below.





In triangle 0BC, $\tan 30° = \dfrac{3}{x}$ from which, $x = \dfrac{3}{\tan 30°} = 5.196$ cm.

Hence, area of hexagon = $6\left[\dfrac{1}{2} \times 6 \times 5.196\right] = 93.53\ cm^2$

and **volume of hexagonal bar** = 93.53 × 300 = **28060 cm^3**

Surface area of bar = $6[0.06 \times 3] + 2[93.53 \times 10^{-4}]$ in metre units

= **1.099 m^2**

16. A block of lead 1.5 m by 90 cm by 750 mm is hammered out to make a square sheet 15 mm thick. Determine the dimensions of the square sheet, correct to the nearest centimetre.

Volume of lead = 1.5 × 0.90 × 0.75 cm^3

Volume of square sheet = (x)(x)(0.015) where x m is the length of each side of the square

Hence, $1.5 \times 0.90 \times 0.75 = 0.015\,x^2$

from which, $$x^2 = \frac{1.5 \times 0.90 \times 0.75}{0.015}$$

and $$x = \sqrt{\frac{1.5 \times 0.90 \times 0.75}{0.015}} = 8.22 \text{ m, correct to the nearest centimetre}$$

Hence, **the dimensions of the square sheet are 8.22 m by 8.22 m**

17. A cylinder is cast from a rectangular piece of alloy 5.20 cm by 6.50 cm by 19.33 cm. If the height of the cylinder is to be 52.0 cm, determine its diameter, correct to the nearest centimetre.

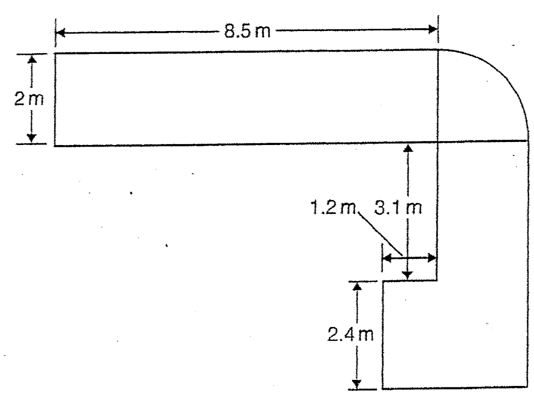
Volume = $5.20 \times 6.50 \times 19.33 = \pi r^2 h = \pi r^2(52.0)$

from which, $$r^2 = \frac{5.20 \times 6.50 \times 19.33}{\pi \times 52.0}$$

and $$\text{radius, } r = \sqrt{\frac{5.20 \times 6.50 \times 19.33}{\pi \times 52.0}} = 2 \text{ cm}$$

and **diameter** = 2 × radius = 2 × 2 = **4 cm**

18. How much concrete is required for the construction of the path shown below, if the path is 12 cm thick?



Area of path = $(8.5 \times 2) + \left(\frac{1}{4}\pi(2)^2\right) + (3.1 \times 2) + (2.4 \times 3.2)$

$= 17 + \pi + 6.2 + 7.68$

$= 34.022\,m^2$

If the concrete is 12 cm thick, i.e. 0.12 m thick,

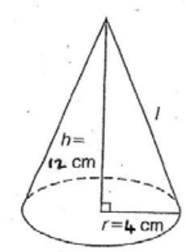
then **volume of concrete** $= 34.022 \times 0.12 =$ $\mathbf{4.08\,m^3}$

EXERCISE 54, Page 130

1. If a cone has a diameter of 80 mm and a perpendicular height of 120 mm, calculate its volume in cm^3 and its curved surface area.

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{8}{2}\right)^2(12)$ = **201.1 cm³**

Curved surface area = πrl



From the diagram, the slant height is calculated using Pythagoras' theorem:

$$l = \sqrt{4^2 + 12^2} = 12.649 \text{ cm}$$

Hence, **curved surface area** $= \pi(4)(12.649) =$ **159.0 cm²**

2. A square pyramid has a perpendicular height of 4 cm. If a side of the base is 2.4 cm long find the volume and total surface area of the pyramid.

Volume of pyramid $= \frac{1}{3}$(area of base) × perpendicular height

$= \frac{1}{3}(2.4 \times 2.4) \times 4 =$ **7.68 cm³**

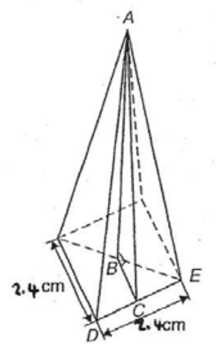
The total surface area consists of a square base and 4 equal triangles.

Area of triangle ADE $= \frac{1}{2} \times$ base × perpendicular height $= \frac{1}{2} \times 2.4 \times$ AC (see diagram below)

The length AC may be calculated using Pythagoras' theorem on triangle ABC, where AB = 4 cm,

BC $= \frac{1}{2} \times 2.4 = 1.2$ cm.

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 1.2^2} = 4.176 \text{ cm}$$



Hence, area of triangle ADE = $\frac{1}{2} \times 2.4 \times 4.176 = 5.011 \text{ cm}^2$

Total surface area of pyramid = $(2.4 \times 2.4) + 4(5.011)$ = **25.81 cm²**

3. A sphere has a diameter of 6 cm. Determine its volume and surface area.

Since diameter = 6 cm, then radius, r = 3 cm.

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 3^3$ = $\mathbf{113.1\ cm^3}$

Surface area of sphere = $4\pi r^2 = 4 \times \pi \times 3^2$ = $\mathbf{113.1\ cm^2}$

4. A pyramid having a square base has a perpendicular height of 25 cm and a volume of 75 cm^3. Determine, in centimetres, the length of each side of the base.

Volume of pyramid = $\frac{1}{3} \times$ area of base $\times$ perpendicular height

i.e. $75 = \frac{1}{3} \times$ area of base $\times 25$

from which, area of base = $\frac{75 \times 3}{25} = 9 \text{ cm}^2$

If each side of the base is x cm, then $x^2 = 9$ from which, $x = \sqrt{9} = 3$ cm

Hence, **the length of each side of the base is 3 cm**

5. A cone has a base diameter of 16 mm and a perpendicular height of 40 mm. Find its volume correct to the nearest cubic millimetre.

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{16}{2}\right)^2(40)$ = **2681 mm³**

6. Determine (a) the volume, and (b) the surface area of a sphere of radius 40 mm.

Since diameter = 6 cm, then radius, r = 3 cm.

(a) **Volume of sphere** $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times 40^3 = \mathbf{268{,}083\,mm^3}$ or $\mathbf{268.083\ cm^3}$

(b) **Surface area of sphere** $= 4\pi r^2 = 4\times\pi\times 40^2 = \mathbf{20{,}106\ mm^2}$ or $\mathbf{201.06\ cm^2}$

7. The volume of a sphere is 325 cm³. Determine its diameter.

Volume of sphere $= \frac{4}{3}\pi r^3$

Hence, $325 = \frac{4}{3}\pi r^3$ from which, $r^3 = \frac{325\times 3}{4\pi}$

and radius, $r = \sqrt[3]{\left(\frac{325\times 3}{4\pi}\right)} = 4.265$ cm

Hence, **diameter** = 2 × radius = 2 × 4.265 = **8.53 cm**

8. Given the radius of the earth is 6380 km, calculate, in engineering notation (a) its surface area in km^2 and (b) its volume in km^3.

(a) **Surface area of earth (i.e. a sphere)** $= 4\pi r^2 = 4\times\pi\times 6380^2 = \mathbf{512\times 10^6\,km^2}$

(b) **Volume of earth (i.e. a sphere)** $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times 6380^3 = \mathbf{1.09\times 10^{12}\,km^3}$

9. An ingot whose volume is $1.5\,\text{m}^3$ is to be made into ball bearings whose radii are 8.0 cm. How many bearings will be produced from the ingot, assuming 5% wastage?

If x is the number of ball bearings then

$$0.95 \times 1.5 \times 10^6 = x\left(\frac{4}{3}\pi(8.0)^3\right)$$

from which, **number of bearings, x** = $\dfrac{0.95 \times 1.5 \times 10^6 \times 3}{4\pi \times 8.0^3}$ = **664**

10. A spherical chemical storage tank has an internal diameter of 5.6 m. Calculate the storage capacity of the tank, correct to the nearest cubic metre. If 1 litre = $1000\,\text{cm}^3$, determine the tank capacity in litres.

Volume of storage tank = $\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi\left(\dfrac{5.6}{2}\right)^3$ = 91.95 = $\mathbf{92\,m^3}$, correct to the nearest cubic metre

Volume of tank = $92 \times 10^6\,\text{cm}^3$

If 1 litre = $1000\,\text{cm}^3$, then **capacity of storage tank** = $\dfrac{92 \times 10^6}{1000}$ litres = **92,000 litres**

EXERCISE 55, Page 134

1. Find the total surface area of a hemisphere of diameter 50 mm.

Total surface area = $\pi r^2 + \frac{1}{2}\left[4\pi r^2\right] = \pi r^2 + 2\pi r^2 = 3\pi r^2$

$$= 3\pi\left(\frac{50}{2}\right)^2 = \mathbf{5890\ mm^2} \quad \text{or} \quad \mathbf{58.90\ cm^2}$$

2. Find (a) the volume, and (b) the total surface area of a hemisphere of diameter 6 cm.

(a) **Volume of hemisphere** = $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = 18\pi = \mathbf{56.55\,cm^3}$

(b) **Total surface area** = $\pi r^2 + \frac{1}{2}\left[4\pi r^2\right] = \pi r^2 + 2\pi r^2 = 3\pi r^2$

$$= 3\pi\left(\frac{6}{2}\right)^2 = \mathbf{84.82\ cm^2}$$

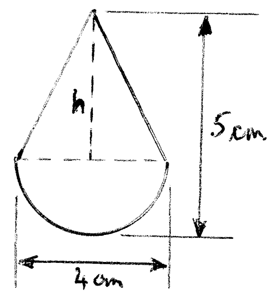
3. Determine the mass of a hemispherical copper container whose external and internal radii are 12 cm and 10 cm, assuming that 1 cm^3 of copper weighs 8.9 g.

Volume of hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left[12^3 - 10^3\right]cm^3$

Mass of copper = volume × density = $\frac{2}{3}\pi\left[12^3 - 10^3\right]cm^3 \times 8.9\,g/cm^3$ = 13570 g = **13.57 kg**

4. A metal plumb bob comprises a hemisphere surmounted by a cone. If the diameter of the hemisphere and cone are each 4 cm and the total length is 5 cm, find its total volume.

The plumb bob is shown sketched below.

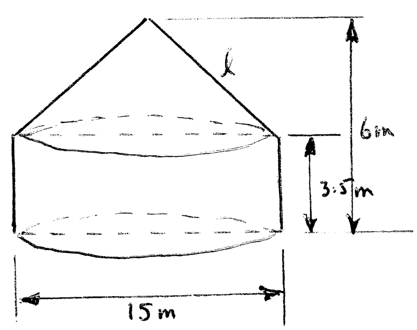


Volume of bob = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi(2)^2(5-2) + \frac{2}{3}\pi(2)^3$

$$= 4\pi + \frac{16}{3}\pi = \mathbf{29.32\,cm^3}$$

5. A marquee is in the form of a cylinder surmounted by a cone. The total height is 6 m and the cylindrical portion has a height of 3.5 m, with a diameter of 15 m. Calculate the surface area of material needed to make the marquee assuming 12% of the material is wasted in the process

The marquee is shown sketched below.



Surface area of material for marquee = $\pi r l + 2\pi r h$ where $l = \sqrt{(7.5^2 + 2.5^2)} = 7.9057$ m

Hence, surface area = $\pi(7.5)(7.9057) + 2\pi(7.5)(3.5)$

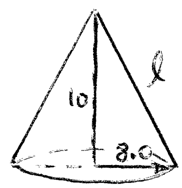
$= 186.2735 + 164.9336 = 351.2071\,m^2$

If 12% of material is wasted then **amount required** = $1.12 \times 351.2071 = \mathbf{393.4\,m^2}$

6. Determine (a) the volume and (b) the total surface area of the following solids:

(i) a cone of radius 8.0 cm and perpendicular height 10 cm
(ii) a sphere of diameter 7.0 cm
(iii) a hemisphere of radius 3.0 cm
(iv) a 2.5 cm by 2.5 cm square pyramid of perpendicular height 5.0 cm
(v) a 4.0 cm by 6.0 cm rectangular pyramid of perpendicular height 12.0 cm
(vi) a 4.2 cm by 4.2 cm square pyramid whose sloping edges are each 15.0 cm
(vii) a pyramid having an octagonal base of side 5.0 cm and perpendicular height 20 cm

(i) A sketch of the cone is shown below.



(a) **Volume of cone** = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8.0)^2(10) = \mathbf{670\,cm^3}$

(b) **Total surface area** = $\pi r^2 + \pi r l$ where $l = \sqrt{(10^2 + 8.0^2)} = 12.80625$ cm

$= \pi(8.0)^2 + \pi(8.0)(12.80625)$

$= 201.062 + 321.856 = \mathbf{523\,cm^2}$

(ii) (a) **Volume of sphere** = $\frac{4}{3}\pi\left(\frac{7.0}{2}\right)^3 = \mathbf{180\,cm^3}$

(b) **Surface area** = $4\pi r^2 = 4\pi\left(\frac{7.0}{2}\right)^2 = \mathbf{154\,cm^2}$

(iii) (a) **Volume of hemisphere** = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3.0)^3 = \mathbf{56.5\,cm^3}$

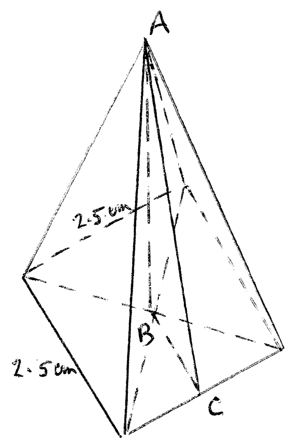
(b) **Surface area** = $\frac{1}{2}(4\pi r^2) + \pi r^2 = 3\pi r^2 = 3\pi(3.0)^2 = \mathbf{84.8\,cm^2}$

(iv) A sketch of the square pyramid is shown below, where AB = 5.0 cm

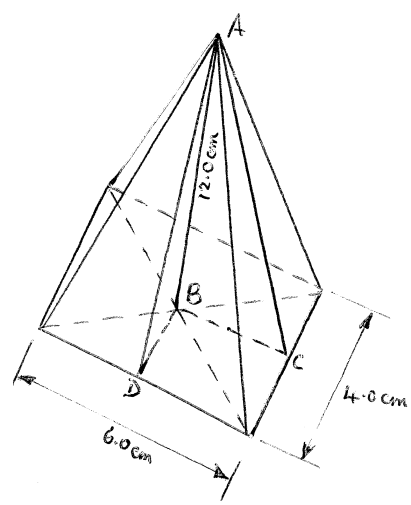
(a) **Volume of pyramid** = $\frac{1}{3}(2.5)^2(5.0) = \mathbf{10.4\,cm^3}$

(b) In the diagram, AC = $\sqrt{\left(AB^2 + BC^2\right)} = \sqrt{\left(5.0^2 + 1.25^2\right)}$ = 5.15388

Surface area = $(2.5)^2 + 4\left[\frac{1}{2} \times 2.5 \times 5.15388\right]$ = 6.25 + 25.7694 = $\mathbf{32.0\,cm^2}$



(v) A sketch of the rectangular pyramid is shown below.



(a) **Volume of rectangular pyramid** = $\frac{1}{3}(6.0 \times 4.0)(12.0)$ = $\mathbf{96.0\,cm^3}$

(b) In the diagram, AC = $\sqrt{\left(12.0^2 + 3.0^2\right)}$ = 12.3693 cm

and AD = $\sqrt{\left(12.0^2 + 2.0^2\right)}$ = 12.1655 cm

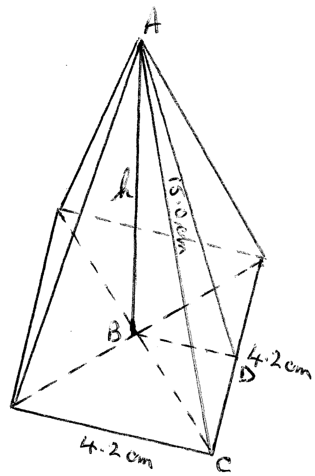
Hence, **surface area** $= (6.0\times4.0)+2\left[\frac{1}{2}\times4.0\times12.3696\right]+2\left[\frac{1}{2}\times6.0\times12.1655\right]$

$= 24 + 49.4784 + 72.993 = \mathbf{146\,cm^2}$

(vi) The square pyramid is shown sketched below.

Diagonal on base $= \sqrt{(4.2^2+4.2^2)} = 5.9397$ cm hence, BC $= \frac{1}{2}\times5.9397 = 2.96985$ cm

Hence, perpendicular height, h $= \sqrt{(15.0^2-2.96985^2)} = 14.703$ cm



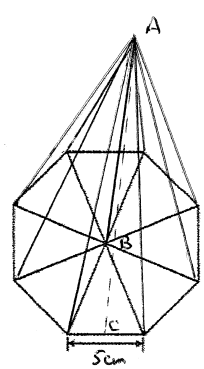
(a) **Volume of pyramid** $= \frac{1}{3}(4.2)^2(14.703) = \mathbf{86.5\,cm^3}$

(b) AD $= \sqrt{(14.703^2+2.1^2)} = 14.8522$

Hence, **surface area** $= (4.2)^2+4\left[\frac{1}{2}\times4.2\times14.8522\right] = 17.64 + 124.75858$

$= \mathbf{142\,cm^2}$

(vii) A pyramid having an octagonal base is shown sketched below.

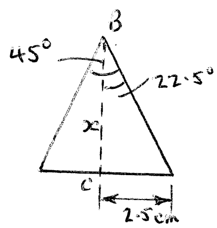


One sector is shown in diagram (p) below, where $\tan 22.5° = \frac{2.5}{x}$

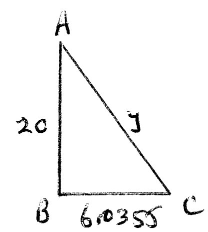
from which, $x = \frac{2.5}{\tan 22.5°} = 6.0355$ cm

Hence, area of whole base = $8\left[\frac{1}{2} \times 5.0 \times 6.0355\right] = 120.71 \text{ cm}^2$

(a) **Volume of pyramid** = $\frac{1}{3}(120.71)(20) = \mathbf{805\,cm^3}$



(p)



(q)

(b) From diagram (q) above, $y = \sqrt{(20^2 + 6.0355^2)} = 20.891$ cm

Total surface area = $120.71 + 8\left[\frac{1}{2} \times 5.0 \times 20.891\right] = 120.71 + 417.817$

$= \mathbf{539\,cm^2}$

7. A metal sphere weighing 24 kg is melted down and recast into a solid cone of base radius 8.0 cm. If the density of the metal is 8000 kg/m^3 determine (a) the diameter of the metal sphere and (b) the perpendicular height of the cone, assuming that 15% of the metal is lost in the process.

Volume of sphere = $\frac{\text{mass}}{\text{density}} = \frac{24\,\text{kg}}{8000\,\text{kg/m}^3} = 0.003\,\text{m}^3 = 0.003 \times 10^6\,\text{cm}^3 = 3000\,\text{cm}^3$

(a) Volume of sphere = $\frac{4}{3}\pi r^3$ i.e. $3000 = \frac{4}{3}\pi r^3$

and radius, $r = \sqrt[3]{\left(\frac{3000 \times 3}{4\pi}\right)} = 8.947$ cm

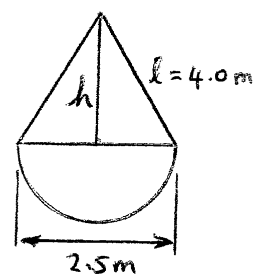
Hence, the **diameter of the sphere, d** = 2r = 2 × 8.947 = **17.9 cm**

(b) Volume of cone = 0.85 × 3000 = 2550 cm^3 = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8.0)^2 h$

from which, **perpendicular height of cone, h** = $\dfrac{2550\times 3}{\pi(8.0)^2}$ = **38.0 cm**

8. A buoy consists of a hemisphere surmounted by a cone. The diameter of the cone and hemisphere is 2.5 m and the slant height of the cone is 4.0 m. Determine the volume and surface area of the buoy.

The buoy is shown in the sketch below.



Height of cone, h = $\sqrt{(4.0^2 - 1.25^2)}$ = 3.80 m.

Volume of buoy = $\dfrac{2}{3}\pi r^3 + \dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi(1.25)^3 + \dfrac{1}{3}\pi(1.25)^2(3.80)$

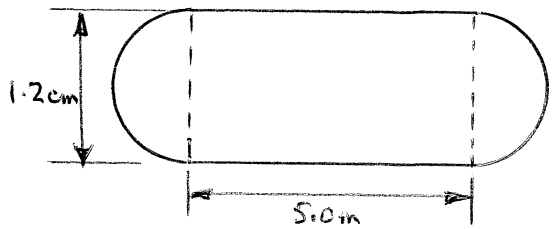
$= 4.0906 + 6.2177 = \mathbf{10.3\ m^3}$

Surface area = $\pi r l + \dfrac{1}{2}(4\pi r^2) = \pi(1.25)(4.0) + 2\pi(1.25)^2$

$= 5\pi + 3.125\pi = 8.125\pi = \mathbf{25.5\ m^2}$

9. A petrol container is in the form of a central cylindrical portion 5.0 m long with a hemispherical section surmounted on each end. If the diameters of the hemisphere and cylinder are both 1.2 m determine the capacity of the tank in litres (1 litre = 1000 cm^3).

The petrol container is shown sketched below.



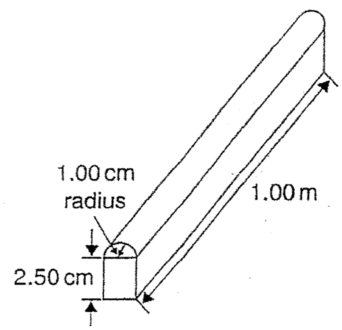
Volume of container = $2\left[\frac{2}{3}\pi r^3\right]+\pi r^2 h=\frac{4}{3}\pi(0.6)^3+\pi(0.6)^2(5.0)$

$$= 0.288\pi + 1.8\pi = 6.55965 \text{ m}^3$$

$$= 6.55965\times10^6 \text{cm}^3$$

and **tank capacity** = $\frac{6.56\times10^6\text{cm}^3}{1000\text{cm}^3/\text{litre}}$ = **6560 litres**

10. The diagram shows a metal rod section. Determine its volume and total surface area.



Volume of rod = $\frac{1}{2}\pi r^2 h+(l\times b\times w)=\frac{1}{2}\pi(1.0)^2(100)+(2.5\times2.0\times100)$

$$= 50\pi + 500 = \mathbf{657.1\ cm^3}$$

Surface area = $\frac{1}{2}(2\pi r h)+2\left[\frac{1}{2}\pi r^2\right]+2(2.50\times2.0)+2(2.5\times100)+(2.0\times100)$

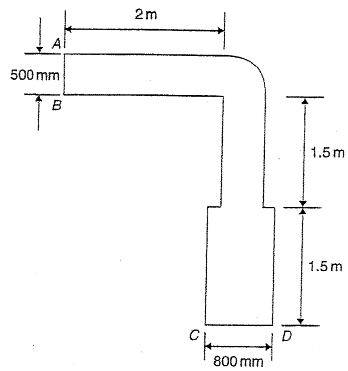
$$= \pi(1.0)(100) + \pi(1.0)^2 + 10 + 500 + 200$$

$$= \mathbf{1027\ cm^2}$$

11. The cross-section of part of a circular ventilation shaft is shown below, ends AB and CD being open. Calculate (a) the volume of the air, correct to the nearest litre, contained in the part of the system shown, neglecting the sheet metal thickness, (given 1 litre = 1000 cm^3),

(b) the cross-sectional area of the sheet metal used to make the system, in square metres, and

(c) the cost of the sheet metal if the material costs £11.50 per square metre, assuming that 25% extra metal is required due to wastage.



(a) In cm^3, **volume of air** $= \pi\left(\frac{50}{2}\right)^2(200) + \frac{1}{4}\left[\frac{4}{3}\pi\left(\frac{50}{2}\right)^3\right] + \pi\left(\frac{50}{2}\right)^2(150) + \pi\left(\frac{80}{2}\right)^2(150)$

$= 125000\pi + 5208.33\pi + 93750\pi + 240000\pi = 463958.33\pi\ cm^3$

$= \frac{463958.33\pi\ cm^3}{1000\,cm^3/\text{litre}}$ = **1458 litre**, correct to the nearest litre

(b) In m^2, cross-sectional area of the sheet metal

$= 2\pi(0.25)(2) + \frac{1}{4}\left[4\pi(0.25)^2\right] + 2\pi(0.25)(1.5) + 2\pi(0.4)(1.5) + \pi(0.4^2 - 0.25^2)$

$= \pi + 0.0625\pi + 0.75\pi + 1.2\pi + 0.0975\pi$

$= 3.11\pi = 9.77035\,m^2$ = $\mathbf{9.77\,m^2}$ correct to 3 significant figures.

(c) Sheet metal required = $9.77035 \times 1.25\,m^2$

Cost of sheet metal = 9.77035 × 1.25 × £11.50 = **£140.45**

EXERCISE 56, Page 136

1. The diameter of two spherical bearings are in the ratio 2 : 5. What is the ratio of their volumes?

Diameters are in the ratio 2:5

Hence, ratio of their volumes = $2^3 : 5^3$ i.e. **8:125**

2. An engineering component has a mass of 400 g. If each of its dimensions are reduced by 30% determine its new mass.

New mass = $(0.7)^3 \times 400 = 0.343 \times 400$ = **137.2 g**

MULTIPLE CHOICE QUESTIONS ON APPLIED MATHEMATICS

EXERCISE 57, Page 140

1. (b) **2.** (d) **3.** (b) **4.** (b) **5.** (a) **6.** (c) **7.** (c) **8.** (d) **9.** (b) **10.** (c) **11.** (a) **12.** (d) **13.** (a)
14. (b) **15.** (a) **16.** (d) **17.** (a) **18.** (b) **19.** (d) **20.** (d) **21.** (a) **22.** (a) **23.** (a) **24.** (a) **25.** (a)
26. (d) **27.** (b) **28.** (c) **29.** (d) **30.** (b) **31.** (a) **32.** (c) **33.** (b) **34.** (b) **35.** (c) **36.** (c) **37.** (b)
38. (d) **39.** (c) **40.** (c) **41.** (a) **42.** (a) **43.** (c) **44.** (b) **45.** (d) **46.** (b) **47.** (d) **48.** (b) **49.** (d)
50. (a) **51.** (a) **52.** (b) **53.** (a) **54.** (a) **55.** (d) **56.** (d) **57.** (c) **58.** (b) **59.** (c) **60.** (c) **61.** (c)
62. (d) **63.** (a) **64.** (a) **65.** (c) **66.** (b) **67.** (b) **68.** (d) **69.** (d) **70.** (a)

CHAPTER 14 SI UNITS AND DENSITY

EXERCISE 58, Page 151

1. Express (a) a length of 52 mm in metres (b) 20,000 mm^2 in square metres

(c) 10,000,000 mm^3 in cubic metres

(a) 52 mm = $\frac{52}{1000}$ m = **0.052 m**

(b) 20000 mm^2 = $20000 \times 10^{-6} m^2$ = $\mathbf{0.02\, m^2}$

(c) 10000000 mm^3 = $10000000 \times 10^{-9} m^3$ = $\mathbf{0.01\, m^3}$

2. A garage measures 5 m by 2.5 m. Determine the area in (a) m^2 (b) mm^2

(a) Area = 5 × 2.5 = $\mathbf{12.5\, m^2}$

(b) Area = $12.5\, m^2$ = $\mathbf{12.5 \times 10^6\, mm^2}$

3. The height of the garage in question 2 is 3 m. Determine the volume in (a) m^3 (b) mm^3

(a) Volume = 5 × 2.5 × 3 = $\mathbf{37.5\, m^3}$

(b) Volume = 37.5 m^3 = $\mathbf{37.5 \times 10^9\, mm^3}$

4. A bottle contains 6.3 litres of liquid. Determine the volume in (a) m^3 (b) cm^3 (c) mm^3

(a) 6.3 litres = 6.3 $\times 1000\, cm^3$ = $6.3 \times 1000 \times 10^{-6} m^3$ = $\mathbf{0.0063\, m^3}$

(b) 6.3 litres = 6.3 $\times 1000\, cm^3$ = **6300 $\mathbf{cm^3}$**

(c) 6.3 litres = 6300 cm^3 = 6300 $\times 10^3 mm^3$ = $\mathbf{6.3 \times 10^6\ mm^3}$

EXERCISE 59, Page 153

1. Determine the density of 200 cm^3 of lead which has a mass of 2280 g.

$$\textbf{Density} = \frac{\text{mass}}{\text{volume}} = \frac{2280\times10^{-3}\,\text{kg}}{200\times10^{-6}\,\text{m}^3} = \mathbf{11{,}400\ kg/m^3}$$

2. The density of iron is 7500 kg/m^3. If the volume of a piece of iron is 200 cm^3, determine its mass.

If density = $\frac{\text{mass}}{\text{volume}}$ then **mass** = density × volume = 7500 kg/m^3 × $200\times10^{-6}\,\text{m}^3$

= **1.5 kg**

3. Determine the volume, in litres, of 14 kg of petrol of density 700 kg/m^3.

If density = $\frac{\text{mass}}{\text{volume}}$ then volume = $\frac{\text{mass}}{\text{density}} = \frac{14\,\text{kg}}{700\,\text{kg/m}^3} = 0.02\,\text{m}^3 = 0.02\times10^6\,\text{cm}^3$

1 litre = 1000 cm^3 hence, **volume** = $\frac{0.02\times10^6}{1000}$ litres = **20 litres**

4. The density of water is 1000 kg/m^3. Determine the relative density of a piece of copper of density 8900 kg/m^3.

Relative density of copper = $\frac{\text{density of copper}}{\text{density of water}} = \frac{8900}{1000}$ = **8.9**

5. A piece of metal 100 mm long, 80 mm wide and 20 mm thick has a mass of 1280 g. Determine the density of the metal.

Volume of metal = 100 × 80 × 20 mm^3 = 100 × 80 × 20$\times10^{-9}\,\text{m}^3$ = $160\times10^{-6}\,\text{m}^3$

Density of the metal = $\dfrac{\text{mass}}{\text{volume}} = \dfrac{1280\times10^{-3}\,\text{kg}}{160\times10^{-6}\,\text{m}^3}$ = $\mathbf{8000\ kg/m^3}$

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6. Some oil has a relative density of 0.80. Determine (a) the density of the oil, and (b) the volume of 2 kg of oil. Take the density of water as 1000 kg/m^3.

(a) Relative density of oil = $\dfrac{\text{density of oil}}{\text{density of water}}$

Hence, density of oil = relative density × density of water = 0.80 × 1000 kg/m^3 = $\mathbf{800\ kg/m^3}$

(b) If density = $\dfrac{\text{mass}}{\text{volume}}$

then **volume** = $\dfrac{\text{mass}}{\text{density}} = \dfrac{2\,\text{kg}}{800\,\text{kg}/\text{m}^3}$ = $\mathbf{0.0025\ m^3}$ or $0.0025\times10^6\,\text{cm}^3$ i.e. $\mathbf{2500\,cm^3}$

EXERCISE 60, Page 153

Answers found from within the text of the chapter, pages 149 to 152.

EXERCISE 61, Page 153

1. (c) **2.** (d) **3.** (b) **4.** (a) **5.** (b) **6.** (c) **7.** (b) **8.** (b) **9.** (c) **10.** (a)

CHAPTER 15 ATOMIC STRUCTURE OF MATTER

EXERCISE 62, Page 159

1. State whether the following are elements, compounds or mixtures:

(a) town gas, (b) water, (c) oil and water, (d) aluminium

(a) Town gas is a **mixture**

(b) Water is a **compound**

(c) Oil and water is a **mixture**

(d) Aluminium is an **element**

2. The solubility of sodium chloride is 0.036 kg in 0.1 kg of water. Determine the amount of water required to dissolve 432 g of sodium chloride.

0.036 kg of soldium chloride dissolves in 0.1 kg,

i.e. 36 g of soldium chloride dissolves in 0.1 kg

Hence, **amount of water to dissolve 432 g of sodium chloride** = $\frac{432}{36} \times 0.1 \text{kg}$ of water = **1.2 kg**

3. Describe, with appropriate sketches, a model depicting the structure of the atom.

See pages 156 of textbook

4. Explain, with the aid of a sketch, what is meant by a crystal, and give two examples of materials with a crystalline structure.

See pages 158 of textbook

EXERCISE 63, Page 159

Answers found from within the text of the chapter, pages 155 to 159.

EXERCISE 64, Page 159

1. (d) **2.** (c) **3.** (a) **4.** (b) **5.** (a) **6.** (b) **7.** (b) **8.** (c)

CHAPTER 16 SPEED AND VELOCITY

EXERCISE 65, Page 162

1. A train covers a distance of 96 km in 1 h 20 min. Determine the average speed of the train (a) in km/h and (b) in m/s

(a) **Average speed** = $\dfrac{\text{distance travelled}}{\text{time taken}} = \dfrac{96\text{ km}}{1\frac{20}{60}\text{ h}} = \dfrac{96}{1.\dot{3}}$ km/h = **72 km/h**

(b) **72 km/h** = $\dfrac{72\text{ km/h} \times 1000\text{ m/km}}{60 \times 60\text{ s/h}} = \dfrac{72}{3.6}\text{ m/s}$ = **20 m/s**

2. A horse trots at an average speed of 12 km/h for 18 minutes; determine the distance covered by the horse in this time

Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

from which, **distance covered** = speed × time taken

= 12 km/h × 18/60 h = **3.6 km**

3. A ship covers a distance of 1365 km at an average speed of 15 km/h. How long does it take to cover this distance ?

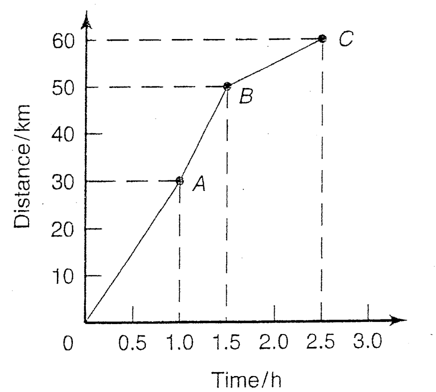
Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

from which, **time taken** = $\dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{1365\text{ km}}{15\text{ km/h}}$ = 91 hours = $\dfrac{91}{24}$ days **= 3 days 19 hours**

EXERCISE 66, Page 164

1. Using the information given in the distance/time graph shown below, determine the average speed when travelling from 0 to A, A to B, B to C, 0 to C and A to C



Average speed from 0 to A = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{30\text{ km}}{1.0\text{ h}}$ = **30 km/h**

Average speed from A to B = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{(50-30)\text{ km}}{(1.5-1.0)\text{ h}} = \frac{20\text{ km}}{0.5\text{ h}}$ = **40 km/h**

Average speed from B to C = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{(60-50)\text{ km}}{(2.5-1.5)\text{ h}} = \frac{10\text{ km}}{1\text{ h}}$ = **10 km/h**

Average speed from 0 to C = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{60\text{ km}}{2.5\text{ h}}$ = **24 km/h**

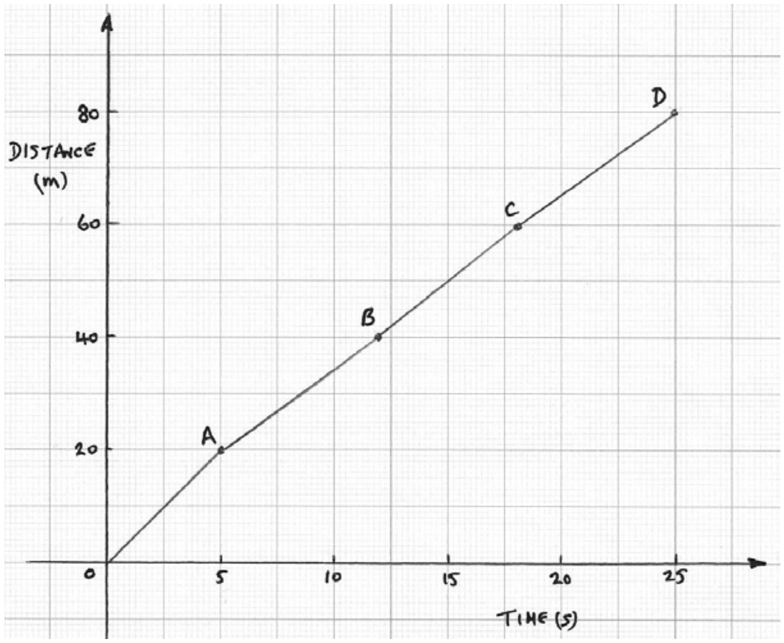
Average speed from A to C = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{(60-30)\text{ km}}{(2.5-1.0)\text{ h}} = \frac{30\text{ km}}{1.5\text{ h}}$ = **20 km/h**

2. The distances travelled by an object from point 0 and the corresponding times taken to reach A, B, C and D, respectively, from the start are as shown:

Points	Start	A	B	C	D
Distance (m)	0	20	40	60	80
Time (s)	0	5	12	18	25

Draw the distance/time graph and hence determine the average speeds from 0 to A, A to B, B to C, C to D and 0 to D

The distance /time graph is shown below.



Average speed from 0 to A = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{20\,\text{m}}{5\,\text{s}}$ = **4 m/s**

Average speed from A to B = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{(40-20)\,\text{m}}{(12-5)\,\text{s}} = \frac{20\,\text{m}}{7\,\text{s}}$ = **2.86 m/s**

Average speed from B to C = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{(60-40)\,\text{m}}{(18-12)\,\text{s}} = \frac{20\,\text{m}}{6\,\text{s}}$ = **3.33 m/s**

Average speed from C to D = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{(80-60)\,\text{m}}{(25-18)\,\text{s}} = \frac{20\,\text{m}}{7\,\text{s}}$ = **2.86 m/s**

Average speed from 0 to D = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{80\,\text{m}}{25\,\text{s}}$ = **3.2 m/s**

3. A train leaves station A and travels via stations B and C to station D. The times the train passes the various stations are as shown:

Station	A	B	C	D
Times	10.55 am	11.40 am	12.15 pm	12.50 pm

The average speeds are: A to B, 56 km/h, B to C, 72 km/h, and C to D, 60 km/h

Calculate the total distance from A to D.

Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$ from which, distance = speed × time

Distance from A to B = $56\dfrac{\text{km}}{\text{h}} \times \dfrac{45}{60}\text{h}$ = 42 km

Distance from B to C = $72\dfrac{\text{km}}{\text{h}} \times \dfrac{35}{60}\text{h}$ = 42 km

Distance from C to D = $60\dfrac{\text{km}}{\text{h}} \times \dfrac{35}{60}\text{h}$ = 35 km

Hence, **distance from A to D** = 42 + 42 + 35 = **119 km**

4. A gun is fired 5 km north of an observer and the sound takes 15 s to reach him. Determine the average velocity of sound waves in air at this place.

Average velocity = $\dfrac{\text{distance travelled}}{\text{time taken}} = \dfrac{5\,\text{km}}{\dfrac{15}{60\times 60}\text{h}}$ = **1200 km/h** or $\dfrac{1200}{3.6}$ = **333.33 m/s**

5. The light from a star takes 2.5 years to reach an observer. If the velocity of light is 330 $\times 10^{6}$ m/s, determine the distance of the star from the observer in kilometres, based on a 365 day year

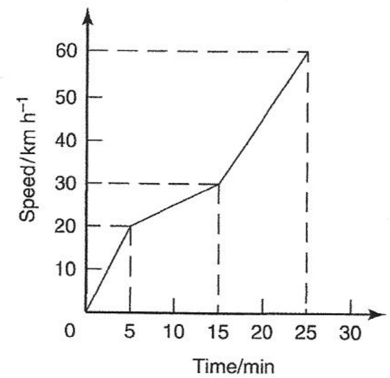
Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$ from which, distance = speed × time

Hence, **distance to the star** = $(330\times 10^6\,\text{m/s})\times(2.5\times 365\times 24\times 60\times 60\,\text{s})$

$$= \mathbf{2.6\times 10^{16}\ m} \quad \text{or} \quad \mathbf{2.6\times 10^{13}\ km}$$

EXERCISE 67, Page 165

1. The speed/time graph for a car journey is shown below. Determine the distance travelled by the car



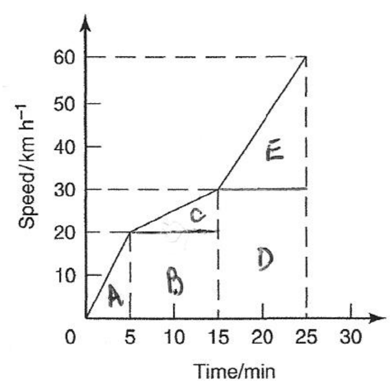
Distance travelled = area under the speed/time graph.

In the speed/time graph shown below,

distance travelled = area A + area B + area C + area D + area E

$$= \left(\frac{1}{2}\times\frac{5}{60}\times 20\right)+\left(\frac{10}{60}\times 20\right)+\left(\frac{1}{2}\times\frac{10}{60}\times 10\right)+\left(\frac{10}{60}\times 30\right)+\left(\frac{1}{2}\times\frac{10}{60}\times 30\right)$$

$$= \frac{5}{6}+3\frac{1}{3}+\frac{5}{6}+5+2.5 = \mathbf{12.5\ km}$$



2. The motion of an object is as follows:

A to B, distance 122 m, time 64 s,

B to C, distance 80 m at an average speed of 20 m/s,

C to D, time 7 s at an average speed of 14 m/s

Determine the overall average speed of the object when travelling from A to D

	Distance	Time
A to B,	122 m	64 s
B to C,	80 m	$\frac{80}{20} = 4$ s
C to D,	7 × 14 m = 98 m	7 s
Totals:	**300 m**	**75 s**

Hence, **the overall average speed of the object when travelling from A to D**

$$= \frac{\text{di tan ce}}{\text{time}} = \frac{300}{75} = \mathbf{4\ m/s}$$

EXERCISE 68, Page 166

Answers found from within the text of the chapter, pages 161 to 166.

EXERCISE 69, Page 167

1. (c) **2.** (g) **3.** (d) **4.** (c) **5.** (e) **6.** (b) **7.** (a) **8.** (b) **9.** (d) **10.** (a) **11.** (a) **12.** (b) **13.** (c) **14.** (d)

CHAPTER 17 ACCELERATION

EXERCISE 70, Page 171

1. A coach increases velocity from 4 km/h to 40 km/h at an average acceleration of 0.2 m/s^2. Find the time taken for this increase in velocity.

Average acceleration, a = $\frac{v-u}{t}$

from which, **time taken for increase in velocity, t** = $\frac{v-u}{a} = \frac{(40-4)\text{km/h}}{0.2\,\text{m/s}^2}$

$$= \frac{36\,\text{km/h} \times \frac{1000\,\text{m/km}}{60\times 60\,\text{s/h}}}{0.2\,\text{m/s}^2}$$

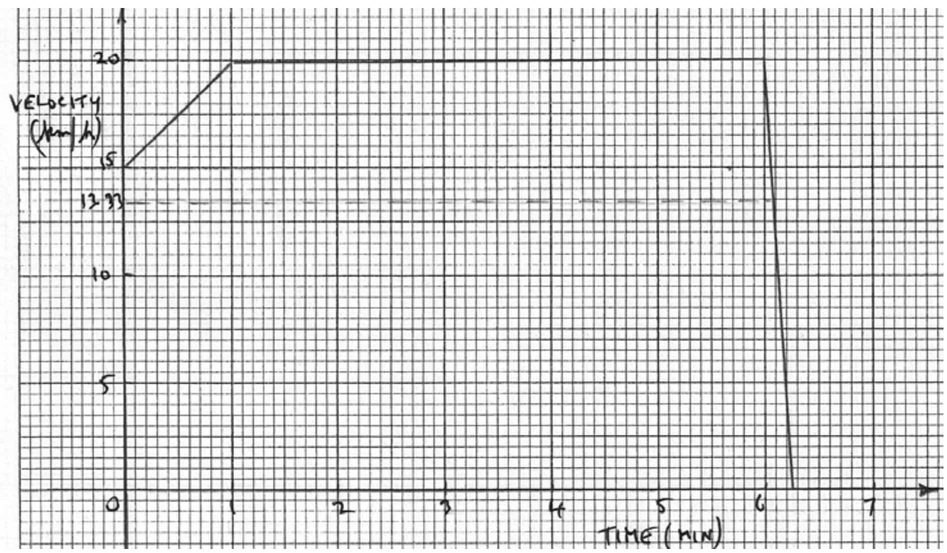
$$= \frac{\frac{36}{3.6}\,\text{m/s}}{0.2\,\text{m/s}^2} = \mathbf{50\ s}$$

2. A ship changes velocity from 15 km/h to 20 km/h in 25 min. Determine the average acceleration in m/s^2 of the ship during this time.

Average acceleration, a = $\frac{v-u}{t} = \frac{\frac{(20-15)}{3.6}\,\text{m/s}}{(25\times 60)\,\text{s}}$ = $\mathbf{9.26\times 10^{-4}\,m/s^2}$

3. A cyclist travelling at 15 km/h changes velocity uniformly to 20 km/h in 1 min, maintains this velocity for 5 min and then comes to rest uniformly during the next 15 s. Draw a velocity/time graph and hence determine the accelerations in m/s^2 (a) during the first minute, (b) for the next 5 minutes, and (c) for the last 10 s.

The velocity/time graph is shown below.



(a) **Average acceleration, a** = $\dfrac{v-u}{t} = \dfrac{(20-15)\,\text{km/h}}{60\,\text{s}} = \dfrac{\dfrac{(20-15)}{3.6}\,\text{m/s}}{1\,\text{min}} = \dfrac{\dfrac{5}{3.6}\,\text{m/s}}{60\,\text{s}}$ = $\mathbf{0.0231\,m/s^2}$

(b) Since the speed does not change then **the average acceleration is zero**

(c) **Average acceleration, a** = $\dfrac{v-u}{t} = \dfrac{(0-13.33)\,\text{km/h}}{10\,\text{s}} = \dfrac{\dfrac{(-13.33)}{3.6}\,\text{m/s}}{10\,\text{s}}$ = $\mathbf{-0.370\,m/s^2}$

4. Assuming uniform accelerations between points draw the velocity/time graph for the data given below, and hence determine the accelerations from A to B, B to C and C to D:

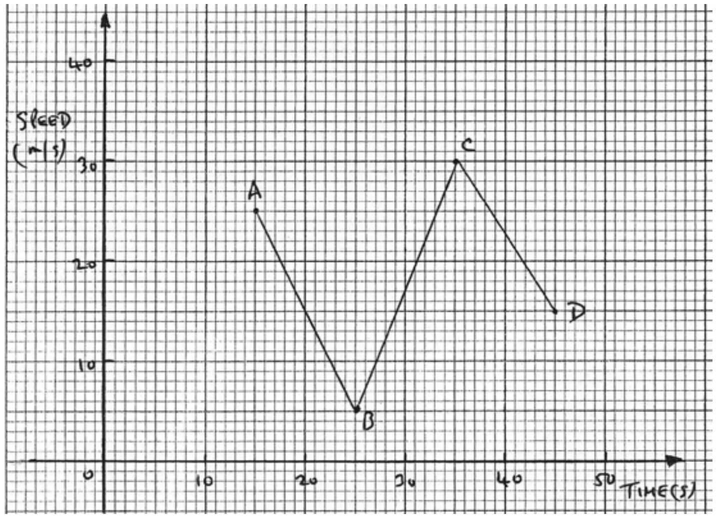
Point	A	B	C	D
Speed (m/s)	25	5	30	15
Time (s)	15	25	35	45

The velocity/time graph is shown below.

Acceleration from A to B = $\dfrac{\text{change in velocity}}{\text{time taken}} = \dfrac{(5-25)\,\text{m/s}}{(25-15)\,\text{s}}$ = $\mathbf{-2\ \ m/s^2}$

Acceleration from B to C = $\dfrac{\text{change in velocity}}{\text{time taken}} = \dfrac{(30-5)\,\text{m/s}}{(35-25)\,\text{s}} = \dfrac{25}{10}\,\text{m/s}^2$ = $\mathbf{2.5\ \ m/s^2}$

Acceleration from C to D = $\frac{\text{change in velocity}}{\text{time taken}} = \frac{(15-30)\,\text{m/s}}{(45-35)\,\text{s}} = \frac{-15}{10}\,\text{m/s}^2$ = **- 1.5 $\mathbf{m/s^2}$**



EXERCISE 71, Page 173

1. An object is dropped from the third floor of a building. Find its approximate velocity 1.25 s later if all forces except that of gravity are neglected.

Velocity after 1.25 s, v = u + at = 0 + (9.8)(1.25) = **12.25 m/s**

2. During free fall, a ball is dropped from point A and is travelling at 100 m/s when it passes point B. Calculate the time for the ball to travel from A to B if all forces except that of gravity are neglected

Velocity, v = u + at   i.e.   100 = 0 + 9.8(t)

from which, **time to travel from A to B, t** = $\frac{100\,m/s}{9.8\,m/s^2}$ = **10.2 s**

3. A piston moves at 10 m/s at the centre of its motion and decelerates uniformly at 0.8 m/s^2. Determine its velocity 3 s after passing the centre of its motion.

Velocity after 3 s, v = u + at = 10 + (- 0.8)(3) = **7.6 m/s**

4. The final velocity of a train after applying its brakes for 1.2 min is 24 km/h. If its uniform retardation is 0.06 m/s^2, find its velocity before the brakes are applied.

Velocity of train, v = u + at

Hence, $\frac{24}{3.6}$ = u + (- 0.06)(1.2 × 60)

i.e. $6.6\dot{6}$ = u – 4.32

from which, **final velocity, u** = 6.666 + 4.32 = $10.98\dot{6}$ m/s

$= 10.98\dot{6} \times 3.6$ km/h = **39.6 km/h**

5. A plane in level flight at 400 km/h starts to descend at a uniform acceleration of 0.6 m/s^2. It levels off when its velocity is 670 km/h. Calculate the time during which it is losing height.

Velocity of plane, v = u + at

Hence, $\frac{670}{3.6} = \frac{400}{3.6} + (0.6)(t)$

from which, **time during which it is losing height, t** = $\frac{\left(\frac{670}{3.6} - \frac{400}{3.6}\right) m/s}{0.6\, m/s^2}$ = 125 s = **2 min 5 s**

6. A lift accelerates from rest uniformly at 0.9 m/s^2 for 1.5 s, travels at constant velocity for 7 s and then comes to rest in 3 s. Determine its velocity when travelling at constant speed and its acceleration during the final 3 s of its travel.

Velocity, v = u + at from which, acceleration, a = $\frac{v - u}{t}$

i.e. $0.9 = \frac{v - 0}{1.5}$

from which, **velocity at constant speed, v** = (0.9)(1.5) = **1.35 m/s**

and **acceleration, a** = $\frac{v - u}{t} = \frac{0 - 1.35}{3}$ = $\mathbf{-\ 0.45\, m/s^2}$

EXERCISE 72, Page 173

Answers found from within the text of the chapter, pages 169 to 173.

EXERCISE 73, Page 173

1. (b)   **2.** (a)   **3.** (c)   **4.** (e)   **5.** (i)   **6.** (g)   **7.** (d)   **8.** (c)   **9.** (a)   **10.** (d)

CHAPTER 18 FORCE, MASS AND ACCELERATION

EXERCISE 74, Page 178

(Take g as 9.81 m/s^2, and express answers to three significant figure accuracy)

1. A car initially at rest, accelerates uniformly to a speed of 55 km/h in 14 s. Determine the accelerating force required if the mass of the car is 800 kg.

Initial velocity, $v_1 = 0$

Final velocity, $v_2 = 55\frac{km}{h} \times 1000\frac{m}{km} \times 1\frac{h}{3600\,s}$ = 15.278 m/s

Time, t = 14 s

Since $v_2 = v_1 + at$ then acceleration, $a = \frac{v_2 - v_1}{t} = \frac{15.278 - 0}{12} = 1.09\,m/s^2$

Hence, **accelerating force, F** = ma = $800\,kg \times 1.09\,m/s^2$ = **873 N**

2. The brakes are applied on the car in question 1 when travelling at 55 km/h and it comes to rest uniformly in a distance of 50 m. Calculate the braking force and the time for the car to come to rest.

Initial velocity, v_1 = 55 km/h = = 15.278 m/s (from above)

Final velocity, $v_2 = 0$

Distance travelled, s = 50 m

$v_2^2 = v_1^2 + 2as$

from which, acceleration, $a = \frac{v_2^2 - v_1^2}{2s} = \frac{0 - 15.278^2}{2 \times 50} = -2.33\,m/s^2$

Hence, **braking force, F** = ma = 800 kg × $2.334\,m/s^2$

= 1867 N = **1.87 kN**

Since $v_2 = v_1 + at$ then **time to come to rest, t** = $\frac{v_2 - v_1}{a} = \frac{15.278 - 0}{2.334}$ = **6.55 s**

3. The tension in a rope lifting a crate vertically upwards is 2.8 kN. Determine its acceleration if the mass of the crate is 270 kg.

$$T - mg = ma$$

i.e. $2800 - 270 \times 9.81 = 270 \times a$

from which, **acceleration, a** = $\frac{2800 - 270 \times 9.81}{270}$ = $\mathbf{0.560\ m/s^2}$

4. A ship is travelling at 18 km/h when it stops its engines. It drifts for a distance of 0.6 km and its speed is then 14 km/h. Determine the value of the forces opposing the motion of the ship, assuming the reduction in speed is uniform and the mass of the ship is 2000 t.

Initial velocity, v_1 = 18 km/h = $18 \frac{km}{h} \times 1000 \frac{m}{km} \times 1 \frac{h}{3600\,s} = \frac{18}{3.6}$ = 5 m/s

Final velocity, $v_2 = \frac{14}{3.6}$ = 3.889 m/s

Distance travelled, s = 0.6 km = 600 m

$v_2^2 = v_1^2 + 2as$

from which, acceleration, a = $\frac{v_2^2 - v_1^2}{2s} = \frac{3.889^2 - 5^2}{2 \times 600} = -\,8.23 \times 10^{-3}\,m/s^2$

Hence, **force opposing motion, F** = ma = $2000 \times 1000\,kg \times 8.23 \times 10^{-3}\,m/s^2$

= 16459 N = **16.46 kN**

5. A cage having a mass of 2 t is being lowered down a mineshaft. It moves from rest with an acceleration of 4 m/s^2, until it is travelling at 15 m/s. It then travels at constant speed for 700 m and finally comes to rest in 6 s. Calculate the tension in the cable supporting the cage during

(a) the initial period of acceleration, (b) the period of constant speed travel, (c) the final retardation period.

(a) **Initial tension in cable**, $T_1 = mg - ma = m(g - a) = 2000(9.81 - 4)$

$= 11620$ N = **11.62 kN**

(b) **Tension in cable during constant speed**, $T_2 = mg - m a_2 = mg - 0 = 2000 \times 9.81$

$= 19620$ N = **19.62 kN**

(c) **Tension in retardation period**, $T_3 = mg - m a_3$

where $a_3 = \dfrac{v-u}{t} = \dfrac{0-15}{6} = -2.5 \text{ m/s}^2$

Hence, **tension**, $T_3 = mg - m a_3 = 19620 - 2000(-2.5)$

$= 24620$ N = **24.62 kN**

EXERCISE 75, Page 180

1. Calculate the centripetal force acting on a vehicle of mass 1 tonne when travelling round a bend of radius 125 m at 40 km/h. If this force should not exceed 750 N, determine the reduction in speed of the vehicle to meet this requirement.

Centripetal acceleration = $\dfrac{v^2}{r}$

where v = $40\dfrac{\text{km}}{\text{h}} \times 1000\dfrac{\text{m}}{\text{km}} \times \dfrac{1\,\text{h}}{3600\,\text{s}}$ = 11.11 m/s and r = 125 m

Hence, centripetal acceleration, a = $\dfrac{11.11^2}{125}$ = 0.988 m/s^2

Centripetal force = ma = 1000 kg × 0.988 m/s^2 = **988 N**

If centripetal force ≤ 750 N ≤ ma_2

then $$a_2 = \frac{750\,\text{N}}{1000\,\text{kg}} = 0.75\,\text{m/s}^2 = \frac{v_2^{\,2}}{r}$$

i.e. $$v_2^{\,2} = 0.75 \times 125 \quad \text{and} \quad v_2 = \sqrt{0.75 \times 125} = 9.682\ \text{m/s}$$

$$9.682\ \text{m/s} = 9.682\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{1000\,\text{m}} \times \frac{3600\,\text{s}}{1\,\text{h}} = 34.86\ \text{km/h}$$

Hence the speed reduces form 40 km/h to 34.86 km/h

2. A speed-boat negotiates an S-bend consisting of two circular arcs of radii 100 m and 150 m. If the speed of the boat is constant at 34 km/h, determine the change in acceleration when leaving one arc and entering the other.

34 km/h = $34\dfrac{\text{km}}{\text{h}} \times \dfrac{1000\,\text{m}}{1\,\text{km}} \times \dfrac{1\,\text{h}}{3600\,\text{s}}$ = 9.444 m/s

Acceleration, $a_1 = \frac{v^2}{r_1} = \frac{9.444^2}{100} = 0.891\,m/s^2$

Acceleration, $a_2 = \frac{v^2}{r_2} = \frac{9.444^2}{150} = 0.595\,m/s^2$

Change of acceleration = $a_1 - a_2 = 0.891 - 0.595 = 0.296\,m/s^2$

i.e. **change in acceleration = $\mathbf{0.3\,m/s^2}$**

3. An object is suspended by a thread 400 mm long and both object and thread move in a horizontal circle with a constant angular velocity of 3.0 rad/s. If the tension in the thread is 36 N, determine the mass of the object.

Centripetal force (i.e. tension in thread) = $\frac{mv^2}{r} = 36$ N

The angular velocity, $\omega = 3.0$ rad/s and radius, r = 400 mm = 0.4 m.

Since linear velocity $v = \omega r$, $v = 3.0 \times 0.4 = 1.2$ m/s, and since $F = \frac{mv^2}{r}$, then $m = \frac{Fr}{v^2}$

i.e. **mass of object, m** = $\frac{36 \times 0.4}{1.2^2}$ = **10 kg**

EXERCISE 76, Page 180

Answers found from within the text of the chapter, pages 175 to 180.

EXERCISE 77, Page 180

1. (c) **2.** (b) **3.** (a) **4.** (d) **5.** (a) **6.** (b) **7.** (b) **8.** (a) **9.** (a) **10.** (d) **11.** (d) **12.** (c)

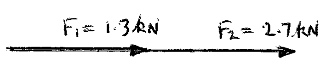
CHAPTER 19 FORCES ACTING AT A POINT

EXERCISE 78, Page 187

1. Determine the magnitude and direction of the resultant of the forces 1.3 kN and 2.7 kN, having the same line of action and acting in the same direction.

The vector diagram of the two forces acting in the same direction is shown in the diagram below, which assumes that the line of action is horizontal. The resultant force F is given by:

$F = F_1 + F_2$ i.e. $\mathbf{F} = (1.3 + 2.7)$ kN = **4.0 kN** in the direction of the original forces.



2. Determine the magnitude and direction of the resultant of the forces 470 N and 538 N having the same line of action but acting in opposite directions.

The vector diagram of the two forces acting in opposite directions is shown in the diagram below.

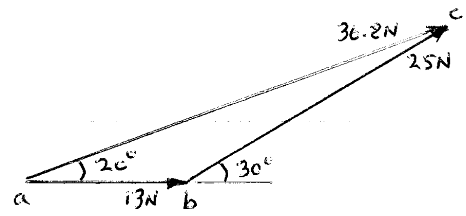
The resultant force F is given by: $F = F_2 - F_1$ i.e. $\mathbf{F} = (538 - 470)$ N = **68 N** in the direction of the 538 N force.

3. Use the triangle of forces method to determine the magnitude and direction of the resultant of the forces 13 N at 0° and 25 N at 30°

With reference to the diagram shown below:

(i) **ab** is drawn 13 units long horizontally

(ii) From b, **bc** is drawn 25 units long, inclined at an angle of 30° to ab.



(iii) By measurement, the resultant **ac** is 36.8 units long inclined at an angle of 20° to ab. That is,

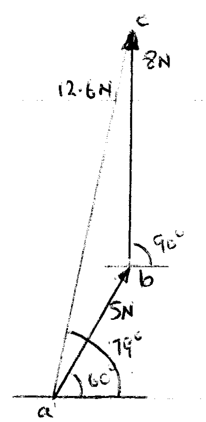
the resultant force is **36.8 N**, inclined at an angle of **20°** to the 13 N force.

4. Use the triangle of forces method to determine the magnitude and direction of the resultant of the forces 5 N at 60° and 8 N at 90°

With reference to the diagram shown below:

(i) **ab** is drawn 5 units long at 60° to the horizontal

(ii) From b, **bc** is drawn 8 units long, inclined at an angle of 90° to the horizontal.



(iii) By measurement, the resultant **ac** is 12.6 units long inclined at an angle of 79° to the horizontal.

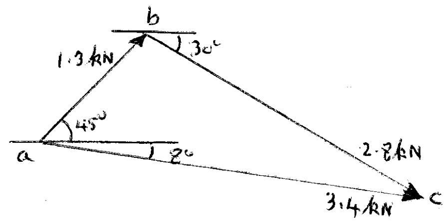
That is, the resultant force is **12.6 N**, inclined at an angle of **79°** to the horizontal.

5. Use the triangle of forces method to determine the magnitude and direction of the resultant of the forces 1.3 kN at 45° and 2.8 kN at - 30°

With reference to the diagram shown below:

(i) **ab** is drawn 1.3 units long at 45° to the horizontal

(ii) From b, **bc** is drawn 2.8 units long, inclined at an angle of - 30° to the horizontal



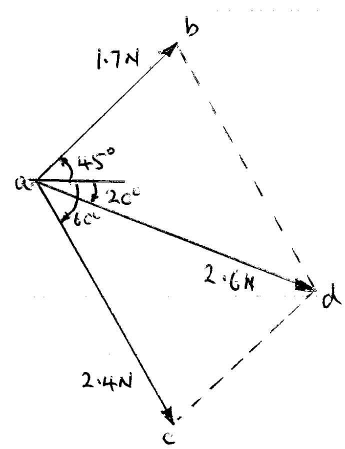
(iii) By measurement, the resultant **ac** is 3.4 units long inclined at an angle of - 8° to ab. That is, the resultant force is **3.4 kN**, inclined at an angle of **- 8°** to the horizontal.

EXERCISE 79, Page 188

1. Use the parallelogram of forces method to determine the magnitude and direction of the resultant of the forces 1.7 N at 45° and 2.4 N at - 60°

With reference to the diagram below:

(i) **ab** is drawn at an angle of 45° and 1.7 units in length



(ii) **ac** is drawn at an angle of - 60° and 2.4 units in length

(iii) **bd** and **cd** are drawn to complete the parallelogram

(iv) **ad** is drawn. By measurement, **ad** is 2.6 units long at an angle of - 20°.

That is, the resultant force is **2.6 N** at an angle of **- 20°**

2. Use the parallelogram of forces method to determine the magnitude and direction of the resultant of the forces 9 N at 126° and 14 N at 223°

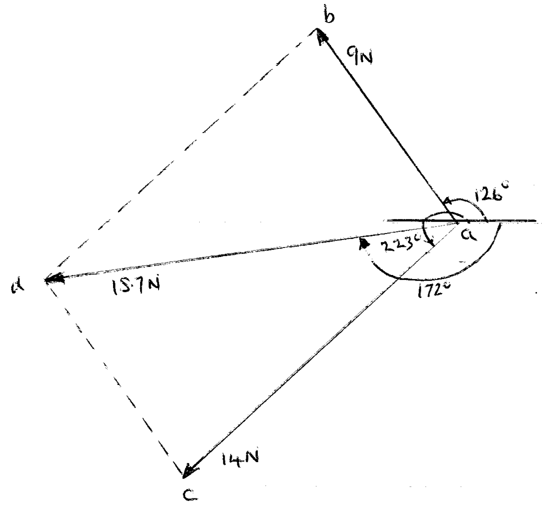
With reference to the diagram below:

(i) **ab** is drawn at an angle of 126° and 9 units in length

(ii) **ac** is drawn at an angle of 223° and 14 units in length

(iii) **bd** and **cd** are drawn to complete the parallelogram

(iv) **ad** is drawn. By measurement, **ad** is 15.7 units long at an angle of **-** 172°.

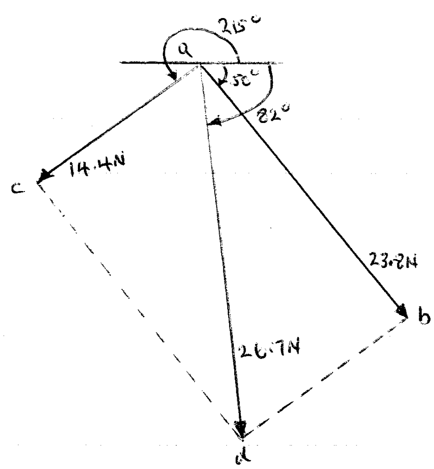


That is, the resultant force is **15.7 N** at an angle of **- 172°**

3. Use the parallelogram of forces method to determine the magnitude and direction of the resultant of the forces 23.8 N at **-** 50° and 14.4 N at 215°

With reference to the diagram below:

(i) **ab** is drawn at an angle of **-** 50° and 23.8 units in length



(ii) **ac** is drawn at an angle of 215° and 14.4 units in length

(iii) **bd** and **cd** are drawn to complete the parallelogram

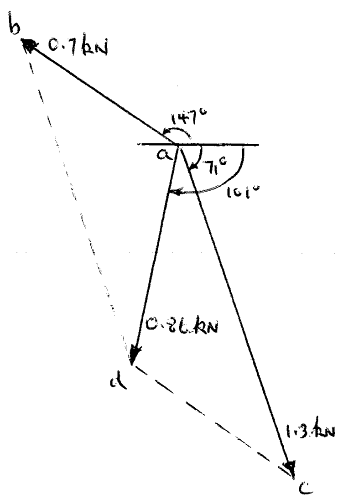
(iv) **ad** is drawn. By measurement, **ad** is 26.7 units long at an angle of - 82°.

That is, the resultant force is **26.7 N** at an angle of **- 82°**

4. Use the parallelogram of forces method to determine the magnitude and direction of the resultant of the forces 0.7 kN at 147° and 1.3 kN at - 71°

With reference to the diagram below:

(i) **ab** is drawn at an angle of 147° and 0.7 units in length



(ii) **ac** is drawn at an angle of - 71° and 1.3 units in length

(iii) **bd** and **cd** are drawn to complete the parallelogram

(iv) **ad** is drawn. By measurement, **ad** is 0.86 units long at an angle of - 101°.

That is, the resultant force is **0.86 kN** at an angle of **- 101°**

5. Use the parallelogram of forces method to determine the magnitude and direction of the resultant of the forces 47 N at 79° and 58 N at 247°

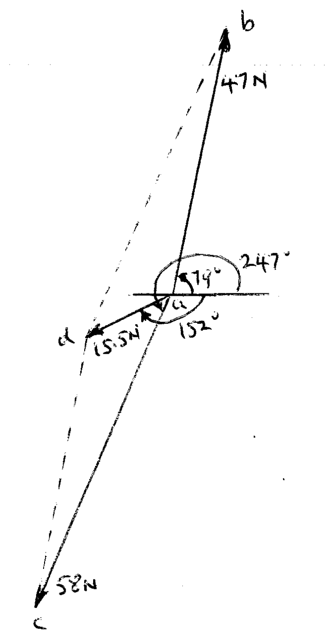
With reference to the diagram below:

(i) **ab** is drawn at an angle of 79° and 47 units in length

(ii) **ac** is drawn at an angle of 247° and 58 units in length

(iii) **bd** and **cd** are drawn to complete the parallelogram

(iv) **ad** is drawn. By measurement, **ad** is 15.5 units long at an angle of - 152°.

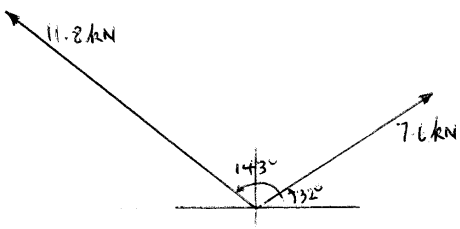


That is, the resultant force is **15.5 N** at an angle of **- 152°**

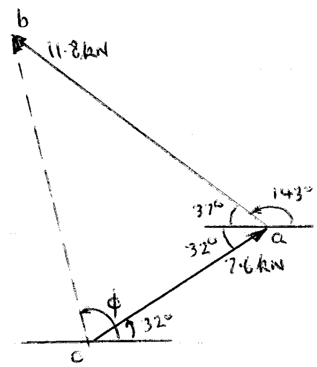
EXERCISE 80, Page 188

1. Forces of 7.6 kN at 32° and 11.8 kN at 143° act at a point. Use the cosine and sine rules to calculate the magnitude and direction of their resultant.

The space diagram is shown in diagram (a) below. A sketch is made of the vector diagram, **oa** representing the 7.6 kN force in magnitude and direction and **ab** representing the 11.8 kN force in



(a)



(b)

magnitude and direction as shown in diagram (b). The resultant is given by length **ob**. By the cosine rule, $ob^2 = oa^2 + ab^2 - 2(oa)(ab)\cos\angle oab$

$$= 7.6^2 + 11.8^2 - 2(7.6)(11.8)\cos(37° + 32°)$$

$$= 57.76 + 139.24 - (64.2769) = 132.723$$

Hence, $ob = \sqrt{132.723} = 11.52$ kN

By the sine rule, $\dfrac{11.8}{\sin\angle aob} = \dfrac{11.52}{\sin 69°}$

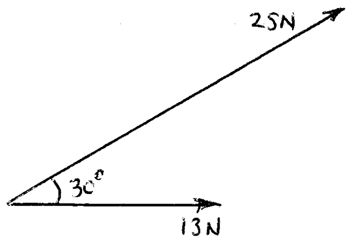
from which, $\sin\angle aob = \dfrac{11.8\sin 69°}{11.52} = 0.95267$

Hence $\angle aob = \sin^{-1}(0.95267) = 73°$. Thus angle ϕ in Figure 4.11(b) is $73° + 32° = 105°$

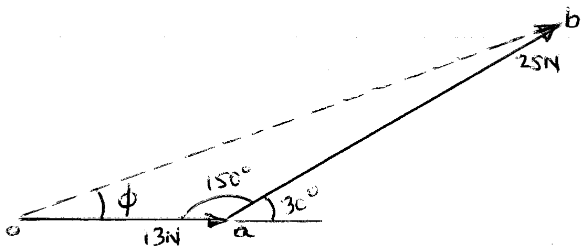
Hence the resultant of the two forces is 11.52 kN acting at an angle of 105° to the horizontal

2. Calculate the resultant of the forces 13 N at 0° and 25 N at 30° by using the cosine and sine rules.

The space diagram is shown in diagram (a). A sketch is made of the vector diagram, **oa** representing the 13 N force in magnitude and direction and **ab** representing the 25 N force in magnitude and



(a)



(b)

direction as shown in diagram (b). The resultant is given by length **ob**. By the cosine rule,

$$ob^2 = oa^2 + ab^2 - 2(oa)(ab)\cos\angle oab$$

$$= 13^2 + 25^2 - 2(13)(25)\cos 150°$$

$$= 169 + 625 - (-562.92) = 1356.917$$

Hence, $ob = \sqrt{1356.917} = 36.84$ N

By the sine rule, $\dfrac{25}{\sin\phi} = \dfrac{36.84}{\sin 150°}$

from which, $\sin\phi = \dfrac{25\sin 150°}{36.84} = 0.339305$

Hence, $\phi = \sin^{-1}(0.339305) = 19.83°$.

Hence, the resultant of the two forces is 36.84 kN acting at an angle of 19.83° to the horizontal

3. Calculate the resultant of the forces 1.3 kN at 45° and 2.8 kN at - 30° by using the cosine and sine rules.

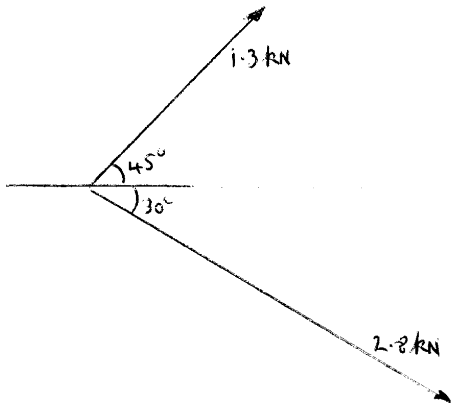
The space diagram is shown in diagram (a). A sketch is made of the vector diagram, **oa** representing the 1.3 kN force in magnitude and direction and **ab** representing the 2.8 kN force in magnitude and direction as shown in diagram (b). The resultant is given by length **ob**. By the cosine rule,

$$0b^2 = 1.3^2 + 2.8^2 - 2(1.3)(2.8)\cos\angle oab$$

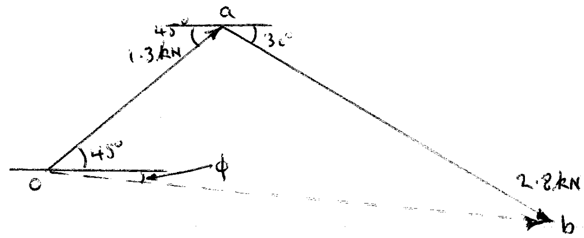
$= 1.3^2 + 2.8^2 - 2(1.3)(2.8)\cos(180° - 45° - 30°)$

$= 1.69 + 7.84 - (-1.8842) = 11.4142$

Hence, $ob = \sqrt{11.4142} = 3.378$ kN



(a)



(b)

By the sine rule, $\dfrac{2.8}{\sin\angle aob} = \dfrac{3.378}{\sin 105°}$

from which, $\sin\angle aob = \dfrac{2.8\sin 105°}{3.378} = 0.8006$

Hence $\angle aob = \sin^{-1}(0.8006) = 53.19°$. Thus angle ϕ in diagram (b) is $53.19° - 45° = 8.19°$

Hence, the resultant of the two forces is 3.38 kN acting at an angle of - 8.19° to the horizontal

4. Calculate the resultant of the forces 9 N at 126° and 14 N at 223° by using the cosine and sine rules.

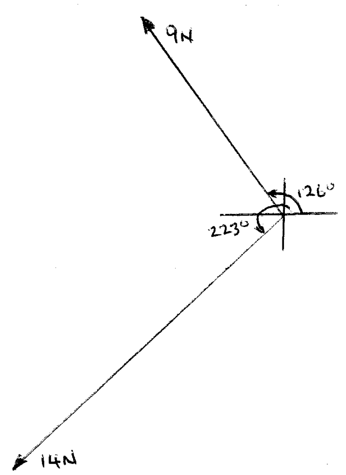
The space diagram is shown in diagram (a). A sketch is made of the vector diagram, **oa** representing the 9 N force in magnitude and direction and **ab** representing the 14 N force in magnitude and direction as shown in diagram (b). The resultant is given by length **ob**. By the cosine rule,

$ob^2 = oa^2 + ab^2 - 2(oa)(ab)\cos\angle oab$

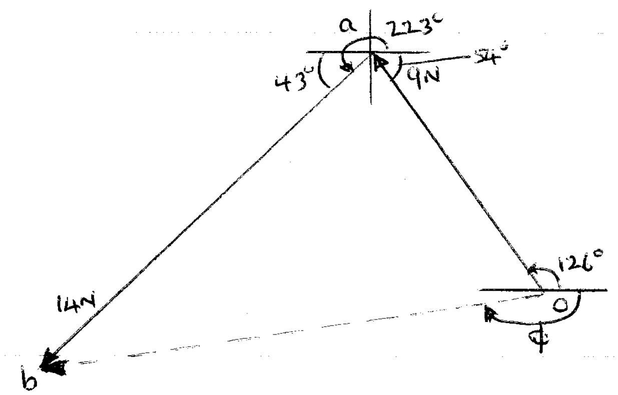
$= 9^2 + 14^2 - 2(9)(14)\cos(180° - 43° - 54°)$

$= 81 + 196 - (30.711) = 246.2889$

Hence, ob = $\sqrt{246.2889}$ = 15.69 N



(a)



(b)

By the sine rule, $\dfrac{14}{\sin\angle aob} = \dfrac{15.69}{\sin 83°}$

from which, $\sin\angle aob = \dfrac{14\sin 83°}{15.69} = 0.88564$

Hence, $\angle aob = \sin^{-1}(0.88564) = 62.33°$. Thus angle ϕ in diagram (b) is 180° - (62.33° - 54°) = 171.67°

Hence, the resultant of the two forces is 15.69 N acting at an angle of - 171.67° to the horizontal

5. Calculate the resultant of the forces 0.7 kN at 147° and 1.3 kN at - 71° by using the cosine and sine rules.

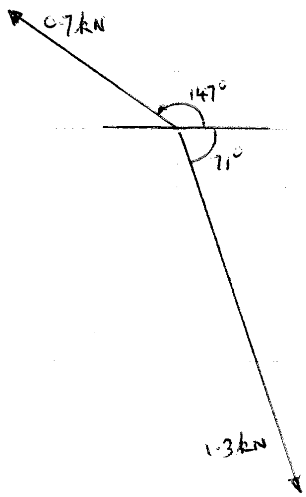
The space diagram is shown in diagram (a). A sketch is made of the vector diagram, **oa** representing the 0.7 kN force in magnitude and direction and **ab** representing the 1.3 kN force in magnitude and direction as shown in diagram (b). The resultant is given by length **ob**. By the cosine rule,

$$ob^2 = oa^2 + ab^2 - 2(oa)(ab)\cos\angle 0ab$$

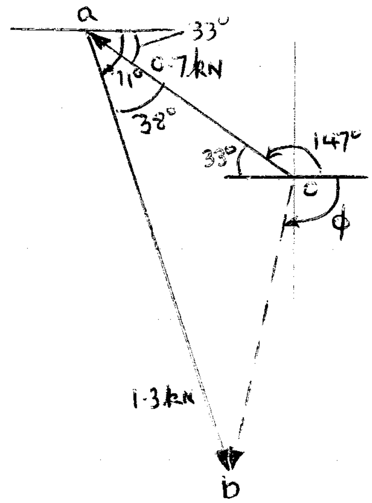
$$= 0.7^2 + 1.3^2 - 2(0.7)(1.3)\cos 38°$$

$= 0.49 + 1.69 - (1.43418) = 0.74582$

Hence, $\text{ob} = \sqrt{0.74582} = 0.8636$ kN



(a)



(b)

By the sine rule, $\dfrac{1.3}{\sin \angle \text{aob}} = \dfrac{0.8636}{\sin 38°}$

from which, $\sin \angle \text{aob} = \dfrac{1.3 \sin 38°}{0.8636} = 0.926772$

Hence $\angle \text{aob} = \sin^{-1}(0.926772) = 67.94°$ or $112.06°$. In this case, the latter answer is seen to be the correct one. Thus angle ϕ in diagram (b) is $180° - (112.06° - 33°) = 100.94°$

Hence, the resultant of the two forces is 0.86 kN acting at an angle of - 100.94° to the horizontal

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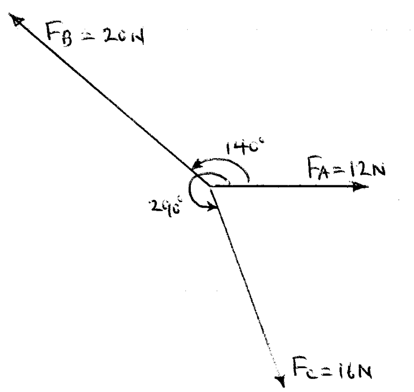
EXERCISE 81, Page 190

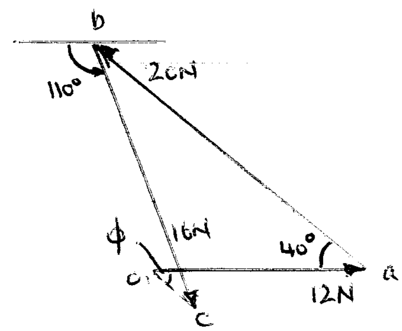
1. Determine graphically the magnitude and direction of the resultant of the following coplanar forces given which are acting at a point: Force A, 12 N acting horizontally to the right, force B, 20 N acting at 140° to force A, force C, 16 N acting 290° to force A.

The space diagram is shown in diagram (a). The vector diagram shown in diagram (b), is produced as follows:

(i) **oa** represents the 12 N force in magnitude and direction



(a)



(b)

(ii) From the nose of **oa**, **ab** is drawn inclined at 140° to **oa** and 20 units long

(iii) From the nose of **ab**, **bc** is drawn 16 units long inclined at 290° to **oa** (i.e. 110° to the horizontal)

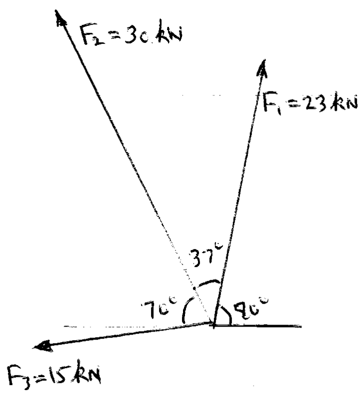
(iv) **oc** represents the resultant; by measurement, the resultant is 3.1 N inclined at ϕ = 45° to the horizontal.

Thus the resultant of the three forces, F_A, F_B and F_C is a force of 3.1 N at - 45° to the horizontal.

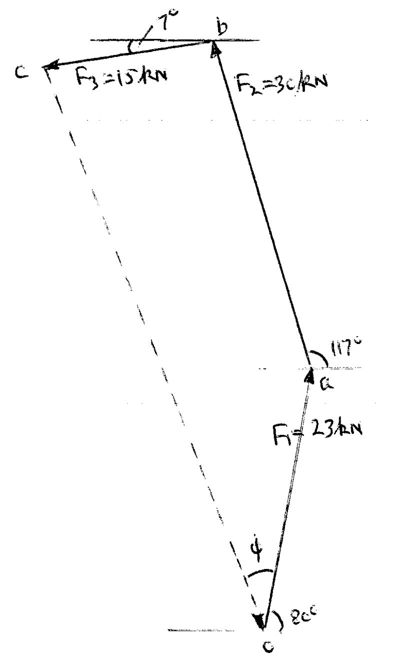
2. Determine graphically the magnitude and direction of the resultant of the following coplanar forces given which are acting at a point: Force 1, 23 kN acting at 80° to the horizontal, force 2, 30 kN acting at 37° to force 1, force 3, 15 kN acting at 70° to force 2.

The space diagram is shown in diagram (a). The vector diagram shown in diagram (b), is produced as follows:

(i) **oa** represents the 23 kN force in magnitude and direction



(a)



(b)

(ii) From the nose of **oa**, **ab** is drawn inclined at 117° to **oa** and 30 units long

(iii) From the nose of **ab**, **bc** is drawn 15 units long inclined at 187° to **oa** (i.e. - 7° to the horizontal)

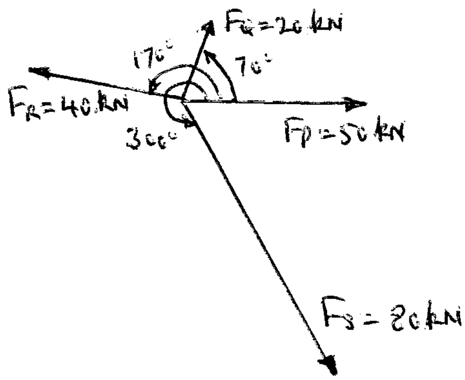
(iv) **oc** represents the resultant; by measurement, the resultant is 53.5 kN inclined at φ = 37° to force 1, i.e. 117° to the horizontal.

Thus the resultant of the three forces, F_1, F_2 and F_3 is a force of 53.5 kN at 117° to the horizontal.

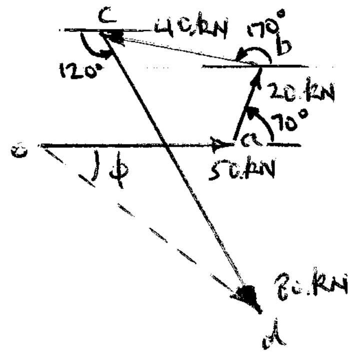
3. Determine graphically the magnitude and direction of the resultant of the following coplanar forces given which are acting at a point: Force P, 50 kN acting horizontally to the right, force Q, 20 kN at 70° to force P, force R, 40 kN at 170° to force P, force S, 80 kN at 300° to force P.

The space diagram is shown in diagram (a). The vector diagram shown in diagram (b), is produced as follows:

(i) **oa** represents the 50 kN force in magnitude and direction



(a)



(b)

(ii) From the nose of **oa**, **ab** is drawn inclined at 70° to **oa** and 20 units long

(iii) From the nose of **ab**, **bc** is drawn 40 units long inclined at 170° to **oa**

(iv) From the nose of **bc**, **cd** is drawn 80 units long inclined at 300° to **oa**

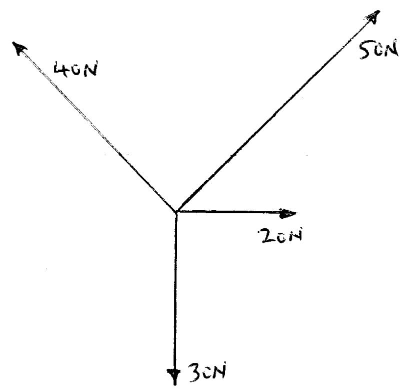
(v) **od** represents the resultant; by measurement, the resultant is 72 kN inclined at $\phi = 37°$ to the horizontal.

Thus the resultant of the three forces, F_P, F_Q, F_R and F_S is a force of 72 kN at - 37° to the horizontal (i.e. to force P).

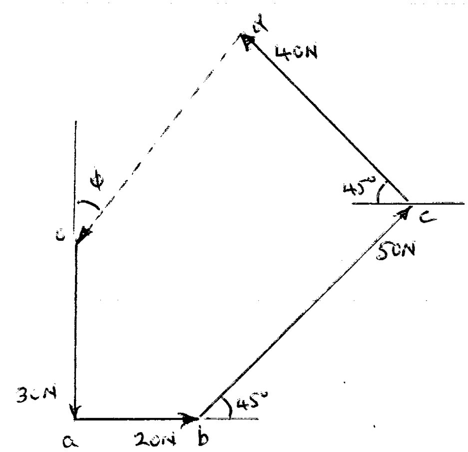
4. Four horizontal wires are attached to a telephone pole and exert tensions of 30 N to the south, 20 N to the east, 50 N to the north-east and 40 N to the north-west. Determine the resultant force on the pole and its direction.

The four forces are shown in the space diagram of diagram (a). The vector diagram is shown in diagram (b), **oa** representing the 30 N force, **ab** representing the 20N force, **bc** the 50 N force, and **cd** the 40 N force. The resultant, **od**, is found by measurement to represent a force of 43.2 N and angle ϕ is 39°.

Thus, the four forces may be represented by a single force of 43.2 N at 39° east of north.



(a)



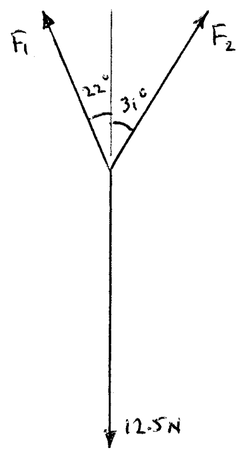
(b)

EXERCISE 82, Page 192

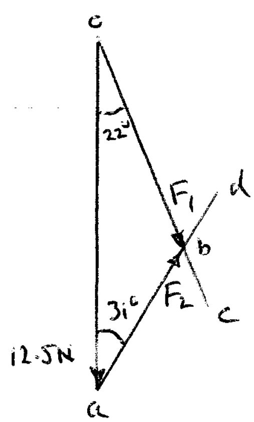
1. A load of 12.5 N is lifted by two strings connected to the same point on the load, making angles of 22° and 31° on opposite sides of the vertical. Determine the tensions in the strings.

The space diagram is shown in diagram (a). Since the system is in equilibrium, the vector diagram must close. The vector diagram, shown in diagram (b), is drawn as follows:

(i) The load of 200 N is drawn vertically as shown by **oa**



(a)



(b)

(ii) The direction only of force F_1 is known, so from point o, **ob** is drawn at 22° to the vertical

(iii) The direction only of force F_2 is known, so from point a, **ab** is drawn at 35° to the vertical

(iv) Lines **ob** and **ab** cross at point b; hence the vector diagram is given by triangle oab. By measurement, **ab** is 5.9 N and **ob** is 8 N.

By calculation, using the sine rule: $\dfrac{12.5}{\sin(180° - 31° - 22°)} = \dfrac{F_1}{\sin 31°}$

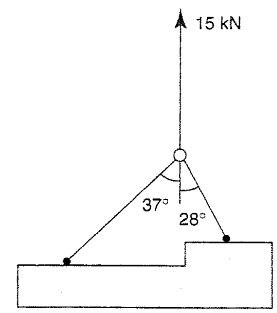
from which, $F_1 = \dfrac{12.5 \sin 31°}{\sin 127°}$ = **8.06 N**

and $\dfrac{12.5}{\sin(180° - 31° - 22°)} = \dfrac{F_2}{\sin 22°}$

from which, $F_2 = \dfrac{12.5 \sin 22°}{\sin 127°}$ = **5.86 N**

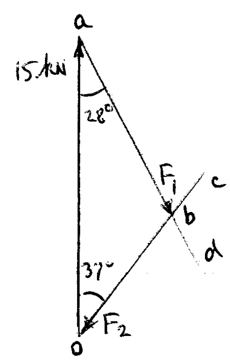
Thus the tensions in the ropes are F_1 = 8.06 N and F_2 = 5.86 N

2. A two-legged sling and hoist chain used for lifting machine parts is shown below. Determine the forces in each leg of the sling if parts exerting a downward force of 15 kN are lifted.



The space diagram is shown above. Since the system is in equilibrium, the vector diagram must close. The vector diagram, shown below, is drawn as follows:

(i) The load of 15 kN is drawn vertically as shown by **oa**



(ii) The direction only of force F_1 is known, so from point a, **ad** is drawn at 28° to the vertical

(iii) The direction only of force F_2 is known, so from point o, **oc** is drawn at 37° to the vertical

(iv) Lines **ad** and **oc** cross at point b; hence the vector diagram is given by triangle oab. By measurement, **ab** is 10 kN and **ob** is 7.8 kN.

By calculation, using the sine rule: $\dfrac{15}{\sin(180° - 28° - 37°)} = \dfrac{F_1}{\sin 37°}$

from which, $F_1 = \dfrac{15\sin 37°}{\sin 115°}$ = **9.96 kN**

and $\dfrac{15}{\sin 115°} = \dfrac{F_2}{\sin 28°}$

from which, $F_2 = \dfrac{15\sin 28°}{\sin 115°}$ = **7.77 kN**

Thus the tensions in the ropes are F_1 = 9.96 kN and F_2 = 7.77 kN

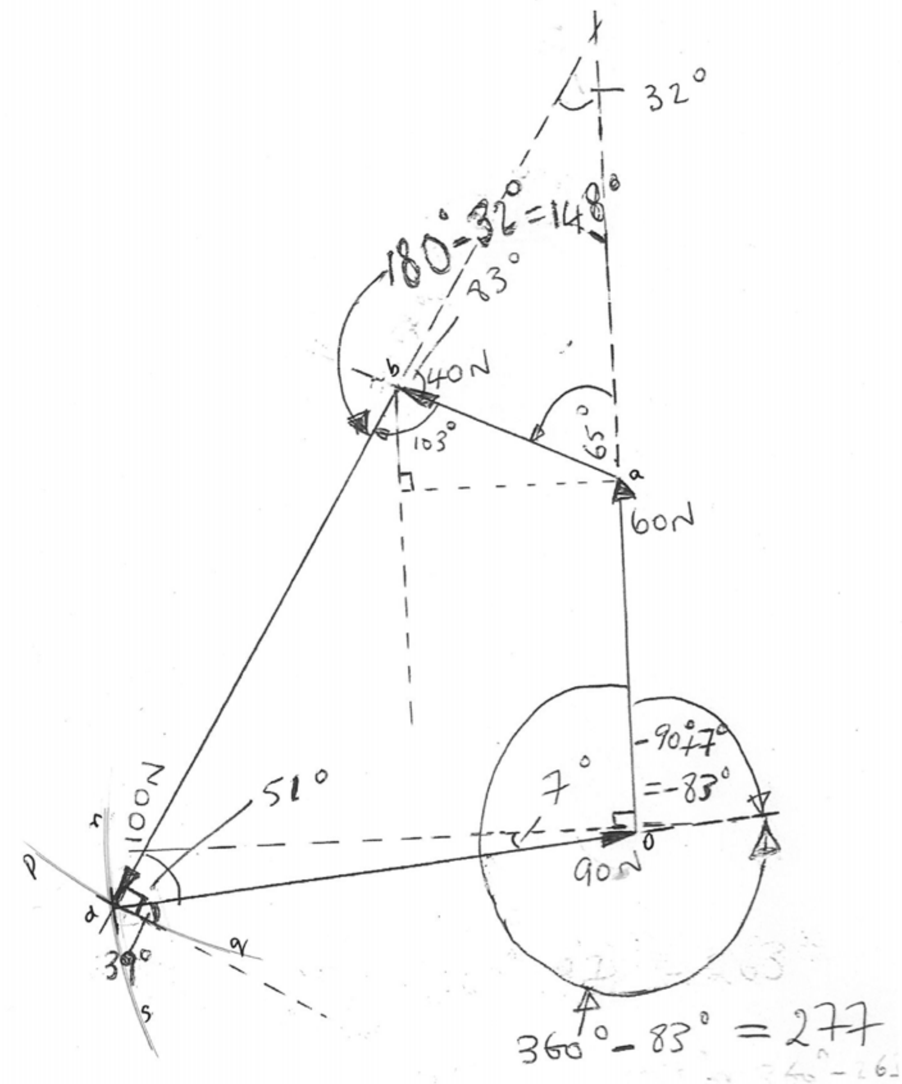
3. Four coplanar forces acting on a body are such that it is in equilibrium. The vector diagram for the forces is such that the 60 N force acts vertically upwards, the 40 N force acts at 65° to the 60 N force, the 100 N force acts from the nose of the 40 N force and the 90 N force acts from the nose of the 100 N force. Determine the direction of the 100 N and 90 N forces relative to the 60 N force.

With reference to the diagram below, **0a** is drawn 60 units long vertically upwards. From point a, **ab** is drawn 40 units long at an angle of 65° to the 60 N force.

The direction of the 100 N force is un known, thus arc pq is drawn with a compass, with centre at b, radius 100 units.

Since the forces are at equilibrium, the polygon of forces must close. Using a compass with centre at 0, arc rs is drawn having a radius 90 units. The point where the arcs intersect is at d.

By measurement, the 100 N force is at an angle of 148° to the 60 N force, and the 90 N force is at an angle of 277° to the 60 N force.


32°
180°-32°=148°
83°
b
40N
103°
65°
a
60N
100N
51°
7°
-90°+7°
=-83°
r
p
d
q
39°
s
90N
o
360°-83° = 277

EXERCISE 83, Page 195

1. Resolve a force of 23.0 N at an angle of 64° into its horizontal and vertical components.

Horizontal component = 23.0 cos 64° = **10.08 N**

Vertical component = 23.0 sin 64° = **20.67 N**

2. Forces of 5 N at 21° and 9 N at 126° act at a point. By resolving these forces into horizontal and vertical components, determine their resultant.

The horizontal component of the 5 N force = 5 cos 21° = 4.6679

and the vertical component of the 5 N force = 5 sin 21° = 1.7918

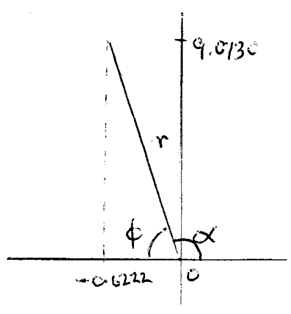
The horizontal component of the 9 N force = 9 cos 126° = - 5.2901

and the vertical component of the 9 N force = 9 sin 126° = 7.2812

Total horizontal component = 4.6679 + (- 5.2901) = - 0.6222

Total vertical component = 1.7918 + 7.2812 = 9.0730

The components are shown sketched in the diagram.



By Pythagoras' theorem, r = $\sqrt{0.6222^2 + 9.0730^2}$ = 9.09,

and by trigonometry, angle ϕ = $\tan^{-1}\dfrac{9.0730}{0.6222}$ = 86.08°

from which, α = 180° - 86.08° = 93.92°

Hence the resultant of the two forces is a force of 9.09 N acting at 93.92° to the horizontal.

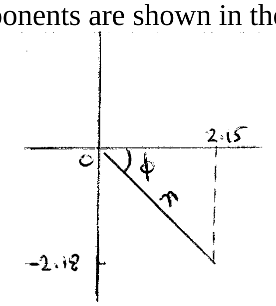
3. Determine the magnitude and direction of the resultant of the following coplanar forces which are acting at a point, by resolution of forces: Force A, 12 N acting horizontally to the right, force B, 20 N acting at 140° to force A, force C, 16 N acting 290° to force A.

A tabular approach using a calculator may be made as shown below:

	Horizontal component
Force A	12 cos 0° = 12.00
Force B	20 cos 140° = - 15.32
Force C	16 cos 290° = 5.47
Total horizontal component	= 2.15

	Vertical component
Force A	12 sin 0° = 0
Force B	20 sin 140° = 12.86
Force C	16 sin 290° = - 15.04
Total vertical component	= - 2.18

The total horizontal and vertical components are shown in the diagram.



Resultant r = $\sqrt{2.15^2 + 2.18^2}$ = 3.06, and

angle $\phi = \tan^{-1}\dfrac{2.18}{2.15}$ = 45.40°

Thus the resultant of the three forces given is 3.06 N acting at an angle of – 45.40° to force A.

4. Determine the magnitude and direction of the resultant of the following coplanar forces which are acting at a point, by resolution of forces: Force 1, 23 kN acting at 80° to the horizontal, force

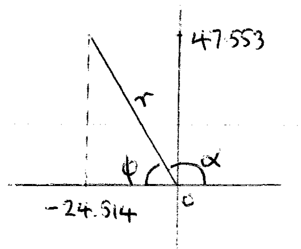
2, 30 kN acting at 37° to force 1, force 3, 15 kN acting at 70° to force 2.

A tabular approach using a calculator may be made as shown below:

	Horizontal component	
Force 1	23 cos 80° = 3.994	
Force 2	30 cos 117° = - 13.620	(Note that force 2 is at 80° + 37° = 117° to the horizontal)
Force 3	15 cos 187° = - 14.888	(Note that force 3 is at 80° + 37° + 70° = 187° to the horizontal)
Total horizontal component	= - 24.514	

	Vertical component
Force 1	23 sin 80° = 22.651
Force 2	30 sin 117° = 26.730
Force 3	15 sin 187° = - 1.828
Total vertical component	= 47.553

The total horizontal and vertical components are shown in the diagram.



Resultant r = $\sqrt{24.514^2 + 47.553^2}$ = 53.50, and

angle $\phi = \tan^{-1}\dfrac{47.553}{24.514} = 62.73°$

from which, $\alpha = 180° - 62.73° = 117.27°$

Thus the resultant of the three forces given is 53.50 kN acting at an angle of 117.27° to the horizontal.

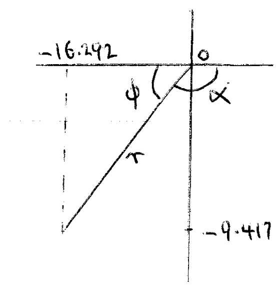
5. Determine, by resolution of forces, the resultant of the following three coplanar forces acting at a point: 10 kN acting at 32° to the horizontal, 15 kN acting at 170° to the horizontal; 20 kN acting at 240° to the horizontal.

A tabular approach using a calculator may be made as shown below:

	Horizontal component
Force 1	$10 \cos 32° = 8.480$
Force 2	$15 \cos 170° = -14.772$
Force 3	$20 \cos 240° = \underline{-10.000}$
Total horizontal component	$= \underline{-16.292}$

	Vertical component
Force 1	$10 \sin 32° = 5.299$
Force 2	$15 \sin 170° = 2.605$
Force 3	$20 \sin 240° = \underline{-17.321}$
Total vertical component	$= \underline{-9.417}$

The total horizontal and vertical components are shown in the diagram.



Resultant r = $\sqrt{16.292^2 + 9.417^2}$ = 18.82, and

angle $\phi = \tan^{-1}\dfrac{9.417}{16.292} = 30.03°$

from which, $\alpha = 180° - 30.03° = 149.97°$

Thus the resultant of the three forces given is 18.82 kN acting at an angle of -149.97° or 210.03° to the horizontal.

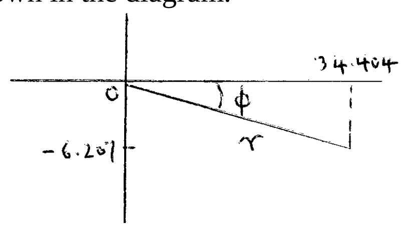
6. The following coplanar forces act at a point: force A, 15 N acting horizontally to the right, force B, 23 N at 81° to the horizontal, force C, 7 N at 210° to the horizontal, force D, 9 N at 265° to the horizontal, and force E, 28 N at 324° to the horizontal. Determine the resultant of the five forces by resolution of the forces.

A tabular approach using a calculator may be made as shown below:

	Horizontal component
Force A	15 cos 0° = 15.000
Force B	23 cos 81° = 3.598
Force C	7 cos 210° = - 6.062
Force D	9 cos 265° = - 0.784
Force E	28 cos 324° = 22.652
Total horizontal component	= 34.404

	Vertical component
Force A	15 sin 0° = 0.000
Force B	23 sin 81° = 22.717
Force C	7 sin 210° = - 3.500
Force D	9 sin 265° = - 8.966
Force E	28 sin 324° = - 16.458
Total vertical component	= - 6.207

The total horizontal and vertical components are shown in the diagram.



Resultant r = $\sqrt{34.404^2 + 6.207^2}$ = 34.96, and

angle $\phi = \tan^{-1}\dfrac{6.207}{34.404} = 10.23°$

Thus the resultant of the five forces given is 34.96 N acting at an angle of – 10.23° to force A.

EXERCISE 84, Page 195

Answers found from within the text of the chapter, pages 183 to 194.

12. 15 N acting horizontally to the right **13.** 10 N at 230° **15.** 5 N at 45°

EXERCISE 85, Page 196

1. (b) **2.** (a) **3.** (b) **4.** (d) **5.** (b) **6.** (c) **7.** (b) **8.** (b) **9.** (c) **10.** (d) **11.** (c) **12.** (d)

13. (d) **14.** (a)

CHAPTER 2 FRACTIONS, DECIMALS AND PERCENTAGES

EXERCISE 5, Page 10

1. Change the improper fraction $\frac{15}{7}$ into a mixed number.

$\frac{15}{7} = \mathbf{2\frac{1}{7}}$ as a mixed number

2. Change the mixed number $2\frac{4}{9}$ into an improper fraction.

$2\frac{4}{9} = 2 + \frac{4}{9}$ and $2 \equiv \frac{18}{9}$ hence, $2\frac{4}{9} = \frac{18}{9} + \frac{4}{9} = \mathbf{\frac{22}{9}}$ as an improper fraction

3. A box contains 165 paper clips. 60 clips are removed from the box. Express this as a fraction in its simplest form.

$\frac{60}{165} = \frac{12}{33}$ by dividing numerator and denominator by 5

$= \mathbf{\frac{4}{11}}$ by dividing numerator and denominator by 5

4. Order the following fractions from the smallest to the largest: $\frac{4}{9}, \frac{5}{8}, \frac{3}{7}, \frac{1}{2}, \frac{3}{5}$

$\frac{4}{9} = 0.44444$ $\quad \frac{5}{8} = 0.62500$ $\quad \frac{3}{7} = 0.42857$ $\quad \frac{1}{2} = 0.50000$ $\quad \frac{3}{5} = 0.60000$

Ordering the fractions gives: $\mathbf{\frac{3}{7}, \frac{4}{9}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}}$

5. Evaluate, in fraction form: $\frac{1}{3} + \frac{2}{5}$

$$\frac{1}{3}+\frac{2}{5}=\frac{(5\times 1)+(3\times 2)}{15}=\mathbf{\frac{11}{15}}$$

6. Evaluate, in fraction form: $\frac{5}{6}-\frac{4}{15}$

$$\frac{5}{6}-\frac{4}{15}=\frac{(5\times 5)-(2\times 4)}{30}=\mathbf{\frac{17}{30}}$$

7. Evaluate, in fraction form: $\frac{1}{2}+\frac{2}{5}$

$$\frac{1}{2}+\frac{2}{5}=\frac{(5\times 1)+(2\times 2)}{10}=\mathbf{\frac{9}{10}}$$

8. Evaluate, in fraction form: $\frac{7}{16}-\frac{1}{4}$

$$\frac{7}{16}-\frac{1}{4}=\frac{(1\times 7)-(4\times 1)}{16}=\mathbf{\frac{3}{16}}$$

9. Evaluate, in fraction form: $\frac{2}{7}+\frac{3}{11}$

$$\frac{2}{7}+\frac{3}{11}=\frac{(11\times 2)+(7\times 3)}{77}=\frac{22+21}{77}=\mathbf{\frac{43}{77}}$$

10. Evaluate, in fraction form: $\frac{2}{9}-\frac{1}{7}+\frac{2}{3}$

$$\frac{2}{9}-\frac{1}{7}+\frac{2}{3}=\frac{(7\times 2)-(9\times 1)+(21\times 2)}{63}=\frac{14-9+42}{63}=\mathbf{\frac{47}{63}}$$

11. Evaluate, in fraction form: $3\frac{2}{5}-2\frac{1}{3}$

$$3\frac{2}{5}-2\frac{1}{3}=\left(3+\frac{2}{5}\right)-\left(2+\frac{1}{3}\right)=3+\frac{2}{5}-2-\frac{1}{3} = 1 + \frac{2}{5}-\frac{1}{3}$$

$$= 1 + \frac{6-5}{15} = 1 + \frac{1}{15} = \mathbf{1\frac{1}{15}}$$

12. Evaluate, in fraction form: $\frac{7}{27}-\frac{2}{3}+\frac{5}{9}$

$$\frac{7}{27}-\frac{2}{3}+\frac{5}{9}=\frac{7-18+15}{27} = \mathbf{\frac{4}{27}}$$

13. Evaluate, in fraction form: $5\frac{3}{13}+3\frac{3}{4}$

$$5\frac{3}{13}+3\frac{3}{4} = 5 + 3 + \frac{3}{13}+\frac{3}{4}=8+\frac{12+39}{52}=8+\frac{51}{52} = \mathbf{8\frac{51}{52}}$$

14. Evaluate, in fraction form: $4\frac{5}{8}-3\frac{2}{5}$

$$4\frac{5}{8}-3\frac{2}{5}=(4-3)+\frac{5}{8}-\frac{2}{5}=1+\frac{25-16}{40}=1+\frac{9}{40} = \mathbf{1\frac{9}{40}}$$

EXERCISE 6, Page 12

1. Evaluate: $\frac{2}{5}\times\frac{4}{7}$

$$\frac{2}{5}\times\frac{4}{7}=\frac{2\times4}{5\times7}=\mathbf{\frac{8}{35}}$$

2. Evaluate: $\frac{3}{4}\times\frac{8}{11}$

$$\frac{3}{4}\times\frac{8}{11}=\frac{3}{1}\times\frac{2}{11}\text{ by cancelling}$$

$$=\frac{3\times2}{1\times11}=\mathbf{\frac{6}{11}}$$

3. Evaluate: $\frac{3}{4}\times\frac{5}{9}$

$$\frac{3}{4}\times\frac{5}{9}=\frac{1}{4}\times\frac{5}{3}\text{ by cancelling}$$

$$=\frac{1\times5}{4\times3}=\mathbf{\frac{5}{12}}$$

4. Evaluate: $\frac{17}{35}\times\frac{15}{68}$

$$\frac{17}{35}\times\frac{15}{68}=\frac{1}{7}\times\frac{3}{4}\text{ by cancelling}$$

$$=\frac{1\times3}{7\times4}=\mathbf{\frac{3}{28}}$$

5. Evaluate: $\frac{3}{5}\times\frac{7}{9}\times1\frac{2}{7}$

$$\frac{3}{5}\times\frac{7}{9}\times1\frac{2}{7}=\frac{3}{5}\times\frac{7}{9}\times\frac{9}{7}=\mathbf{\frac{3}{5}}\quad\text{by cancelling}$$

6. Evaluate: $\frac{1}{4}\times\frac{3}{11}\times1\frac{5}{39}$

$$\frac{1}{4}\times\frac{3}{11}\times1\frac{5}{39}=\frac{1}{4}\times\frac{3}{11}\times\frac{44}{39}=\frac{1}{4}\times\frac{1}{1}\times\frac{4}{13}=\frac{1}{1}\times\frac{1}{1}\times\frac{1}{13}=\mathbf{\frac{1}{13}}\quad\text{by cancelling}$$

7. Evaluate: $\frac{2}{9}\div\frac{4}{27}$

$$\frac{2}{9}\div\frac{4}{27}=\frac{2}{9}\times\frac{27}{4}=\frac{1}{1}\times\frac{3}{2}=\mathbf{\frac{3}{2}}\text{ or }\mathbf{1\frac{1}{2}}\quad\text{by cancelling}$$

8. Evaluate: $\frac{3}{8}\div\frac{45}{64}$

$$\frac{3}{8}\div\frac{45}{64}=\frac{3}{8}\times\frac{64}{45}=\frac{1}{1}\times\frac{8}{15}=\mathbf{\frac{8}{15}}\quad\text{by cancelling}$$

9. Evaluate: $\frac{3}{8}\div\frac{5}{32}$

$$\frac{3}{8}\div\frac{5}{32}=\frac{3}{8}\times\frac{32}{5}=\frac{3}{1}\times\frac{4}{5}=\frac{12}{5}=\mathbf{2\frac{2}{5}}\quad\text{by cancelling}$$

10. Evaluate: $2\frac{1}{4}\times1\frac{2}{3}$

$$2\frac{1}{4}\times1\frac{2}{3}=\frac{9}{4}\times\frac{5}{3}=\frac{3}{4}\times\frac{5}{1}=\frac{15}{4}=\mathbf{3\frac{3}{4}}\quad\text{by cancelling}$$

11. Evaluate: $1\frac{1}{3}\div2\frac{5}{9}$

$$1\frac{1}{3}\div2\frac{5}{9}=\frac{4}{3}\div\frac{23}{9}=\frac{4}{3}\times\frac{9}{23}=\frac{4}{1}\times\frac{3}{23}=\mathbf{\frac{12}{23}}\quad\text{by cancelling}$$

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12. Evaluate: $2\frac{3}{4} \div 3\frac{2}{3}$

$$2\frac{3}{4} \div 3\frac{2}{3} = \frac{11}{4} \div \frac{11}{3} = \frac{11}{4} \times \frac{3}{11} = \frac{1}{4} \times \frac{3}{1} = \mathbf{\frac{3}{4}} \text{ by cancelling}$$

13. Evaluate: $\frac{1}{9} \times \frac{3}{4} \times 1\frac{1}{3}$

$$\frac{1}{9} \times \frac{3}{4} \times 1\frac{1}{3} = \frac{1}{9} \times \frac{3}{4} \times \frac{4}{3} = \frac{1}{9} \times \frac{1}{1} \times \frac{1}{1} = \mathbf{\frac{1}{9}}$$

14. Evaluate: $3\frac{1}{4} \times 1\frac{3}{5} \div \frac{2}{5}$

$$3\frac{1}{4} \times 1\frac{3}{5} \div \frac{2}{5} = \frac{13}{4} \times \frac{8}{5} \div \frac{2}{5} = \frac{13}{4} \times \frac{8}{5} \times \frac{5}{2} = \frac{13}{1} \times \frac{2}{1} \times \frac{1}{2} = \frac{13}{1} \times \frac{1}{1} \times \frac{1}{1} = \mathbf{13}$$

15. If a storage tank is holding 450 litres when it is three-quarters full, how much will it contain when it is two-thirds full?

If 450 litres is $\frac{3}{4}$ full then $\frac{1}{4}$ full would be 450 ÷ 3 = 150 litres.

Thus, a full tank would have 4 × 150 = 600 litres.

$\frac{2}{3}$ of the tank will contain $\frac{2}{3} \times 600$ = **400 litres**

16. A tank contains 24,000 litres of oil. Initially, $\frac{7}{10}$ of the contents are removed, then $\frac{3}{5}$ of the remainder is removed. How much oil is left in the tank?

If $\frac{7}{10}$ is removed then $\frac{3}{10} \times 24,000$ litres remains, i.e. 7200 litres.

If $\frac{3}{5}$ of this is then removed, then $\frac{2}{5} \times 7200$ litres remains, i.e. **2880 litres**

EXERCISE 7, Page 13

1. Evaluate: $2\frac{1}{2}-\frac{3}{5}\times\frac{20}{27}$

$$2\frac{1}{2}-\frac{3}{5}\times\frac{20}{27}=\frac{5}{2}-\frac{3}{5}\times\frac{20}{27}=\frac{5}{2}-\frac{1}{1}\times\frac{4}{9} \text{ by cancelling}$$

$$=\frac{5}{2}-\frac{4}{9} \qquad \text{(M)}$$

$$=\frac{45-8}{18}=\frac{37}{18}=\mathbf{2\frac{1}{18}} \qquad \text{(S)}$$

2. Evaluate: $\frac{1}{3}-\frac{3}{4}\times\frac{16}{27}$

$$\frac{1}{3}-\frac{3}{4}\times\frac{16}{27}=\frac{1}{3}-\frac{1}{1}\times\frac{4}{9} \text{ by cancelling}$$

$$=\frac{1}{3}-\frac{4}{9} \qquad \text{(M)}$$

$$=\frac{3-4}{9}=\mathbf{-\frac{1}{9}} \qquad \text{(S)}$$

3. Evaluate: $\frac{1}{2}+\frac{3}{5}\div\frac{9}{15}-\frac{1}{3}$

$$\frac{1}{2}+\frac{3}{5}\div\frac{9}{15}-\frac{1}{3}=\frac{1}{2}+\left(\frac{3}{5}\times\frac{15}{9}\right)-\frac{1}{3}=\frac{1}{2}+\frac{1}{1}-\frac{1}{3}=\frac{3+6-2}{6}=\frac{7}{6}=\mathbf{1\frac{1}{6}}$$

4. Evaluate: $\frac{1}{5}+2\frac{2}{3}\div\frac{5}{9}-\frac{1}{4}$

$$\frac{1}{5}+2\frac{2}{3}\div\frac{5}{9}-\frac{1}{4}=\frac{1}{5}+\frac{8}{3}\times\frac{9}{5}-\frac{1}{4} \qquad \text{(D)}$$

$$=\frac{1}{5}+\frac{8}{1}\times\frac{3}{5}-\frac{1}{4} \text{ by cancelling}$$

$$= \frac{1}{5} + \frac{24}{5} - \frac{1}{4} \qquad \text{(M)}$$

$$= \frac{4+96-5}{20} = \frac{95}{20} = 4\frac{15}{20} = \mathbf{4\frac{3}{4}} \qquad \text{(A/S)}$$

5. Evaluate: $\frac{4}{5} \times \frac{1}{2} - \frac{1}{6} \div \frac{2}{5} + \frac{2}{3}$

$$\frac{4}{5} \times \frac{1}{2} - \frac{1}{6} \div \frac{2}{5} + \frac{2}{3} = \frac{4}{5} \times \frac{1}{2} - \frac{1}{6} \times \frac{5}{2} + \frac{2}{3} \qquad \text{(D)}$$

$$= \frac{2}{5} \times \frac{1}{1} - \frac{1}{6} \times \frac{5}{2} + \frac{2}{3} \text{ by cancelling}$$

$$= \frac{2}{5} - \frac{5}{12} + \frac{2}{3} \qquad \text{(M)}$$

$$= \frac{24-25+40}{60} = \frac{39}{60} = \mathbf{\frac{13}{20}} \qquad \text{(A/S)}$$

6. Evaluate: $\frac{3}{5} - \left(\frac{2}{3} - \frac{1}{2}\right) \div \left(\frac{5}{6} \times \frac{3}{2}\right)$

$$\frac{3}{5} - \left(\frac{2}{3} - \frac{1}{2}\right) \div \left(\frac{5}{6} \times \frac{3}{2}\right) = \frac{3}{5} - \frac{4-3}{6} \div \frac{15}{12} \qquad \text{(B)}$$

$$= \frac{3}{5} - \frac{1}{6} \times \frac{12}{15} \qquad \text{(D)}$$

$$= \frac{3}{5} - \frac{1}{1} \times \frac{2}{15} \text{ by cancelling}$$

$$= \frac{3}{5} - \frac{2}{15} \qquad \text{(M)}$$

$$= \frac{9-2}{15} = \mathbf{\frac{7}{15}} \qquad \text{(S)}$$

7. Evaluate: $\frac{1}{2} \text{ of } \left(4\frac{2}{5} - 3\frac{7}{10}\right) + \left(3\frac{1}{3} \div \frac{2}{3}\right) - \frac{2}{5}$

$$\frac{1}{2}\text{ of }\left(4\frac{2}{5}-3\frac{7}{10}\right)+\left(3\frac{1}{3}\div\frac{2}{3}\right)-\frac{2}{5}=\frac{1}{2}\times\left(\frac{22}{5}-\frac{37}{10}\right)+\left(\frac{10}{3}\div\frac{2}{3}\right)-\frac{2}{5} \qquad \text{(O)}$$

$$=\frac{1}{2}\times\frac{44-37}{10}+\frac{10}{3}\times\frac{3}{2}-\frac{2}{5} \qquad \text{(B/D)}$$

$$=\frac{1}{2}\times\frac{7}{10}+\frac{5}{1}\times\frac{1}{1}-\frac{2}{5} \quad \text{by cancelling}$$

$$=\frac{7}{20}+\frac{5}{1}-\frac{2}{5} \qquad \text{(M)}$$

$$=\frac{7+100-8}{20}=\frac{99}{20}=\mathbf{4\frac{19}{20}} \qquad \text{(A/S)}$$

8. Evaluate: $\dfrac{6\frac{2}{3}\times1\frac{2}{5}-\frac{1}{3}}{6\frac{3}{4}\div1\frac{1}{2}}$

$$\frac{6\frac{2}{3}\times1\frac{2}{5}-\frac{1}{3}}{6\frac{3}{4}\div1\frac{1}{2}}=\frac{\frac{20}{3}\times\frac{7}{5}-\frac{1}{3}}{\frac{27}{4}\div\frac{3}{2}}=\frac{\frac{4}{3}\times\frac{7}{1}-\frac{1}{3}}{\frac{27}{4}\times\frac{2}{3}} \quad \text{by cancelling}$$

$$=\frac{\frac{4}{3}\times\frac{7}{1}-\frac{1}{3}}{\frac{9}{2}\times\frac{1}{1}} \quad \text{by cancelling}$$

$$=\frac{\frac{28}{3}-\frac{1}{3}}{\frac{9}{2}}$$

$$=\frac{\frac{28-1}{3}}{\frac{9}{2}}=\frac{\frac{27}{3}}{\frac{9}{2}}=\frac{9}{\frac{9}{2}}=9\times\frac{2}{9}=\mathbf{2}$$

EXERCISE 8, Page 14

1. In a box of 333 paper clips, 9 are defective. Express the non-defective paper clips as a ratio of the defective paper clips, in its simplest form.

Non-defective paper clips = 333 – 9 = 324

Non-defective paper clips as a ratio of the defective paper clips = 324:9

= **36:1** by dividing by 9

2. A gear wheel having 84 teeth is in mesh with a 24 tooth gear. Determine the gear ratio in its simplest form.

Gear ratio = 84:24

= 42:12 = 21:6 = **7:2** or **3.5:1** in its simplest form

3. In a box of 2000 nails, 120 are defective. Express the non-defective nails as a ratio of the defective ones, in its simplest form.

Non-defective nails = 2000 – 120 = 1880

Non-defective nails as a ratio of the defective nails = 1880:120

= 188:12 = 94:6 = **47:3** in its simplest form

4. A metal pipe 3.36 m long is to be cut into two in the ratio 6 to 15. Calculate the length of each piece.

Number of parts = 6 + 15 = 21

Length of 1 part = 3.36 m ÷ 21 = 336 cm ÷ 21 = $\frac{336}{21} = \frac{112}{7}$ = 16 cm

Hence, **6 parts** = 6 × 16 = **96 cm** and **15 parts** = 15 × 16 = **240 cm**

5. On the instructions for cooking a turkey it says that it needs to be cooked 45 minutes for every kilogram. How long will it take to cook a 7 kg turkey?

If 1 kg takes 45 minutes, then 7 kg takes $7 \times 45 = 315$ minutes

$= \mathbf{5\ hours\ 15\ minutes}$ or $\mathbf{5\frac{1}{4}\ hours}$

6. In a will, £6440 is to be divided between three beneficiaries in the ratio 4:2:1. Calculate the amount each receives.

Number of parts = 4 + 2 + 1 = 7

Amount for each part = $\dfrac{£6440}{7=}$ = £920

Hence, 4 parts = 4 × £920 = **£3680**, 4 parts = 2 × £920 = **£1840** and 1 part = 1 × £920 = **£920**

7. A local map has a scale of 1:22,500. The distance between two motorways is 2.7 km. How far are they apart on the map?

Distance apart on map $= \dfrac{2.7\,\text{km}}{22500} = \dfrac{2700\,\text{m}}{22500} = \dfrac{2700 \times 100\,\text{cm}}{22500}$

$= \mathbf{12\ cm}$

8. A machine produces 320 bolts in a day. Calculate the number of bolts produced by 4 machines in 7 days.

The machine produces 320 bolts in 1 day

If there were 4 machines, then 4 × 320 bolts would be produced daily, i.e. 1280 bolts.

In 7 days, **number of bolts produced** = 7 × 1280 = **8960 bolts**

EXERCISE 9, Page 15

1. Express 14.1794 correct to 2 decimal places

14.1794 = **14.18**, correct to 2 decimal places

2. Express 2.7846 correct to 4 significant figures

2.7846 = **2.785**, correct to 4 significant figures

3. Express 65.3792 correct to 2 decimal places

65.3792 = **65.38**, correct to 2 decimal places

4. Express 43.2746 correct to 4 significant figures

43.2746 = **43.27**, correct to 4 significant figures

5. Express 1.2973 correct to 3 decimal places

1.2973 = **1.297**, correct to 3 decimal places

6. Express 0.0005279 correct to 3 significant figures.

0.0005279 = **0.000528**, correct to 3 significant figures.

EXERCISE 10, Page 15

1. Evaluate 37.69 + 42.6, correct to 3 significant figures.

$$\begin{array}{r} 37.69 \\ +\ \underline{42.60} \\ \underline{80.29} \\ {\scriptstyle 1\,1} \end{array}$$

Hence, 37.69 + 42.6 = 80.29 = **80.3**, correct to 3 significant figures

2. Evaluate 378.1 – 48.85, correct to 1 decimal place.

$$\begin{array}{r} 378.10 \\ -\ \underline{48.85} \\ \underline{329.25} \end{array}$$

Hence, 378.1 – 48.85 = 329.25 = **329.3**, correct to 1 decimal place

3. Evaluate 68.92 + 34.84 – 31.223, correct to 4 significant figures.

$$\begin{array}{r} 68.92 \\ +\ \underline{34.84} \\ \underline{103.76} \end{array} \qquad\qquad \begin{array}{r} 103.760 \\ -\ \underline{31.223} \\ \underline{72.337} \end{array}$$

Hence, 68.92 + 34.84 – 31.223 = 72.537 = **72.54**, correct to 4 significant figures

4. Evaluate 67.841 – 249.55 + 56.883, correct to 2 decimal places.

$$\begin{array}{r} 67.841 \\ +\ \underline{56.883} \\ \underline{124.724} \end{array} \qquad\qquad \begin{array}{r} 249.550 \\ -\ \underline{124.724} \\ \underline{124.826} \end{array} \qquad 124.724 - 249.550 = -\,(249.55 - 124.724)$$

Hence, 67.841 – 249.55 + 56.883 = - 124.826 = **- 124.83**, correct to 2 decimal places

5. Evaluate 483.24 – 120.44 – 67.49, correct to 4 significant figures.

$$\begin{array}{r} 120.44 \\ +\ \underline{67.49} \\ \underline{187.93} \end{array} \qquad\qquad \begin{array}{r} 483.24 \\ -\ \underline{187.93} \\ \underline{295.31} \end{array}$$

Hence, 483.24 – 120.44 – 67.49 = 295.31 = **295.3**, correct to 4 significant figures

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EXERCISE 11, Page 17

1. Evaluate without using a calculator: 3.57×1.4

$$\begin{array}{r} 357 \\ \times \quad \underline{14} \\ 1428 \\ \underline{3570} \\ \underline{4998} \end{array}$$

$357 \times 14 = 4998$, hence $3.57 \times 1.4 = \mathbf{4.998}$

2. Evaluate without using a calculator: 67.92×0.7

$$\begin{array}{r} 6792 \\ \times \quad \underline{7} \\ \underline{47544} \end{array}$$

$6792 \times 7 = 47544$, hence $67.92 \times 0.7 = \mathbf{47.544}$

3. Evaluate without using a calculator: $548.28 \div 1.2$

$548.28 \div 1.2 = \dfrac{548.28}{1.2}$ The denominator is multiplied by 10 to change it into an integer. The numerator is also multiplied by 10 to keep the fraction the same.

Thus, $\dfrac{548.28}{1.2} = \dfrac{548.28 \times 10}{1.2 \times 10} = \dfrac{5482.8}{12}$

The long division is similar to the long division of integers.

$$\begin{array}{r} 456.9 \\ 12\overline{)5482.8} \\ \underline{48} \\ 68 \\ \underline{60} \\ 82 \\ \underline{72} \\ 108 \\ \underline{108} \\ 0 \end{array}$$

Hence, $548.28 \div 1.2 = \mathbf{456.9}$

4. Evaluate without using a calculator: 478.3 ÷ 1.1, correct to 5 significant figures

478.3 ÷ 1.1= $\frac{478.3}{1.1}$ The denominator is multiplied by 10 to change it into an integer. The numerator is also multiplied by 10 to keep the fraction the same.

Thus, $\frac{478.3}{1.1} = \frac{478.3 \times 10}{1.1 \times 10} = \frac{4783}{11}$

The long division is similar to the long division of integers.

$$
\begin{array}{r}
434.818.. \\
11\,\big)\overline{4783.000} \\
\underline{44} \\
38 \\
\underline{33} \\
53 \\
\underline{44} \\
90 \\
\underline{88} \\
20 \\
\underline{11} \\
90 \\
\underline{88} \\
2
\end{array}
$$

Hence, 478.3 ÷ 1.1= 434.818.. = **434.82**, correct to 5 significant figures

5. Evaluate without using a calculator: 563.48 ÷ 0.9, correct to 4 significant figures

563.48 ÷ 0.9 = $\frac{563.48}{0.9}$ The denominator is multiplied by 10 to change it into an integer. The numerator is also multiplied by 10 to keep the fraction the same.

Thus, $\frac{563.48}{0.9} = \frac{563.48 \times 10}{0.9 \times 10} = \frac{5634.8}{9}$

The long division is similar to the long division of integers.

$$\begin{array}{r} 626.088.. \\ 9\overline{)5634.800} \\ \underline{54} \\ 23 \\ \underline{18} \\ 54 \\ \underline{54} \\ 0\ 8 \\ \underline{0\ 0} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

Hence, $563.48 \div 0.9 = 626.088..$ = **626.1**, correct to 4 significant figures

6. Express $\frac{4}{9}$ as a decimal fraction correct to 3 significant figures.

$$\begin{array}{r} 0.4444 \\ 9\overline{)4.000} \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

Hence, $\frac{4}{9} = 0.4444$ = **0.444**, correct to 4 significant figures

7. Express $\frac{17}{27}$ as a decimal fraction, correct to 5 decimal place

$$\begin{array}{r} 0.629629 \\ 27\overline{)17.000000} \\ \underline{162} \\ 80 \\ \underline{54} \\ 260 \\ \underline{243} \\ 170 \\ \underline{162} \\ 80 \\ \underline{54} \\ 26 \end{array}$$

Thus, $\frac{17}{27}$ = **0.62963, correct to 5 decimal places.**

8. Express $1\frac{9}{16}$ as a decimal fraction correct to 4 significant figures

$$\begin{array}{r} 0.5625 \\ 16\overline{)9.00000} \end{array}$$

Thus, $1\frac{9}{16}$ = **1.563, correct to 4 significant figures.**

9. Express $13\frac{31}{37}$ as a decimal fraction, correct to 2 decimal places

$$\begin{array}{r@{}l} & 0.837 \\ 37 & \overline{)31.00} \\ & \underline{296} \\ & 140 \\ & \underline{111} \\ & 290 \\ & \underline{259} \\ & 31 \end{array}$$

Thus, $13\frac{31}{37}$ = 13.837 = **13.84**, correct to 2 decimal places

10. Evaluate 421.8 ÷ 17, (a) correct to 4 significant figures and (b) correct to 3 decimal places.

$$\begin{array}{r@{}l} & 24.811764 \\ 17 & \overline{)421.800000} \\ & \underline{34} \\ & 81 \\ & \underline{68} \\ & 138 \\ & \underline{136} \\ & 20 \\ & \underline{17} \\ & 30 \\ & 17 \\ & 130 \\ & \underline{119} \\ & 110 \\ & \underline{102} \\ & 80 \\ & \underline{68} \\ & 12 \end{array}$$

(a) 421.8 ÷ 17 = **24.81, correct to 4 significant figures**

(b) 421.8 ÷ 17 = **24.812, correct to 3 decimal places**

11. Evaluate $\dfrac{0.0147}{2.3}$, (a) correct to 5 decimal places and (b) correct to 2 significant figures.

$$\frac{0.0147}{2.3} = \frac{0.147}{23}$$

$$\begin{array}{r}
0.0063913 \\
23\overline{)0.1470000} \\
\underline{138} \\
90 \\
69 \\
210 \\
\underline{207} \\
30 \\
\underline{23} \\
70 \\
\underline{69} \\
1
\end{array}$$

(a) 0.0147 ÷ 2.3 = **0.00639, correct to 5 decimal places**

(b) 0.0147 ÷ 2.3 = **0.0064, correct to 2 significant figures**

12. Evaluate (a) $\dfrac{12.\dot{6}}{1.5}$ (b) $5.\dot{2} \times 12$

(a) $\dfrac{12.\dot{6}}{1.5} = \dfrac{12.6666}{1.5} = \dfrac{126.666}{15}$

$$\begin{array}{r}
8.4444 \\
15\overline{)126.666} \\
\underline{120} \\
66 \\
\underline{60} \\
66 \\
\underline{60} \\
66 \\
\underline{60} \\
66 \\
\underline{60} \\
6
\end{array}$$

Hence, $\dfrac{12.\dot{6}}{1.5} = 8.4444 = \mathbf{8.\dot{4}}$

(b) $5.\dot{2} \times 12 = 5.2222 \times 12$

$$\begin{array}{r} 5222222 \\ \times \quad \underline{\quad\quad 12} \\ \underline{6266664} \end{array}$$

Hence, $5.\dot{2} \times 12 = 62.6666 = \mathbf{62.\dot{6}}$

EXERCISE 12, Page 18

1. Express 0.0032 as a percentage

$0.0032 = 0.0032 \times 100\% = \mathbf{0.32\%}$

2. Express 1.734 as a percentage

$1.734 = 1.734 \times 100\% = \mathbf{173.4\%}$

3. Express 0.057 as a percentage

$0.057 = 0.057 \times 100\% = \mathbf{5.7\%}$

4. Express 20% as a decimal number

$20\% = \frac{20}{100} = \mathbf{0.20}$ as a decimal number

5. Express 1.25% as a decimal number

$1.25\% = \frac{1.25}{100} = \mathbf{0.0125}$ as a decimal number

6. Express $\frac{11}{16}$ as a percentage

$$\frac{11}{16} = \frac{11}{16} \times 100\% = \frac{1100}{16}\%$$

$$
\begin{array}{r}
68.75 \\
16\overline{)1100.00} \\
\underline{96} \\
140 \\
\underline{128} \\
120 \\
\underline{112} \\
80 \\
\underline{80} \\
0
\end{array}
$$

Hence, $\frac{11}{16}$ = **68.75%**

7. Express as percentages, correct to 3 significant figures:

(a) $\frac{7}{33}$ (b) $\frac{19}{24}$ (c) $1\frac{11}{16}$

(a) $\frac{7}{33}=\frac{7}{33}\times 100\%=\frac{700}{33}$ = 21.21212…% = **21.2%**, correct to 3 significant figures.

(b) $\frac{19}{24}=\frac{19}{24}\times 100\%=\frac{1900}{24}=79.1666...\%$ = **79.2%**, correct to 3 significant figures.

(c) $1\frac{11}{16}=1.6875=1.6875\times 100\%$ = 168.75% = **169%**, correct to 3 significant figures.

8. Place the following in order of size, the smallest first, expressing each as percentages, correct to 1 decimal place: (a) $\frac{12}{21}$ (b) $\frac{9}{17}$ (c) $\frac{5}{9}$ (d) $\frac{6}{11}$

(a) $\frac{12}{21}$ = 0.5714 = 57.1% (b) $\frac{9}{17}$0.5294 = 52.9% (c) $\frac{5}{9}$ = 0.5555 = 55.6%

(d) $\frac{6}{11}$ = 0.5454 = 54.5%

Hence, the order is: **(b), (d), (c) and (a)**

9. Express 31.25% as a fraction in its simplest form

$31.25\% = \frac{31.25}{100} = \frac{3125}{10000} = \frac{125}{400} = \mathbf{\frac{5}{16}}$

10. Express 56.25% as a fraction in its simplest form.

$56.25\% = \frac{56.25}{100} = \frac{5625}{10000} = \mathbf{\frac{9}{16}}$

11. Calculate 43.6% of 50 kg

43.6% of 50 kg = $\frac{43.6}{100} \times 50$ kg = **21.8 kg**

12. Determine 36% of 27 m

36% of 27 m = $\frac{36}{100} \times 27$ m = **9.72 m**

13. Calculate correct to 4 significant figures:

(a) 18% of 2758 tonnes (b) 47% of 18.42 grams (c) 147% of 14.1 seconds

(a) 18% of 2758 = $\frac{18}{100} \times 2758 = 18 \times 27.58$ = 496.44 t = **496.4 t**, correct to 4 significant figures.

(b) 47% of 18.42 = $\frac{47}{100} \times 18.42 = 47 \times 0.1842$ = 8.6574 g = **8.657 g**, correct to 4 significant figures.

(c) 147% of 14.1 = $\frac{147}{100} \times 14.1 = 147 \times 0.141$ = 20.727 s, = **20.73 s**, correct to 4 significant figures.

14. Express: (a) 140 kg as a percentage of 1 t (b) 47 s as a percentage of 5 min

(c) 13.4 cm as a percentage of 2.5 m

(a) 140 kg as a percentage of 1 t $\frac{140}{1000} \times 100\% = \frac{140}{10}$ = **14%**

(b) 47 s as a percentage of 5 min $= \frac{47}{5\times 60}\times 100\% = \frac{47}{300}\times 100\% = \frac{47}{3}$ $=$ **$15.\dot{6}$ %**

(c) 13.4 cm as a percentage of 2.5 m $= \frac{13.4}{250}\times 100\% = \frac{13.4}{2.5} = \frac{134}{25}$ $=$ **5.36%**

15. Express 325 mm as a percentage of 867 mm, correct to 2 decimal places.

325 mm as a percentage of 867 mm $= \frac{325}{867}\times 100\%$ $=$ **37.49%**

16. Express 408 g as a percentage of 2.40 kg.

408 g as a percentage of 2.40 kg = 408 g as a percentage of 2400 g

$$= \frac{408}{2400}\times 100\% = \mathbf{17\%}$$

17. When signing a new contract, a Premiership footballer's pay increases from £15,500 to £21,500 per week. Calculate the percentage pay increase, correct 3 significant figures.

Percentage pay increase $= \frac{21500 - 15500}{15500}\times 100\% = \frac{6000}{15500}\times 100\%$ $=$ **38.7%**

18. A metal rod 1.80 m long is heated and its length expands by 48.6 mm. Calculate the percentage increase in length.

Percentage increase in length $= \frac{48.6\,\text{mm}}{1.80\times 10^3\,\text{mm}}\times 100\%$ $=$ **2.7%**

19. A machine part has a length of 36 mm. The length is incorrectly measured as 36.9 mm. Determine the percentage error in the measurement.

Percentage error in the measurement $= \frac{36.9 - 36}{36}\times 100\% = \frac{0.9}{36}\times 100\%$ $=$ **2.5% too high**

20. A resistor has a value of 820 Ω ± 5%. Determine the range of resistance values expected.

5% of 820 = $\frac{5}{100} \times 820 = 41$

The lowest value expected is 820 – 5% of 820 i.e. 820 – 41 = 779 Ω

The highest value expected is 820 + 5% of 820 i.e. 820 + 41 = 861 Ω

Hence, **range of values expected is: 779 Ω to 861 Ω**

21. For each of the following resistors, determine the (i) minimum value, (ii) maximum value:

(a) 680 Ω ± 20% (b) 47 kΩ ± 5%

(a) 20% of 680 Ω = $\frac{20}{100} \times 680 = 136$ Ω

Hence, (i) **minimum value** = 680 – 136 = **544 Ω**

(ii) **maximum value** = 680 + 136 = **816 Ω**

(b) 5% of 47 kΩ = $\frac{5}{100} \times 47 = 2.35$ kΩ

Hence, (i) **minimum value** = 47 – 2.35 = **44.65 kΩ**

(ii) **maximum value** = 47 + 2.35 = **49.35 kΩ**

22. An engine speed is 2400 rev/min. The speed is increased by 8%. Calculate the new speed.

8% of 2400 rev/min = $\frac{8}{100} \times 2400 = 192$ rev/min

New speed = 2400 + 192 = **2592 rev/min**

CHAPTER 20 WORK, ENERGY AND POWER

EXERCISE 86, Page 202

1. Determine the work done when a force of 50 N pushes an object 1.5 km in the same direction as the force.

Work done = force × distance moved in the direction of the force

= 50 N × 1500 m = 75000 J (since 1 J = 1 Nm)

i.e. **work done = 75 kJ**

2. Calculate the work done when a mass of weight 200 N is lifted vertically by a crane to a height of 100 m.

When work is done in lifting then:

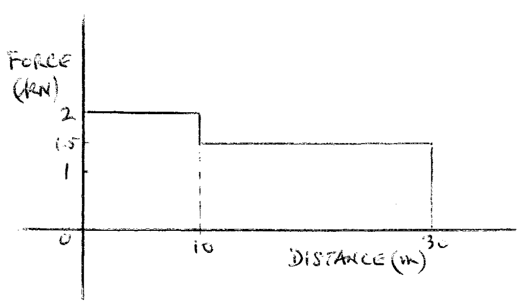
work done = (weight of the body) × (vertical distance moved)

Weight is the downward force due to the mass of an object. Hence

work done = 200 N × 100 m = 20000 J = **20 kJ**

3. A motor supplies a constant force of 2 kN to move a load 10 m. The force is then changed to a constant 1.5 kN and the load is moved a further 20 m. Draw the force/distance graph for the complete operation, and, from the graph, determine the total work done by the motor.

The force/distance graph is shown below.



Total work done = area under the force/distance graph

$= (2000 \times 10) + (1500 \times 20)$

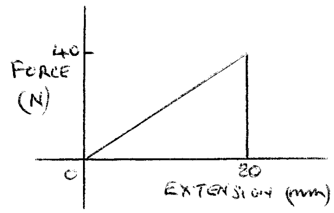
$= 20000 + 30000$

$= 50000$ J = **50 kJ**

4. A spring, initially relaxed, is extended 80 mm. Draw a work diagram and hence determine the work done if the spring requires a force of 0.5 N/mm of stretch.

Force = 0.5 N/mm × 80 mm = 40 N

The work diagram is shown below.



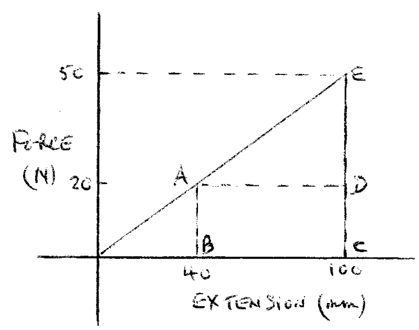
Total work done = area under the diagram

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 80 \text{ mm} \times 40 \text{ N}$$

$$= \frac{1}{2} \times (80 \times 10^{-3} \text{ m}) \times 40 \text{ N} = \mathbf{1.6\ J}$$

5. A spring requires a force of 50 N to cause an extension of 100 mm. Determine the work done in extending the spring (a) from 0 to 100 mm, and (b) from 40 mm to 100 mm.

The work diagram is shown below.



(a) The work done in extending the spring from 0 to 100 mm

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (100 \times 10^{-3}\,\text{m}) \times 50\ \text{N} = \mathbf{2.5\ J}$$

(b) The work done in extending the spring from 40 mm to 100 mm

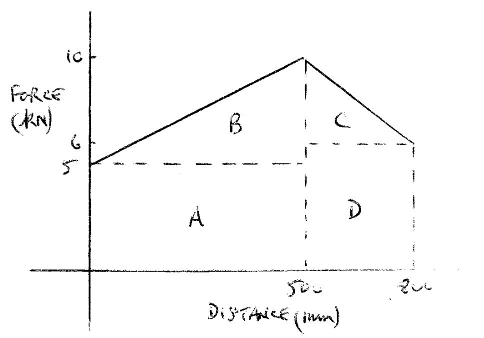
$$= \text{area ABCE} = \text{area ABCD} + \text{area ADE}$$

$$= (60 \times 10^{-3}\,\text{m})(20\ \text{N}) + \frac{1}{2}(60 \times 10^{-3}\,\text{m})(30\ \text{N})$$

$$= 1.2 + 0.9 = \mathbf{2.1\ J}$$

6. The resistance to a cutting tool varies during the cutting stroke of 800 mm as follows: (i) the resistance increases uniformly from an initial 5000 N to 10,000 N as the tool moves 500 mm, and (ii) the resistance falls uniformly from 10,000 N to 6000 N as the tool moves 300 mm. Draw the work diagram and calculate the work done in one cutting stroke.

The work diagram is shown below.



Work done in one cutting stroke = area under the diagram

$$= A + B + C + D$$

$$= (500 \times 10^{-3} \times 5) + \frac{1}{2}(500 \times 10^{-3} \times 5) + \frac{1}{2}(300 \times 10^{-3} \times 4)$$

$$+ (300 \times 10^{-3} \times 6)$$

$$= 2.5\ \text{kJ} + 1.25\ \text{kJ} + 0.6\ \text{kJ} + 1.8\ \text{kJ}$$

$$= \mathbf{6.15\ kJ}$$

EXERCISE 87, Page 204

1. A machine lifts a mass of weight 490.5 N through a height of 12 m when 7.85 kJ of energy is supplied to it. Determine the efficiency of the machine.

Work done in lifting mass = force × distance moved

= weight of body × distance moved

= 490.5 N × 12 m = 5886 J = useful energy output

Energy input = 7.85 kJ = 7850 J

Efficiency, $\boldsymbol{\eta} = \dfrac{\text{useful output energy}}{\text{input energy}} = \dfrac{5886}{7850} = \mathbf{0.75}$ or **75%**

2. Determine the output energy of an electric motor which is 60% efficient if it uses 2 kJ of electrical energy.

Efficiency, $\eta = \dfrac{\text{useful output energy}}{\text{input energy}}$ thus $\dfrac{60}{100} = \dfrac{\text{output energy}}{2000\text{ J}}$

from which, **output energy** $= \dfrac{60}{100} \times 2000 = 1200\text{ J} = \mathbf{1.2\ kJ}$

3. A machine that is used for lifting a particular mass is supplied with 5 kJ of energy. If the machine has an efficiency of 65% and exerts a force of 812.5 N to what height will it lift the mass?

Efficiency, $\eta = \dfrac{\text{useful output energy}}{\text{input energy}}$ i.e. $\dfrac{65}{100} = \dfrac{\text{output energy}}{5000\text{ J}}$

from which, output energy $= \dfrac{65}{100} \times 5000 = 3250\text{ J}$

Work done = force × distance moved

hence 3250 J = 812.5 N × height

from which, **height** = $\frac{3250\,J}{812.5\,N}$ = **4 m**

4. A load is hoisted 42 m and requires a force of 100 N. The efficiency of the hoist gear is 60% and that of the motor is 70%. Determine the input energy to the hoist.

Output energy = work done = force × distance = 100 N × 42 m = 4200 J

For the gearing, efficiency = $\frac{\text{output energy}}{\text{input energy}}$ i.e. $\frac{60}{100} = \frac{4200}{\text{input energy}}$

from which, the input energy to the gears = 4200 × $\frac{100}{60}$ = 7000 J

The input energy to the gears is the same as the output energy of the motor. Thus, for the motor,

efficiency = $\frac{\text{output energy}}{\text{input energy}}$ i.e. $\frac{70}{100} = \frac{7000}{\text{input energy}}$

Hence, **input energy to the hoist** = 7000 × $\frac{100}{70}$ = 10000 J = **10 kJ**

EXERCISE 88, Page 208

1. The output power of a motor is 10 kW. How much work does it do in 1 minute?

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

from which, **work done** = power × time = 10000 W × 60 s = 600000 J = **600 kJ**

2. Determine the power required to lift a load through a height of 20 m in 12.5 s if the force required is 2.5 kN.

Work done = force × distance moved = 2500 N × 20 m = 50000 J

Power = $\frac{\text{work done}}{\text{time taken}} = \frac{50000\text{ J}}{12.5\text{ s}}$ = **4000 W** or **4 kW**

3. 25 kJ of work is done by a force in moving an object uniformly through 50 m in 40 s. Calculate (a) the value of the force, and (b) the power.

(a) Work done = force × distance

hence 25000 J = force × 50 m

from which, **force** = $\frac{25000\text{ J}}{50\text{ m}}$ = **500 N**

(b) **Power** = $\frac{\text{work done}}{\text{time taken}} = \frac{25000\text{ J}}{40\text{ s}}$ = **625 W**

4. A car towing another at 54 km/h exerts a steady pull of 800 N. Determine (a) the work done in $\frac{1}{4}$ hr, and (b) the power required.

(a) Work done = force × distance moved.

The distance moved in 15 min, i.e. $\frac{1}{4}$ h, at 54 km/h = $\frac{54}{4}$ = 13.5 km.

Hence, **work done** = 800 N × 13500 m = **10800 kJ** or **10.8 MJ**

(b) **Power required** = $\frac{\text{work done}}{\text{time taken}} = \frac{10.8 \times 10^6\,\text{J}}{15 \times 60\,\text{s}}$ = **12000 W** or **12 kW**

5. To what height will a mass of weight 500 N be raised in 20 s by a motor using 4 kW of power?

Work done = force × distance. Hence, work done = 500 N × height.

Power = $\frac{\text{work done}}{\text{time taken}}$, from which, work done = power × time taken

= 4000 W × 20 s = 80000 J

Hence, 80000 = 500 N × height, from which, **height** = $\frac{80000\,\text{J}}{500\,\text{N}}$ = **160 m**

6. The output power of a motor is 10 kW. Determine (a) the work done by the motor in 2 hours, and (b) the energy used by the motor if it is 72% efficient.

(a) Work done = power × time taken

= 10 kW × 2 h = **20 kWh** = 20000 × 60 × 60 Ws

= **72 MJ**

(b) Efficiency = $\frac{\text{output energy}}{\text{input energy}}$ i.e. $\frac{72}{100} = \frac{72}{\text{input energy}}$

Hence, **energy used by the motor** = 72 × $\frac{100}{72}$ = **100 MJ**

7. A car is travelling at a constant speed of 81 km/h. The frictional resistance to motion is 0.60 kN. Determine the power required to keep the car moving at this speed.

Power = force × velocity

= 0.60 kN × 81 km/h = 600 N × $\frac{81000}{60 \times 60}\frac{\text{m}}{\text{s}}$ = 13500 N m/s

= 13500 J/s = **13.5 kW**

8. A constant force of 2.0 kN is required to move the table of a shaping machine when a cut is being made. Determine the power required if the stroke of 1.2 m is completed in 5.0 s.

Work done in each cutting stroke = force × distance

$$= 2000 \text{ N} \times 1.2 \text{ m} = 2400 \text{ J}$$

Power required = $\dfrac{\text{work done}}{\text{time taken}} = \dfrac{2400\text{ J}}{5\text{ s}}$ = **480 W**

9. The variation of force with distance for a vehicle that is decelerating is as follows:

Distance (m)	600	500	400	300	200	100	0
Force (kN)	24	20	16	12	8	4	0

If the vehicle covers the 600 m in 1.2 minutes, find the power needed to bring the vehicle to rest.

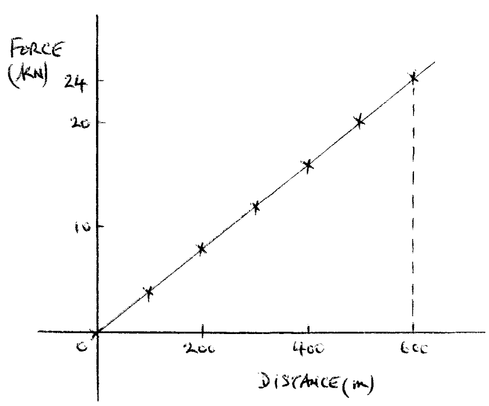
The force/distance graph is shown below.

Work done = area under the force/distance graph

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 600 \text{ m} \times 24 \text{ kN}$$

$$= 7200 \text{ kJ}$$

Power needed to bring the vehicle to rest = $\dfrac{\text{work done}}{\text{time taken}} = \dfrac{7200\text{ kJ}}{1.2 \times 60\text{ s}}$ = **100 kW**



10. A cylindrical bar of steel is turned in a lathe. The tangential cutting force on the tool is 0.5 kN and the cutting speed is 180 mm/s. Determine the power absorbed in cutting the steel.

Power absorbed in cutting the steel = force × velocity

= 0.5 kN ×180 mm/s

= 500 N × 0.180 m/s

= 90 J/s = **90 W**

EXERCISE 89, Page 211

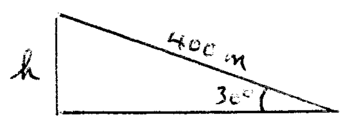
1. An object of mass 400 g is thrown vertically upwards and its maximum increase in potential energy is 32.6 J. Determine the maximum height reached, neglecting air resistance.

Potential energy = mgh

i.e. $32.6 = (0.4 \text{ kg})(9.81 \text{ m/s}^2)(\text{h})$

from which, **maximum height, h** = $\dfrac{32.6}{(0.4)(9.81)}$ = **8.31 m**

2. A ball bearing of mass 100 g rolls down from the top of a chute of length 400 m inclined at an angle of 30° to the horizontal. Determine the decrease in potential energy of the ball bearing as it reaches the bottom of the chute.



With reference the above diagram, $\sin 30° = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{h}{400}$

from which, $h = 400 \sin 30° = 200 \text{ m}$

Hence, increase in potential energy = mgh

$= 0.1 \text{ kg} \times 9.81 \text{ m/s}^2 \times 200 \text{ m}$

= 196.2 J

3. A vehicle of mass 800 kg is travelling at 54 km/h when its brakes are applied. Find the kinetic energy lost when the car comes to rest.

Kinetic energy = $\dfrac{1}{2}\text{mv}^2 = \dfrac{1}{2}(800 \text{ kg})\left(\dfrac{54}{3.6}\text{m/s}\right)^2$

i.e. **kinetic energy lost** = **90000 J** or **90 kJ**

4. Supplies of mass 300 kg are dropped from a helicopter flying at an altitude of 60 m. Determine the potential energy of the supplies relative to the ground at the instant of release, and its kinetic energy as it strikes the ground.

Potential energy of supplies at release = mgh

$$= (300 \text{ kg})(9.81 \text{ m/s}^2)(60 \text{ m})$$

$$= 176580 \text{ J} = \mathbf{176.6 \text{ kJ}}$$

By the principle of conservation of energy,

kinetic energy as the supplies strikes the ground = potential energy at release = **176.6 kJ**

5. A shell of mass 10 kg is fired vertically upwards with an initial velocity of 200 m/s. Determine its initial kinetic energy and the maximum height reached, correct to the nearest metre, neglecting air resistance.

Initial kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2}(10 \text{ kg})(200 \text{ m/s})^2 = \mathbf{200 \text{ kJ}}$

At the maximum height, the velocity of the canister is zero and all the kinetic energy has been converted into potential energy. Hence,

potential energy = initial kinetic energy = 200000 J

Then, $200000 = mgh = (10)(9.81)(h)$

from which, height $h = \frac{200000}{(10)(9.81)} = 2039$ m

i.e. **the maximum height reached is 2039 m** or **2.039 km**

6. The potential energy of a mass is increased by 20.0 kJ when it is lifted vertically through a height of 25.0 m. It is now released and allowed to fall freely. Neglecting air resistance, find its kinetic energy and its velocity after it has fallen 10.0 m.

Potential energy of mass = 20.0 kJ = mgh

from which, mass, $m = \frac{20000}{gh} = \frac{20000}{(9.81)(25.0)} = 81.55$ kg

Potential energy after falling 10.0 m = mgh = (81.55)(9.81)(10.0)

= 8000 J

Kinetic energy = potential energy = 8000 J = **8 kJ**

Kinetic energy = $\frac{1}{2}mv^2$ i.e. $8000 = \frac{1}{2}(81.55)v^2$

from which, $v^2 = \frac{2 \times 8000}{81.55}$

and **velocity after falling 10.0 m, v** = $\sqrt{\frac{2 \times 8000}{81.55}}$ = **14.0 m/s**

7. A pile-driver of mass 400 kg falls freely through a height of 1.2 m on to a pile of mass 150 kg. Determine the velocity with which the driver hits the pile. If, at impact, 2.5 kJ of energy are lost due to heat and sound, the remaining energy being possessed by the pile and driver as they are driven together into the ground a distance of 150 mm, determine (a) the common velocity after impact, (b) the average resistance of the ground.

The potential energy of the pile-driver is converted into kinetic energy.

Thus, potential energy = kinetic energy, i.e. $mgh = \frac{1}{2}mv^2$

from which, velocity $v = \sqrt{2gh} = \sqrt{(2)(9.81)(1.2)} = 4.85$ m/s.

Hence, **the pile-driver hits the pile at a velocity of 4.85 m/s**

(a) Before impact, kinetic energy of pile driver = $\frac{1}{2}mv^2 = \frac{1}{2}(400)(4.85)^2$

= 4704.5 J = 4.705 kJ

Kinetic energy after impact = 4.705 – 2.5 = 2.205 kJ

Thus the pile-driver and pile together have a mass of 400 + 150 = 550 kg and possess kinetic energy of 2.205 kJ

Hence, $2205 = \frac{1}{2}mv^2 = \frac{1}{2}(550)v^2$

from which, velocity $v = \sqrt{\left(\frac{2 \times 2205}{550}\right)} = 2.83$ m/s

Thus, **the common velocity after impact is 2.83 m/s**

(b) The kinetic energy after impact is absorbed in overcoming the resistance of the ground, in a distance of 150 mm.

Kinetic energy = work done = resistance × distance

i.e. $2205 = \text{resistance} \times 0.150$

from which, $\text{resistance} = \frac{2205}{0.150} = 14700$ N

Hence, **the average resistance of the ground is 14.70 kN**

EXERCISE 90, Page 211

Answers found from within the text of the chapter, pages 198 to 211.

EXERCISE 91, Page 212

1. (b) **2.** (c) **3.** (c) **4.** (a) **5.** (d) **6.** (c) **7.** (a) **8.** (d) **9.** (c) **10.** (b) **11.** (b) **12.** (a)
13. (d) **14.** (a) **15.** (d)

CHAPTER 21 SIMPLY SUPPORTED BEAMS

EXERCISE 92, Page 215

1. Determine the moment of a force of 25 N applied to a spanner at an effective length of 180 mm from the centre of a nut.

Moment, M = force × distance

= 25 N × 0.18 m = **4.5 N m**

2. A moment of 7.5 N m is required to turn a wheel. If a force of 37.5 N applied to the rim of the wheel can just turn the wheel, calculate the effective distance from the rim to the hub of the wheel.

Moment, M = force × distance

from which, **distance from rim to hub** = $\frac{\text{moment, M}}{\text{force, F}} = \frac{7.5\text{ N m}}{37.5\text{ N}}$ = 0.2 m = **200 mm**

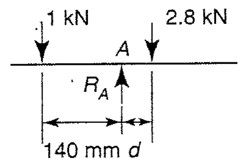
3. Calculate the force required to produce a moment of 27 N m on a shaft, when the effective distance from the centre of the shaft to the point of application of the force is 180 mm.

Moment, M = force × distance

from which, **force** = $\frac{\text{moment, M}}{\text{distance, d}} = \frac{27\text{ N m}}{180\times10^{-3}\text{ m}}$ = **150 N**

EXERCISE 93, Page 217

1. Determine distance d and the force acting at the support A for the force system shown below, when the system is in equilibrium.



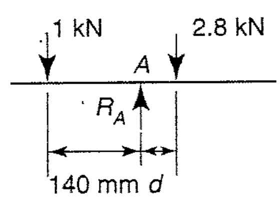
Clockwise moment = anticlockwise moment

Hence, $2.8 \times d = 1 \times 140$

i.e. distance, $\mathbf{d} = \dfrac{1 \times 140}{2.8} = \mathbf{50\ mm}$

Force at support A, $\mathbf{R_A} = 1 + 2.8 = \mathbf{3.8\ kN}$

2. If the 1 kN force shown below is replaced by a force F at a distance of 250 mm to the left of R_A, find the value of F for the system to be in equilibrium.



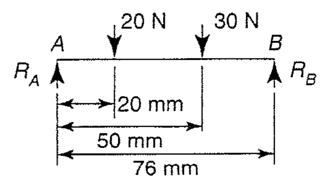
Clockwise moment = anticlockwise moment

Hence, if d = 50 mm from above, then

$2.8 \times 50 = F \times 250$

and force, $\mathbf{F} = \dfrac{2.8 \times 50}{250} = 0.56\ \text{kN} = \mathbf{560\ N}$

3. Determine the values of the forces acting at A and B for the force system shown below.



At equilibrium, $R_A + R_B = 20 + 30 = 50$ N (1)

Taking moments about point A gives:

clockwise moment = anticlockwise moment

Hence, $20 \times 20 + 30 \times 50 = R_B \times 76$

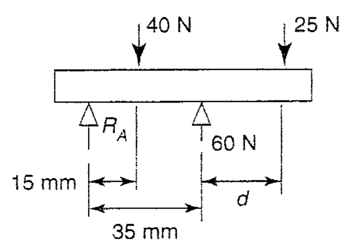
i.e. $400 + 1500 = 76 R_B$

from which, force acting at B, $\mathbf{R_B} = \frac{1900}{76} = \mathbf{25\ N}$

From equation (1), $R_A + 25 = 50$

from which, $\mathbf{R_A} = 50 - 25 = \mathbf{25\ N}$

4. The forces acting on a beam are as shown below. Neglecting the mass of the beam, find the value of R_A and distance d when the beam is in equilibrium.



At equilibrium, $R_A + 60 = 40 + 25$

from which, $\mathbf{R_A} = 40 + 25 - 60 = \mathbf{5\ N}$

Taking moments about the 60 N force gives:

clockwise moment = anticlockwise moment

Hence, $25 \times d + R_A \times 35 = 40 \times (35 - 15)$

i.e. $25d + 5 \times 35 = 40 \times 20$

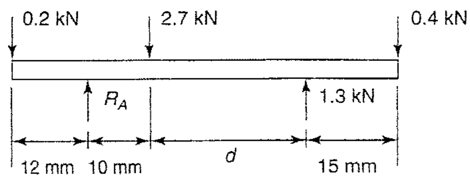
i.e. $25d + 175 = 800$

i.e. $25d = 800 - 175$

from which, distance, **d** = $\frac{800 - 175}{25} = \frac{625}{25}$ = **25 mm**

EXERCISE 94, Page 220

1. Calculate the force R_A and distance d for the beam shown below. The mass of the beam should be neglected and equilibrium conditions assumed.



At equilibrium, $0.2 + 2.7 + 0.4 = R_A + 1.3$

from which, $\mathbf{R_A} = 0.2 + 2.7 + 0.4 - 1.3 = \mathbf{2.0\ kN}$

Taking moments about the 2.7 kN force gives:

clockwise moment = anticlockwise moment

Hence, $0.4 \times (d + 15) + R_A \times 10 = 1.3 \times d + 0.2 \times (12 + 10)$

i.e. $0.4d + 6 + 2.0 \times 10 = 1.3d + 4.4$

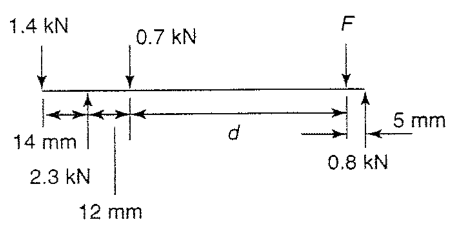
i.e. $0.4d + 6 + 20 = 1.3d + 4.4$

i.e. $6 + 20 - 4.4 = 1.3d - 0.4d$

and $21.6 = 0.9d$

from which, distance, $\mathbf{d} = \frac{21.6}{0.9} = \mathbf{24\ mm}$

2. For the force system shown below, find the values of F and d for the system to be in equilibrium.



At equilibrium, $1.4 + 0.7 + F = 2.3 + 0.8$

i.e. force, $\mathbf{F} = 2.3 + 0.8 - 1.4 - 0.7 = \mathbf{1.0\ kN}$

Taking moments about the 0.7 kN force gives:

clockwise moment = anticlockwise moment

Hence, $F \times d + 2.3 \times 12 = 0.8 \times (d + 5) + 1.4 \times (14 + 12)$

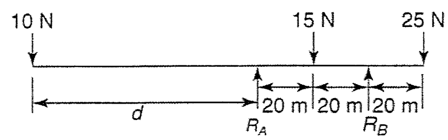
i.e. $1.0 \times d + 27.6 = 0.8d + 4 + 36.4$

i.e. $d - 0.8d = 4 + 36.4 - 27.6$

and $0.2d = 12.8$

from which, distance, $\mathbf{d} = \frac{12.8}{0.2} = \mathbf{64\ mm}$

3. For the force system shown below, determine distance d for the forces R_A and R_B to be equal, assuming equilibrium conditions.



For equilibrium, $R_A + R_B = 10 + 15 + 25 = 50$ N

Hence, if $R_A = R_B$ then $\mathbf{R_A} = \mathbf{R_B} = \frac{50}{2} = \mathbf{25\ N}$

Taking moments about the R_A gives:

clockwise moment = anticlockwise moment

Hence, $15 \times 20 + 25 \times (20 + 20 + 20) = R_B \times (20 + 20) + 10 \times d$

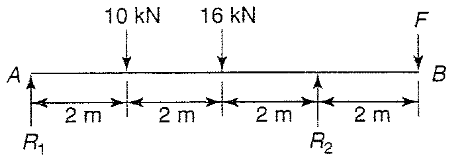
i.e. $300 + 1500 = 25 \times 40 + 10d$

i.e. $1800 = 1000 + 10d$

and $10d = 1800 - 1000 = 800$

from which, distance, $\mathbf{d} = \frac{800}{10} = \mathbf{80\ m}$

4. A simply supported beam AB is loaded as shown below. Determine the load F in order that the reaction at A is zero.



If $R_1 = 0$, then taking moments about R_2 gives:

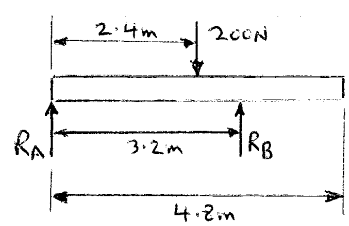
clockwise moment = anticlockwise moment

i.e. $F \times 2 = 16 \times 2 + 10 \times (2 + 2)$

i.e. $2F = 32 + 40 = 72$

from which, **load, F** $= \dfrac{72}{2}$ = **36 kN**

5. A uniform wooden beam, 4.8 m long, is supported at its left-hand end and also at 3.2 m from the left-hand end. The mass of the beam is equivalent to 200 N acting vertically downwards at its centre. Determine the reactions at the supports.



The beam is shown above.

Taking moments about the left-hand support gives:

clockwise moment = anticlockwise moment

i.e. $200 \times 2.4 = R_B \times 3.2$

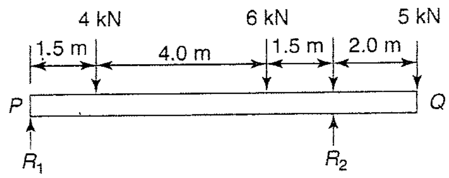
from which, $\mathbf{R_B} = \frac{200 \times 2.4}{3.2} = \mathbf{150\ N}$

For equilibrium, $R_A + R_B = 200$

Hence, $\mathbf{R_A} = 200 - R_B$

$= 200 - 150 = \mathbf{50\ N}$

6. For the simply supported beam PQ shown below, determine (a) the reaction at each support, (b) the maximum force which can be applied at Q without losing equilibrium.



(a) Taking moments about the left-hand support gives:

clockwise moment = anticlockwise moment

i.e. $4 \times 1.5 + 6 \times (1.5 + 4.0) + 5 \times (1.5 + 4.0 + 1.5 + 2.0) = R_2 \times (1.5 + 4.0 + 1.5)$

i.e. $6 + 33 + 45 = 7 R_2$

from which, $\mathbf{R_2} = \frac{6+33+45}{7} = \frac{84}{7} = \mathbf{12\ kN}$

For equilibrium, $R_1 + R_2 = 4 + 6 + 5 = 15$ kN

Hence, $R_1 + 12 = 15$

from which, $\mathbf{R_1} = 15 - 12 = \mathbf{3\ kN}$

(b) Let the force at Q be R_Q

Taking moments about R_2 gives:

clockwise moment = anticlockwise moment

i.e. $R_Q \times 2.0 = 6 \times 1.5 + 4 \times 5.5$

i.e. $2 R_Q = 9 + 22 = 31$

from which, $\mathbf{R_Q} = \frac{31}{2} = \mathbf{15.5\ kN}$

EXERCISE 95, Page 221

Answers found from within the text of the chapter, pages 214 to 220.

EXERCISE 96, Page 232

1. (a) **2.** (c) **3.** (a) **4.** (d) **5.** (a) **6.** (d) **7.** (c) **8.** (a) **9.** (d) **10.** (c)

CHAPTER 22 LINEAR AND ANGULAR MOTION

EXERCISE 97, Page 226

1. A pulley driving a belt has a diameter of 360 mm and is turning at 2700/π revolutions per minute. Find the angular velocity of the pulley and the linear velocity of the belt assuming that no slip occurs.

Angular velocity ω = 2πn, where n is the speed of revolution in revolutions per second, i.e.

$n = \dfrac{2700}{60\pi}$ revolutions per second.

Thus, **angular velocity,** $\omega = 2\pi\left(\dfrac{2700}{60\pi}\right) =$ **90 rad/s**

The linear velocity of a point on the rim, v = ωr, where r is the radius of the wheel, i.e.

$r = \dfrac{360}{2} = 180$ mm = 0.18 m

Thus, **linear velocity, v** = ωr = 90 × 0.18 = **16.2 m/s**

2. A bicycle is travelling at 36 km/h and the diameter of the wheels of the bicycle is 500 mm. Determine the angular velocity of the wheels of the bicycle and the linear velocity of a point on the rim of one of the wheels.

Linear velocity, v = 36 km/h = $\dfrac{36\times1000}{3600}$ m/s = **10 m/s**

(Note that changing from km/h to m/s involves dividing by 3.6)

Radius of wheel, $r = \dfrac{500}{2} = 250$ mm = 0.25 m

Since, v = ωr, then **angular velocity,** $\omega = \dfrac{v}{r} = \dfrac{10}{0.25} =$ **40 rad/s**

EXERCISE 98, Page 227

1. A flywheel rotating with an angular velocity of 200 rad/s is uniformly accelerated at a rate of $5\ \text{rad/s}^2$ for 15 s. Find the final angular velocity of the flywheel both in rad/s and revolutions per minute.

Angular velocity, ω_1 = 200 rad/s, angular acceleration, α = 5 rad/s^2 and time, t = 15 s.

Final angular velocity, $\omega_2 = \omega_1 + \alpha t$

$$= 200 + (5)(15) = 200 + 75 = \mathbf{275\ rad/s}$$

In **revolutions per minute**, 275 rad/s = $275 \times \dfrac{60}{2\pi} = \dfrac{\mathbf{8250}}{\boldsymbol{\pi}}$ rev/min or **2626 rev/min**

2. A disc accelerates uniformly from 300 revolutions per minute to 600 revolutions per minute in 25 s. Determine its angular acceleration and the linear acceleration of a point on the rim of the disc, if the radius of the disc is 250 mm.

Initial angular velocity, $\omega_1 = 300 \times \dfrac{2\pi}{60} = 10\pi$ rad/s

and final angular velocity, $\omega_2 = 600 \times \dfrac{2\pi}{60} = 20\pi$ rad/s

$\omega_2 = \omega_1 + \alpha t$ from which,

$$\textbf{angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{20\pi - 10\pi}{25} = \frac{10\pi}{25} = \mathbf{0.4\pi\ rad/s^2} \text{ or } \mathbf{1.257\ rad/s^2}$$

Linear acceleration, a = $r\alpha$ = (0.25)(0.4π) = **0.1π m/s^2** or **0.314 m/s^2**

EXERCISE 99, Page 229

1. A grinding wheel makes 300 revolutions when slowing down uniformly from 1000 rad/s to 400 rad/s. Find the time for this reduction in speed.

Angle turned through, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$ hence $300 \times 2\pi = \left(\frac{1000 + 400}{2}\right)t$

i.e. $600\pi = 700t$

from which, **time, t** $= \frac{600\pi}{700}$ = **2.693 s**

2. Find the angular retardation for the grinding wheel in question 1.

$\omega_2 = \omega_1 + \alpha t$ from which,

angular acceleration, $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{400 - 1000}{2.693} = \frac{-600}{2.693}$ = - 222.8 rad/s^2

i.e. **angular retardation is 222.8 rad/s^2**

3. A disc accelerates uniformly from 300 revolutions per minute to 600 revolutions per minute in 25 s. Calculate the number of revolutions the disc makes during this accelerating period.

Angle turned through,

$$\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t = \left(\frac{\frac{300 \times 2\pi}{60} + \frac{600 \times 2\pi}{60}}{2}\right)(25) \text{ rad}$$

However, there are 2π radians in 1 revolution, hence,

number of revolutions $= \left(\frac{\frac{300 \times 2\pi}{60} + \frac{600 \times 2\pi}{60}}{2}\right)\left(\frac{25}{2\pi}\right) = \frac{1}{2}\left(\frac{900}{60}\right)(25) =$ **187.5 revolutions**

4. A pulley is accelerated uniformly from rest at a rate of 8 rad/s^2. After 20 s the acceleration stops and the pulley runs at constant speed for 2 min, and then the pulley comes uniformly to rest after a further 40 s. Calculate: (a) the angular velocity after the period of acceleration,

(b) the deceleration,

(c) the total number of revolutions made by the pulley.

(a) **Angular velocity after acceleration period,** $\omega_2 = \omega_1 + \alpha t = 0 + (8)(20) =$ **160 rad/s**

(b) $\omega_3 = \omega_2 + \alpha t$ from which,

angular acceleration, $\alpha = \dfrac{\omega_3 - \omega_2}{t} = \dfrac{0-160}{40} = -\,4$ rad/s^2

i.e. **angular deceleration is 4 rad/s^2**

(c) Initial angle turned through, $\theta_1 = \left(\dfrac{\omega_1 + \omega_2}{2}\right)t = \left(\dfrac{0+160}{2}\right)(20) = 1600$ rad $= \dfrac{1600}{2\pi}$ rev

At constant speed, angle turned through, $\theta_2 = 160$ rad/s $\times (2 \times 60)$s $= 19200$ rad $= \dfrac{19200}{2\pi}$ rev

Angle turned through during deceleration, $\theta_3 = \left(\dfrac{160+0}{2}\right)(40) = 3200$ rad $= \dfrac{3200}{2\pi}$ rev

Hence, total number of revolutions made by the pulley $= \theta_1 + \theta_2 + \theta_3$

$$= \frac{1600}{2\pi} + \frac{19200}{2\pi} + \frac{3200}{2\pi}$$

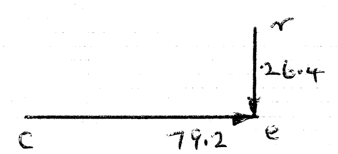
$$= \frac{24000}{2\pi} = \mathbf{\frac{12000}{\pi}\ rev} \text{ or } \mathbf{3820\ rev}$$

EXERCISE 100, Page 231

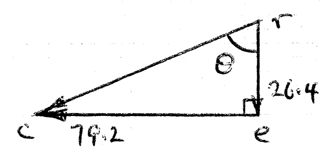
1. A car is moving along a straight horizontal road at 79.2 km/h and rain is falling vertically downwards at 26.4 km/h. Find the velocity of the rain relative to the driver of the car.

The space diagram is shown in diagram (a). The velocity diagram is shown in diagram (b) and the velocity of the rain relative to the driver is given by vector **rc** where **rc** = **re** + **ec**

$$|\mathbf{rc}| = \sqrt{(79.2^2 + 26.4^2)} = 83.5 \text{ km/h} \qquad \text{and} \quad \theta = \tan^{-1}\left(\frac{79.2}{26.4}\right) = 71.6°$$



(a)

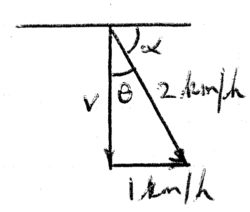


(b)

i.e. **the velocity of the rain relative to the driver is 83.5 km/h at 71.6° to the vertical**.

2. Calculate the time needed to swim across a river 142 m wide when the swimmer can swim at 2 km/h in still water and the river is flowing at 1 km/h. At what angle to the bank should the swimmer swim?

The swimmer swims at 2 km/h relative to the water, and as he swims the movement of the water carries him downstream. He must therefore aim against the flow of the water – at an angle θ shown in the triangle of velocities shown below where v is the swimmers true speed.



$$v = \sqrt{2^2 - 1^2} = \sqrt{3} \text{ km/h} = \sqrt{3}\left(\frac{1000}{60}\right) \text{m/min} = 28.87 \text{ m/min}$$

Hence, if the width of the river is 142 m, **the swimmer will take** $\frac{142}{28.87}$ = 4.919 minutes

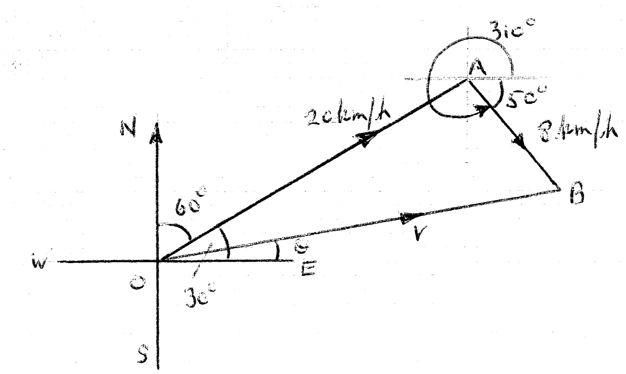
= **4 min 55 s**

In the above diagram, $\sin\theta = \frac{1}{2}$ from which, $\theta = 30°$

Hence, **the swimmer needs to swim at an angle of 60° to the bank** (shown as angle α in the diagram.

3. A ship is heading in a direction N 60° E at a speed which in still water would be 20 km/h. It is carried off course by a current of 8 km/h in a direction of E 50° S. Calculate the ship's actual speed and direction.

In the triangle of velocities shown below (triangle 0AB), 0A represents the velocity of the ship in still water, AB represents the velocity of the water relative to the earth, and 0B is the velocity of the ship relative to the earth.



Total horizontal component of v = 20 cos 30° + 8 cos 310° = 22.46

Total vertical component of v = 20 sin 30° + 8 sin 310° = 3.87

Hence, $v = \sqrt{\left(22.46^2 + 3.87^2\right)}$ = 22.79 km/h,

and $\theta = \tan^{-1}\left(\frac{3.87}{22.46}\right) = 9.78°$

Hence, **the ships actual speed is 22.79 km/h in a direction E 9.78° N**

EXERCISE 101, Page 231

Answers found from within the text of the chapter, pages 224 to 231.

EXERCISE 102, Page 231

1. (b) **2.** (c) **3.** (a) **4.** (c) **5.** (a) **6.** (d) **7.** (c) **8.** (b) **9.** (d) **10.** (c) **11.** (b) **12.** (d) **13.** (a)

CHAPTER 23 FRICTION

EXERCISE 103, Page 235

1. The coefficient of friction of a brake pad and a steel disc is 0.82. Determine the normal force between the pad and the disc if the frictional force required is 1025 N.

Frictional force, F = $\mu \times$ normal force

i.e. $F = \mu \times N$

Hence, $1025 = 0.82 \times N$

from which, **normal force, N** = $\frac{1025}{0.82}$ = **1250 N**

2. A force of 0.12 kN is needed to push a bale of cloth along a chute at a constant speed. If the normal force between the bale and the chute is 500 N, determine the dynamic coefficient of friction.

As the bale of cloth is moving at constant speed, the force applied must be that required to overcome frictional forces, i.e. frictional force, F = 120 N;

the normal force is 500 N, and since $F = \mu N$,

$$\mu = \frac{F}{N} = \frac{120}{500} = 0.24$$

i.e. **the dynamic coefficient of friction is 0.24**

3. The normal force between a belt and its driver wheel is 750 N. If the static coefficient of friction is 0.9 and the dynamic coefficient of friction is 0.87, calculate (a) the maximum force which can be transmitted, and (b) maximum force which can be transmitted when the belt is running at a constant speed.

(a) **Maximum force that can be transmitted** = μN = (0.9)(750)

= **675 N**

(b) Maximum force which can be transmitted when the belt is running at a constant speed

= μN = (0.87)(750)

= **652.5 N**

EXERCISE 104, Page 236

Answers found from within the text of the chapter, pages 233 to 236.

EXERCISE 105, Page 236

1. (c) **2.** (c) **3.** (f) **4.** (e) **5.** (i) **6.** (c) **7.** (h) **8.** (b) **9.** (d) **10.** (a)

CHAPTER 24 SIMPLE MACHINES

EXERCISE 106, Page 240

1. A simple machine raises a load of 825 N through a distance of 0.3 m. The effort is 250 N and moves through a distance of 3.3 m. Determine: (a) the force ratio, (b) the movement ratio, (c) the efficiency of the machine at this load.

Force ratio $= \dfrac{\text{load}}{\text{effort}} = \dfrac{825\text{ N}}{250\text{ N}} = \mathbf{3.3}$

Movement ratio $= \dfrac{\text{distance moved by the effort}}{\text{distance moved by the load}} = \dfrac{3.3\text{ m}}{0.3\text{ m}} = \mathbf{11}$

Efficiency $= \dfrac{\text{force ratio}}{\text{movement ratio}} \times 100\% = \dfrac{3.3}{11} \times 100 = \mathbf{30\%}$

2. The efficiency of a simple machine is 50%. If a load of 1.2 kN is raised by an effort of 300 N, determine the movement ratio.

Force ratio $= \dfrac{\text{load}}{\text{effort}} = \dfrac{1200\text{ N}}{300\text{ N}} = 4$

Efficiency $= \dfrac{\text{force ratio}}{\text{movement ratio}}$ from which, **movement ratio** $= \dfrac{\text{force ratio}}{\text{efficiency}} = \dfrac{4}{\frac{50}{100}} = \dfrac{4}{0.5} = \mathbf{8}$

3. An effort of 10 N applied to a simple machine moves a load of 40 N through a distance of 100 mm, the efficiency at this load being 80%. Calculate: (a) the movement ratio, (b) the distance moved by the effort.

Force ratio $= \dfrac{\text{load}}{\text{effort}} = \dfrac{40\text{ N}}{10\text{ N}} = 4$

(a) Efficiency = $\frac{\text{force ratio}}{\text{movement ratio}}$ from which, **movement ratio** = $\frac{\text{force ratio}}{\text{efficiency}} = \frac{4}{\frac{80}{100}} = \frac{4}{0.8}$ = **5**

(b) Movement ratio = $\frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$

from which, **the distance moved by the effort** = movement ratio × distance moved by the load

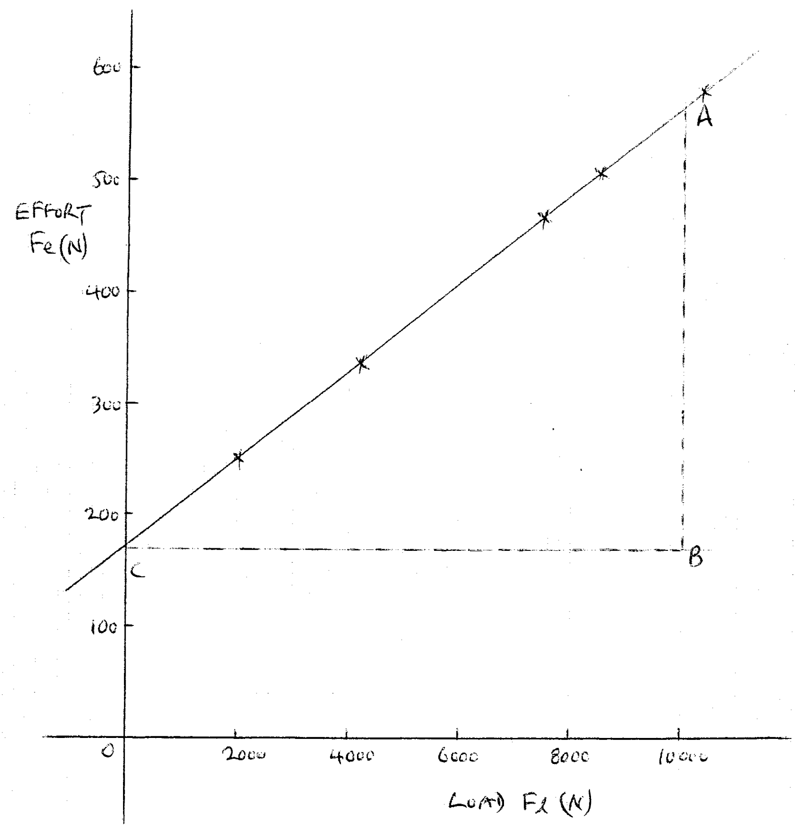
= 5 × 100 = **500 mm**

4. The effort required to raise a load using a simple machine, for various values of load is as shown:

Load F_l (N)	2050	4120	7410	8240	10300
Effort F_e (N)	252	340	465	505	580

If the movement ratio for the machine is 30, determine (a) the law of the machine, (b) the limiting force ratio, (c) the limiting efficiency.

The load/effort graph is shown below.



(a) The law of the machine is $F_e = a F_l + b$

where gradient of curve, $a = \frac{AB}{BC} = \frac{570 - 170}{10000} = \frac{400}{10000} = \frac{4}{100} = 0.04$

and intercept, b = 170.

Hence, the law of the machine is: $\mathbf{F_e = 0.04 F_l + 170}$

(b) **Limiting force ratio** $= \frac{1}{a} = \frac{1}{0.04} = \mathbf{25}$

(c) **Limiting efficiency** $= \frac{1}{a \times \text{movement ratio}} = \frac{1}{0.04 \times 30} \times 100\% = \mathbf{83.3\%}$

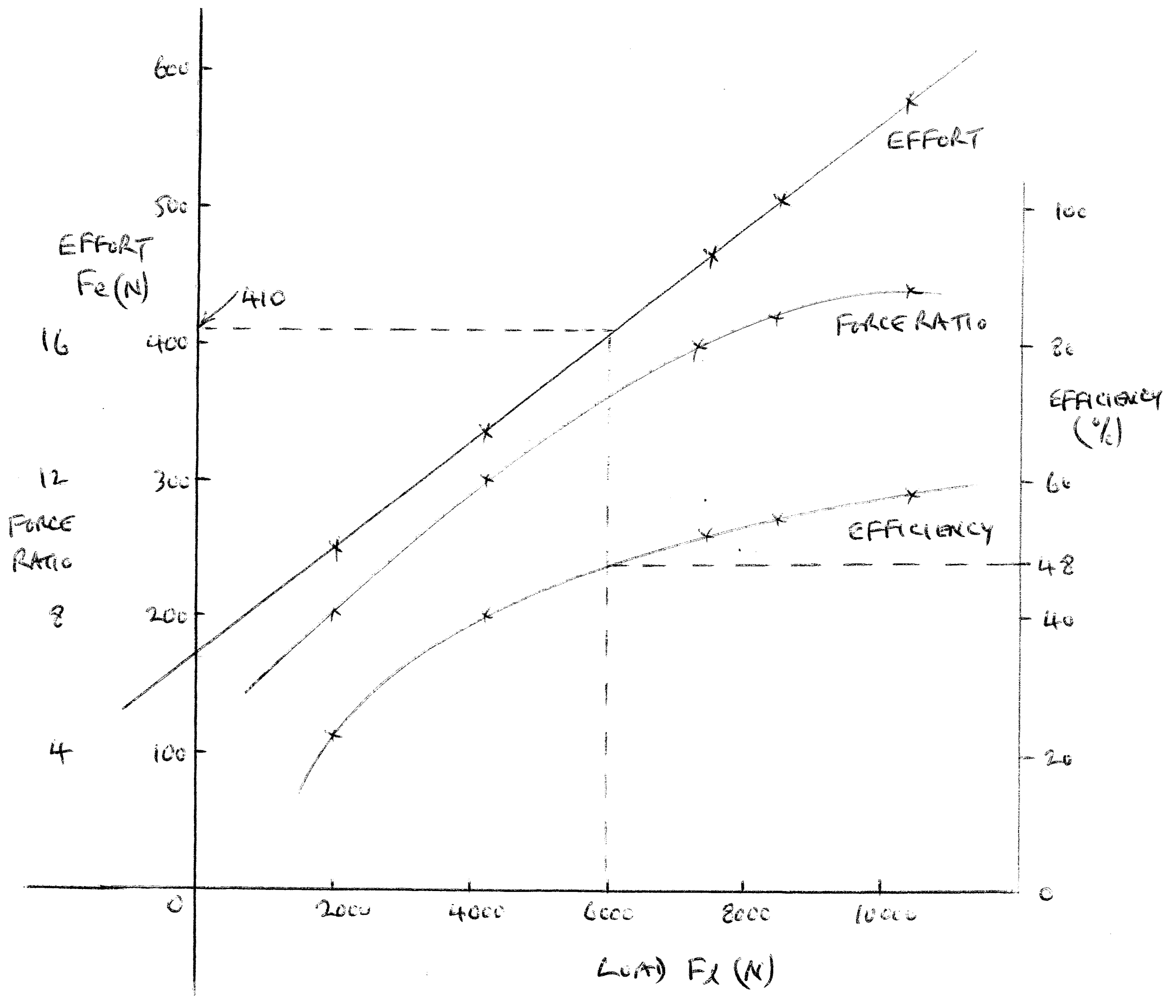
5. For the data given in question 4, determine the values of force ratio and efficiency for each value of the load. Hence plot graphs of effort, force ratio and efficiency to a base of load. From the graphs, determine the effort required to raise a load of 6 kN and the efficiency at this load.

Load F_l (N)	2050	4120	7410	8240	10300
Effort F_e (N)	252	340	465	505	580
Force ratio $= \frac{\text{load}}{\text{effort}}$	8.13	12.12	15.94	16.32	17.76
Efficiency $= \frac{\text{force ratio}}{\text{movement ratio}}$	27.1%	40.4%	53.1%	54.4%	59.2%

Graphs of load/effort, load/force ratio and load/efficiency are shown below.

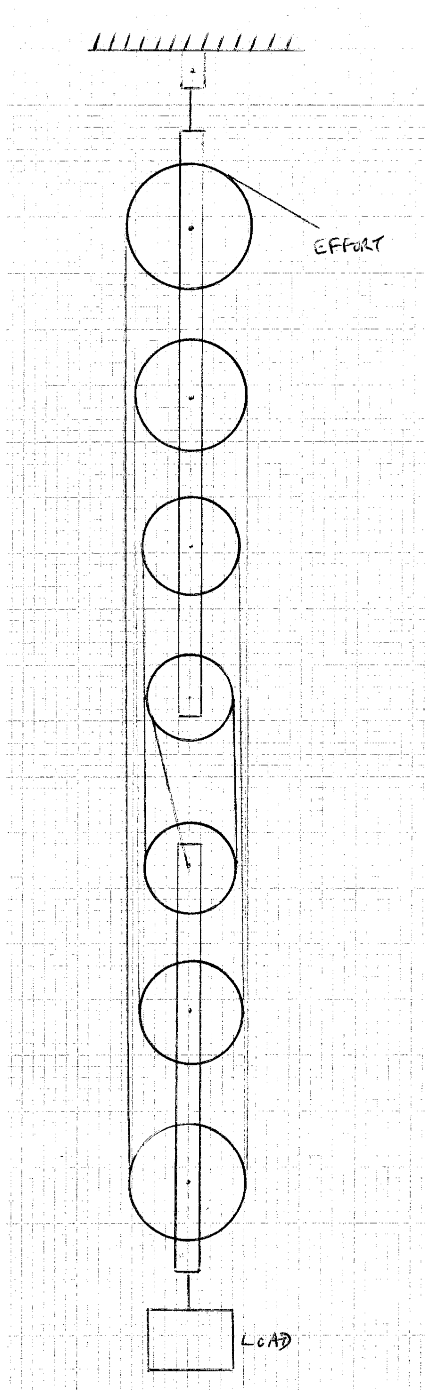
From the graph, when the load is 6 kN, i.e. 6000 N

effort = 410 N and **efficiency = 48%**


600
500
400
300
200
100
EFFORT
Fe(N)
410
16
12
FORCE
RATIO
8
4
EFFORT
FORCE RATIO
EFFICIENCY
100
80
60
48
40
20
0
EFFICIENCY
(%)
0
2000
4000
6000
8000
10000
LOAD Fl (N)

EXERCISE 107, Page 242

1. A pulley system consists of four pulleys in an upper block and three pulleys in a lower block. Make a sketch of this arrangement showing how a movement ratio of 7 may be obtained. If the force ratio is 4.2, what is the efficiency of the pulley.



Efficiency = $\dfrac{\text{force ratio}}{\text{movement ratio}} = \dfrac{4.2}{7} \times 100\%$ = **60%**

2. A three-pulley lifting system is used to raise a load of 4.5 kN. Determine the effort required to raise this load when losses are neglected. If the actual effort required is 1.6 kN, determine the efficiency of the pulley system at this load.

Load = 4.5 kN and movement ratio = n = 3

When losses are neglected, efficiency = 100% = $\dfrac{\text{force ratio}}{\text{movement ratio}}$

from which, force ratio = $\dfrac{\text{load}}{\text{effort}}$ = movement ratio

i.e. $\dfrac{4.5\,\text{kN}}{\text{effort}} = 3$

and **effort** = $\dfrac{4.5\,\text{kN}}{3}$ = **1.5 kN**

If the actual effort required is 1.6 kN, **efficiency** = $\dfrac{\text{force ratio}}{\text{movement ratio}} \times 100\%$

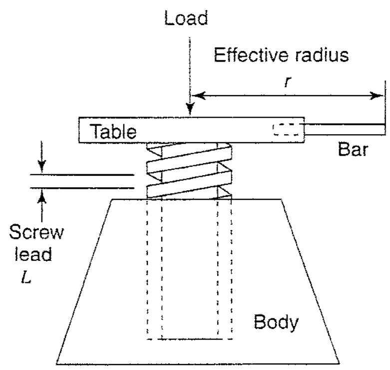
$$= \frac{\dfrac{\text{load}}{\text{effort}}}{\text{movement ratio}} \times 100\% = \frac{\dfrac{4.5}{1.6}}{3} \times 100\%$$

= **93.75%**

EXERCISE 108, Page 243

1. Sketch a simple screw-jack. The single-start screw of such a jack has a lead of 6 mm and the effective length of the operating bar from the centre of the screw is 300 mm. Calculate the load which can be raised by an effort of 150 N if the efficiency at this load is 20%.

A simple screw-jack is shown below, where lead, L = 6 mm and radius, r = 300 mm



Movement ratio = $\frac{2\pi r}{L} = \frac{2\pi(300)}{6} = 100\pi$

Efficiency = $\frac{\text{force ratio}}{\text{movement ratio}}$ i.e. $\frac{20}{100} = \frac{\text{force ratio}}{100\pi}$

from which, force ratio = $\frac{20(100\pi)}{100} = 20\pi$

Force ratio = $\frac{\text{load}}{\text{effort}}$ from which, **load** = force ratio × effort = 20π × 150 N

= 3000π = 9425 N = **9.425 kN**

2. A load of 1.7 kN is lifted by a screw-jack having a single-start screw of lead 5 mm. The effort is applied at the end of an arm of effective length 320 mm from the centre of the screw. Calculate the effort required if the efficiency at this load is 25%.

Movement ratio = $\dfrac{2\pi r}{L} = \dfrac{2\pi(320)}{5} = 128\pi$

Efficiency = $\dfrac{\text{force ratio}}{\text{movement ratio}}$ i.e. $\dfrac{25}{100} = \dfrac{\text{force ratio}}{128\pi}$

from which, force ratio = $\dfrac{25(128\pi)}{100} = 32\pi$

Force ratio = $\dfrac{\text{load}}{\text{effort}}$ from which, **effort** = $\dfrac{\text{load}}{\text{force ratio}} = \dfrac{1.7\,\text{kN}}{32\pi} = \dfrac{1700}{32\pi}$

= **16.91 N**

EXERCISE 109, Page 245

1. The driver gear of a gear system has 28 teeth and meshes with a follower gear having 168 teeth. Determine the movement ratio and the speed of the follower when the driver gear rotates at 60 revolutions per second.

Movement ratio $= \dfrac{\text{teeth on follower}}{\text{teeth on driver}} = \dfrac{168}{28} = \mathbf{6}$

Also, movement ratio $= \dfrac{\text{speed of driver}}{\text{speed of follower}}$ i.e. $6 = \dfrac{60 \text{ rev/s}}{\text{speed of follower}}$

from which, **the speed of the follower** $= \dfrac{60}{6} = \mathbf{10\ rev/s}$

2. A compound gear train has a 30-tooth driver gear A, meshing with a 90-tooth follower gear B. Mounted on the same shaft as B and attached to it is a gear C with 60 teeth, meshing with a gear D on the output shaft having 120 teeth. Calculate the movement and force ratios if the overall efficiency of the gears is 72%.

The speed of D = $\text{speed of A} \times \dfrac{T_A}{T_B} \times \dfrac{T_C}{T_D}$

Movement ratio $= \dfrac{\text{speed of A}}{\text{speed of D}} = \dfrac{T_B}{T_A} \times \dfrac{T_D}{T_C} = \dfrac{90}{30} \times \dfrac{120}{60} = \mathbf{6}$

The efficiency of any simple machine $= \dfrac{\text{force ratio}}{\text{movement ratio}} \times 100\%$

from which, **force ratio** = efficiency × movement ratio

$$= \frac{72}{100} \times 6 = \mathbf{4.32}$$

3. A compound gear train is as shown on page 223. The movement ratio is 6 and the numbers of teeth on gears A, C and D are 25, 100 and 60, respectively. Determine the number of teeth on gear B and the force ratio when the efficiency is 60%.

Movement ratio = $\dfrac{\text{speed of A}}{\text{speed of D}} = \dfrac{T_B}{T_A} \times \dfrac{T_D}{T_C}$

i.e. $$6 = \frac{T_B}{25} \times \frac{60}{100}$$

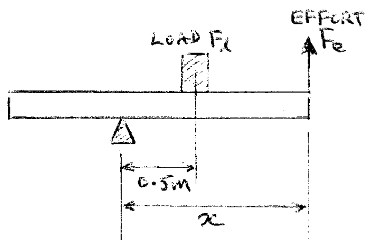
from which, **number of teeth on B**, $\mathbf{T_B} = \dfrac{6 \times 25 \times 100}{60}$ = **250**

Efficiency = $\dfrac{\text{force ratio}}{\text{movement ratio}}$ i.e. $\dfrac{60}{100} = \dfrac{\text{force ratio}}{6}$

from which, **force ratio** = $\dfrac{60 \times 6}{100}$ = **3.6**

EXERCISE 110, Page 246

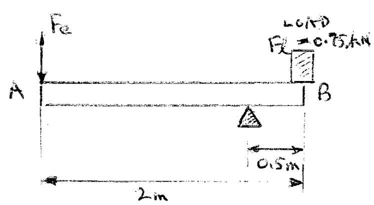
1. In a second-order lever system, the force ratio is 2.5. If the load is at a distance of 0.5 m from the fulcrum, find the distance that the effort acts from the fulcrum if losses are negligible.



Force ratio = $\frac{\text{distance of effort from fulcrum}}{\text{distance of load from fulcrum}}$ i.e. $2.5 = \frac{x}{0.5}$

Hence, **the distance that the effort acts from the fulcrum, x** = 2.5 × 0.5 = **1.25 m**

2. A lever AB is 2 m long and the fulcrum is at a point 0.5 m from B. Find the effort to be applied at A to raise a load of 0.75 kN at B when losses are negligible.

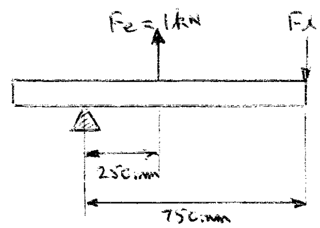


Clockwise moment = anticlockwise moment

$$(F_e)(1.5) = (0.5)(0.75)$$

Hence, **effort at A,** $\mathbf{F_e} = \frac{(0.5)(0.75)}{1.5}$ = **0.25 kN** or **250 N**

3. The load on a third-order lever system is at a distance of 750 mm from the fulcrum and the effort required to just move the load is 1 kN when applied at a distance of 250 mm from the fulcrum. Determine the value of the load and the force ratio if losses are negligible.



Clockwise moment = anticlockwise moment

$$(F_l)(750) = (1)(250)$$

i.e. $$\mathbf{F_l} = \frac{250}{750} = \frac{1}{3}\,\text{kN} = \mathbf{333.3\ N}$$

$$\textbf{Force ratio} = \frac{\text{distance of effort from fulcrum}}{\text{distance of load from fulcrum}} = \frac{250}{750} = \mathbf{\frac{1}{3}}$$

EXERCISE 111, Page 246

Answers found from within the text of the chapter, pages 238 to 246.

EXERCISE 112, Page 247

1. (b) **2.** (f) **3.** (c) **4.** (d) **5.** (b) **6.** (a) **7.** (b) **8.** (d) **9.** (c) **10.** (d) **11.** (d) **12.** (b)

CHAPTER 25 THE EFFECTS OF FORCES ON MATERIALS

EXERCISE 113, Page 253

1. A rectangular bar having a cross-sectional area of 80 mm ^{2} has a tensile force of 20 kN applied to it. Determine the stress in the bar.

Stress σ = $\frac{\text{force F}}{\text{area A}} = \frac{20\times10^3}{80\times10^{-6}} = 250\times10^6$ Pa = **250 MPa**

2. A circular section cable has a tensile force of 1 kN applied to it and the force produces a stress of 7.8 MPa in the cable. Calculate the diameter of the cable.

Stress σ = $\frac{\text{force F}}{\text{area A}}$ hence, cross-sectional area, A = $\frac{\text{force F}}{\text{stress}\ \sigma} = \frac{1\times10^3}{7.8\times10^6} = 128.2\times10^{-6}\,\text{m}^2$

Circular area = $\pi r^2 = 128.2\times10^{-6}\,\text{m}^2$

from which, $r^2 = \frac{128.2\times10^{-6}}{\pi}$ and radius r = $\sqrt{\frac{128.2\times10^{-6}}{\pi}} = 6.388\times10^{-3}$ m = 6.388 mm

and **diameter d** = 2 × r = 2 × 6.388 = **12.78 mm**

3. A square-sectioned support of side 12 mm is loaded with a compressive force of 10 kN. Determine the compressive stress in the support.

Stress σ = $\frac{\text{force F}}{\text{area A}} = \frac{10\times10^3}{12\times12\times10^{-6}} = 69.44\times10^6$ Pa = **69.44 MPa**

4. A bolt having a diameter of 5 mm is loaded so that the shear stress in it is 120 MPa. Determine the value of the shear force on the bolt.

Stress $\sigma = \dfrac{\text{force F}}{\text{area A}}$ hence, force = stress × area = stress × πr^2

$$= 120\times10^6 \times \pi\left(\frac{5\times10^{-3}}{2}\right)^2 = 2356 \text{ N or } \mathbf{2.356\ kN}$$

5. A split pin requires a force of 400 N to shear it. The maximum shear stress before shear occurs is 120 MPa. Determine the minimum diameter of the pin.

Stress $\sigma = \dfrac{\text{force F}}{\text{area A}}$ hence, cross-sectional area, A = $\dfrac{\text{force F}}{\text{stress }\sigma} = \dfrac{400}{120\times10^6} = 3.3333\times10^{-6}\text{m}^2$

Circular area = $\pi r^2 = 3.3333\times10^{-6}\text{m}^2$

from which, $r^2 = \dfrac{3.3333\times10^{-6}}{\pi}$ and radius r = $\sqrt{\dfrac{3.3333\times10^{-6}}{\pi}} = 1.030\times10^{-3}\text{m} = 1.030 \text{ mm}$

and **diameter d** = 2 × r = 2 × 1.030 = **2.06 mm**

6. A tube of outside diameter 60 mm and inside diameter 40 mm is subjected to a tensile load of 60 kN. Determine the stress in the tube.

Area of tube end (annulus) = $\pi\left(\dfrac{D^2}{4} - \dfrac{d^2}{4}\right) = \pi\left(\dfrac{\left(60\times10^{-3}\right)^2}{4} - \dfrac{\left(40\times10^{-3}\right)^2}{4}\right) = 1.5708 \times 10^{-3}\text{mm}^2$

Stress $\boldsymbol{\sigma} = \dfrac{\text{force F}}{\text{area A}} = \dfrac{60\times10^3}{1.5708\times10^{-3}} = 38.20\times10^6 \text{ Pa} = \mathbf{38.2\ MPa}$

EXERCISE 114, Page 255

1. A wire of length 4.5 m has a percentage strain of 0.050% when loaded with a tensile force. Determine the extension in the wire.

Original length of wire = 4.5 m = 4500 mm and strain = $\frac{0.050}{100}$ = 0.00050

Strain $\varepsilon = \frac{\text{extension x}}{\text{original length L}}$ hence, **extension x** = εL = (0.00050)(4500) = **2.25 mm**

2. A metal bar 2.5 m long extends by 0.05 mm when a tensile load is applied to it. Determine (a) the strain, (b) the percentage strain.

(a) **Strain ε** = $\frac{\text{extension}}{\text{original lengh}} = \frac{0.05\,\text{mm}}{2.5\times10^3\,\text{mm}} = \frac{0.05}{2500}$ = **0.00002**

(b) Percentage strain = 0.00002 × 100 = **0.002%**

3. An 80 cm long bar contracts axially by 0.2 mm when a compressive load is applied to it. Determine the strain and the percentage strain.

Strain ε = $\frac{\text{contraction}}{\text{original lengh}} = \frac{0.2\,\text{mm}}{800\,\text{mm}}$ = **0.00025**

Percentage strain = 0.00025 × 100 = **0.025%**

4. A pipe has an outside diameter of 20 mm, an inside diameter of 10 mm and length 0.30 m and it supports a compressive load of 50 kN. The pipe shortens by 0.6 mm when the load is applied. Determine (a) the compressive stress, (b) the compressive strain in the pipe when supporting this load.

Compressive force F = 50 kN = 50000 N, and cross-sectional area A = $\frac{\pi}{4}\left(D^2 - d^2\right)$, where D = outside diameter = 20 mm and d = inside diameter = 10 mm.

Hence, A = $\frac{\pi}{4}(20^2 - 10^2)\,\text{mm}^2 = \frac{\pi}{4}(20^2 - 10^2)\times 10^{-6}\,\text{m}^2 = 2.3562\times 10^{-4}\,\text{m}^2$

(a) Compressive stress, $\sigma = \frac{F}{A} = \frac{50000\,\text{N}}{2.3562\times 10^{-4}\,\text{m}^2} = 212.2 \times 10^6\,\text{Pa}$ = **212.2 MPa**

(b) Contraction of pipe when loaded, x = 0.6 mm = 0.0006 m, and original length L = 0.30 m.

Hence, compressive strain, $\varepsilon = \frac{x}{L} = \frac{0.0006}{0.3}$ = **0.002** (or **0.20%**)

5. A rectangular block of plastic material 400 mm long by 15 mm wide by 300 mm high has its lower face fixed to a bench and a force of 150 N is applied to the upper face and in line with it. The upper face moves 12 mm relative to the lower face. Determine (a) the shear stress, and (b) the shear strain in the upper face, assuming the deformation is uniform.

(a) Shear stress, $\tau = \frac{\text{force}}{\text{area parallel to the force}}$

Area of any face parallel to the force = 400 mm × 15 mm

$= (0.4 \times 0.015)\,\text{m}^2 = 0.006\,\text{m}^2$

Hence, **shear stress,** $\tau = \frac{150\,\text{N}}{0.006\,\text{m}^2}$ = **25000 Pa** or **25 kPa**

(b) Shear strain, $\gamma = \frac{x}{L}$

$= \frac{12}{300}$ = **0.04** (or **4%**)

EXERCISE 115, Page 258

1. A wire is stretched 1.5 mm by a force of 300 N. Determine the force that would stretch the wire 4 mm, assuming the elastic limit of the wire is not exceeded.

Hooke's law states that extension x is proportional to force F, provided that the limit of proportionality is not exceeded, i.e. $x \alpha F$ or $x = kF$ where k is a constant.

When x = 1.5 mm, F = 300 N, thus 1.5 = k(300), from which, constant $k = \frac{1.5}{300} = 0.005$

When x = 4 mm, then 4 = kF i.e. 4 = 0.005 F

from which, force $F = \frac{4}{0.005} = 800$ N

Thus to stretch the wire 4 mm, a force of 800 N is required.

2. A rubber band extends 50 mm when a force of 300 N is applied to it. Assuming the band is within the elastic limit, determine the extension produced by a force of 60 N.

Hooke's law states that extension x is proportional to force F, provided that the limit of proportionality is not exceeded, i.e. $x \alpha F$ or $x = kF$ where k is a constant.

When x = 50 mm, F = 300 N, thus 50 = k(300), from which, constant $k = \frac{50}{300} = \frac{1}{6}$

When F = 60 N, then x = k(60) i.e. $x = \left(\frac{1}{6}\right)(60) = 10$ mm

Thus, a force of 60 N stretches the wire 10 mm.

3. A force of 25 kN applied to a piece of steel produces an extension of 2 mm. Assuming the elastic limit is not exceeded, determine (a) the force required to produce an extension of 3.5 mm, (b) the extension when the applied force is 15 kN.

From Hooke’s law, extension x is proportional to force F within the limit of proportionality, i.e.

x α F or x = kF, where k is a constant. If a force of 25 kN produces an extension of 2 mm, then

2 = k(25), from which, constant k = $\frac{2}{25}$ = 0.08

(a) When an extension x = 3.5 mm, then 3.5 = k(F), i.e. 3.5 = 0.08 F,

from which, **force F =** $\frac{3.5}{0.08}$ **= 43.75 kN**

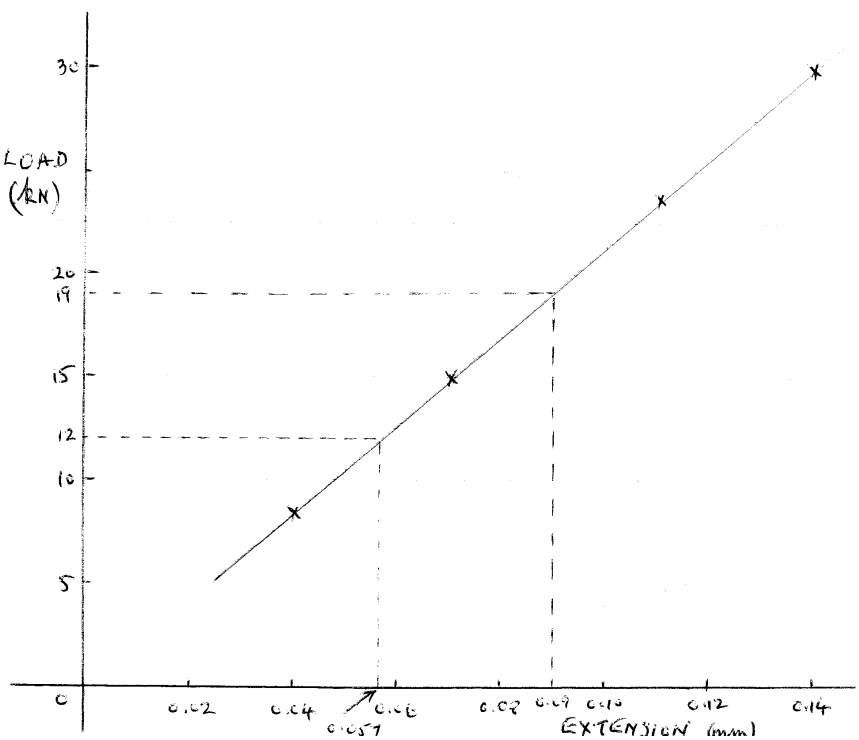
(b) When force F = 15 kN, then **extension x** = k(15) = (0.08)(15) = **1.2 mm**

4. A test to determine the load/extension graph for a specimen of copper gave the following results:

Load (kN)	8.5	15.0	23.5	30.0
Extension (mm)	0.04	0.07	0.11	0.14

Plot the load/extension graph, and from the graph determine (a) the load at an extension of 0.09 mm, and (b) the extension corresponding to a load of 12.0 kN.

A graph of load/extension is shown below.



(a) When the extension is 0.09 mm, the **load is 19 kN**

(b) When the load is 12.0 kN, the extension is **0.057 mm**

5. A circular section bar is 2.5 m long and has a diameter of 60 mm. When subjected to a compressive load of 30 kN it shortens by 0.20 mm. Determine Young's modulus of elasticity for the material of the bar.

Force, F = 30 kN = 30000 N and cross-sectional area A = $\pi r^2 = \pi\left(\dfrac{60\times10^{-3}}{2}\right)^2 = 2.8274\times10^{-3}\,m^2$

Stress $\sigma = \dfrac{F}{A} = \dfrac{30000}{2.8274\times10^{-3}}$ = 10.61 MPa

Bar shortens by 0.20 mm = 0.00020 m

Strain $\varepsilon = \dfrac{x}{L} = \dfrac{0.00020}{2.5} = 0.00008$

Modulus of elasticity, E = $\dfrac{\text{stress}}{\text{strain}} = \dfrac{10.61\times10^6}{0.00008} = 132.6\times10^9$ = **132.6 GPa**

6. A bar of thickness 20 mm and having a rectangular cross-section carries a load of 82.5 kN. Determine (a) the minimum width of the bar to limit the maximum stress to 150 MPa, (b) the modulus of elasticity of the material of the bar if the 150 mm long bar extends by 0.8 mm when carrying a load of 200 kN.

(a) Force, F = 82.5 kN = 82500 N and cross-sectional area A = $(20x)10^{-6}\,m^2$, where x is the width of the rectangular bar in millimetres.

Stress $\sigma = \dfrac{F}{A}$, from which, $A = \dfrac{F}{\sigma} = \dfrac{82500\,N}{150\times10^6\,Pa} = 5.5\times10^{-4}\,m^2 = 5.5\times10^{-4}\times10^6\,mm^2$

$= 5.5\times10^2\,mm^2 = 550\,mm^2$

Hence, 550 = 20x, from which, **width of bar**, $x = \dfrac{550}{20}$ = **27.5 mm**

(b) Stress $\sigma = \frac{F}{A} = \frac{200000}{550 \times 10^{-6}}$ = 363.64 MPa

Extension of bar = 0.8 mm

Strain $\varepsilon = \frac{x}{L} = \frac{0.8}{150}$ = 0.005333

Modulus of elasticity, E = $\frac{\text{stress}}{\text{strain}} = \frac{363.64 \times 10^6}{0.005333}$ = 68.2×10^9 = **68.2 GPa**

7. A metal rod of cross-sectional area 100 mm^2 carries a maximum tensile load of 20 kN. The modulus of elasticity for the material of the rod is 200 GPa. Determine the percentage strain when the rod is carrying its maximum load.

Stress $\sigma = \frac{F}{A} = \frac{20000}{100 \times 10^{-6}}$ = 200 MPa

Modulus of elasticity, E = $\frac{\text{stress}}{\text{strain}}$ from which, strain = $\frac{\text{stress}}{E} = \frac{200 \times 10^6}{200 \times 10^9}$ = 0.001

Hence, **percentage strain, ε** = 0.001 × 100% = **0.10%**

EXERCISE 116, Page 259

Answers found from within the text of the chapter, pages 250 to 259.

EXERCISE 117, Page 260

1. (c) **2.** (c) **3.** (a) **4.** (b) **5.** (c) **6.** (c) **7.** (b) **8.** (d) **9.** (b) **10.** (c) **11.** (f) **12.** (h)

13. (d)

CHAPTER 26 LINEAR MOMENTUM AND IMPULSE

EXERCISE 118, Page 265

1. Determine the momentum in a mass of 50 kg having a velocity of 5 m/s.

Momentum = mass × velocity = 50 kg × 5 m/s = **250 kg m/s downwards**

2. A milling machine and its component have a combined mass of 400 kg. Determine the momentum of the table and component when the feed rate is 360 mm/min.

Momentum = mass × velocity = 400 kg × $\frac{360\times10^{-3}}{60}$ m/s = **2.4 kg m/s downwards**

3. The momentum of a body is 160 kg m/s when the velocity is 2.5 m/s. Determine the mass of the body.

Momentum = mass × velocity

Hence, 160 = mass × 2.5

from which, **mass** = $\frac{160}{2.5}$ = **64 kg**

4. Calculate the momentum of a car of mass 750 kg moving at a constant velocity of 108 km/h.

Momentum = mass × velocity

Mass = 750 kg and velocity = 108 km/h = $\frac{108}{3.6}$ m/s = 30 m/s.

Hence, **momentum** = 750 kg × 30 m/s = **22,500 kg m/s**

5. A football of mass 200 g has a momentum of 5 kg m/s. What is the velocity of the ball in km/h.

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

Hence, $5 = 0.2 \times v$

from which, **velocity, v** $= \dfrac{5}{0.2} = \mathbf{25\ m/s}$

$= 25 \times 3.6 \text{ km/h} = \mathbf{90\ km/h}$

6. A wagon of mass 8 t is moving at a speed of 5 m/s and collides with another wagon of mass 12 t, which is stationary. After impact, the wagons are coupled together. Determine the common velocity of the wagons after impact.

Mass $m_1 = 8$ t = 8000 kg, $m_2 = 12000$ kg and velocity $u_1 = 5$ m/s, $u_2 = 0$.

Total momentum before impact $= m_1 u_1 + m_2 u_2$

$= (8000 \times 5) + (12000 \times 0) = 40000$ kg m/s

Let the common velocity of the wagons after impact be v m/s

Since total momentum before impact = total momentum after impact:

$$40000 = m_1 v + m_2 v$$

$$= v(m_1 + m_2) = v(20000)$$

Hence $v = \dfrac{40000}{20000} = 2$ m/s

i.e. **the common velocity after impact is 2 m/s in the direction in which the 8 t wagon is initially travelling.**

7. A car of mass 800 kg was stationary when hit head-on by a lorry of mass 2000 kg travelling at 15 m/s. Assuming no brakes are applied and the car and lorry move as one, determine the speed of the wreckage immediately after collision.

Mass $m_1 = 800$ kg, $m_2 = 2000$ kg and velocity $u_1 = 0$, $u_2 = 15$ m/s

Total momentum before impact = $m_1u_1 + m_2u_2$

$$= (800 \times 0) + (2000 \times 15) = 30000 \text{ kg m/s}$$

Let the common velocity of the wagons after impact be v m/s

Since total momentum before impact = total momentum after impact:

$$30000 = m_1v + m_2v$$

$$= v(m_1 + m_2) = v(2800)$$

Hence $$v = \frac{30000}{2800} = 10.71 \text{ m/s}$$

i.e. **the speed of the wreckage immediately after collision is 10.71 m/s in the direction in which the lorry is initially travelling.**

8. A body has a mass of 25 g and is moving with a velocity of 30 m/s. It collides with a second body which has a mass of 15 g and which is moving with a velocity of 20 m/s. Assuming that the bodies both have the same speed after impact, determine their common velocity (a) when the speeds have the same line of action and the same sense, and (b) when the speeds have the same line of action but are opposite in sense.

Mass m_1 = 25 g = 0.025 kg, m_2 = 15 g = 0.015 kg, velocity u_1 = 30 m/s and u_2 = 20 m/s.

(a) When the velocities have the same line of action and the same sense, both u_1 and u_2 are considered as positive values

Total momentum before impact = $m_1u_1 + m_2u_2 = (0.025 \times 30) + (0.015 \times 20)$

$$= 0.75 + 0.30 = 1.05 \text{ kg m/s}$$

Let the common velocity after impact be v m/s

Total momentum before impact = total momentum after impact

i.e. $$1.05 = m_1v + m_2v = v(m_1 + m_2)$$

$$1.05 = v(0.025 + 0.015)$$

from which, **common velocity,** $\mathbf{v} = \frac{1.05}{0.040}$ = **26.25 m/s in the direction in which the bodies are initially travelling**

(b) When the velocities have the same line of action but are opposite in sense, one is considered as positive and the other negative. Taking the direction of mass m_1 as positive gives:

velocity u_1 = +30 m/s and u_2 = - 20 m/s

Total momentum before impact = $m_1u_1 + m_2u_2 = (0.025 \times 30) + (0.015 \times -20)$

$= 0.75 - 0.30 = +0.45$ kg m/s

and since it is positive this indicates a momentum in the same direction as that of mass m_1.

If the common velocity after impact is v m/s then

$$0.45 = v(m_1 + m_2) = v(0.040)$$

from which, **common velocity,** $\mathbf{v} = \frac{0.45}{0.040}$ = **11.25 m/s in the direction that the 25 g mass is initially travelling.**

EXERCISE 119, Page 267

1. The sliding member of a machine tool has a mass of 200 kg. Determine the change in momentum when the sliding speed is increased from 10 mm/s to 50 mm/s.

Change of linear momentum = mass × change of velocity

Hence, **change in momentum** = 200 kg × (50 – 10) $\times 10^{-3}$ m/s

= **8 kg m/s**

2. A force of 48 N acts on a body of mass 8 kg for 0.25 s. Determine the change in velocity.

Impulse = applied force × time = change in linear momentum

i.e. 48 N × 0.25 s = mass × change in velocity

= 8 kg × change in velocity

from which, **change in velocity** $= \frac{48\,\text{N} \times 0.25\,\text{s}}{8\,\text{kg}}$ = **1.5 m/s** (since 1 N = 1 kg m/s^2)

3. The speed of a car of mass 800 kg is increased from 54 km/h to 63 km/h in 2 s. Determine the average force in the direction of motion necessary to produce the change in speed.

Change of momentum = applied force × time

i.e. mass × change of velocity = applied force × time

i.e. 800 kg × $\left(\frac{63}{3.6} - \frac{54}{3.6}\right)$ m/s = applied force × 2 s

from which, **applied force** = $\frac{800 \times \frac{9}{3.6}}{2}$ = **1000 N** or **1kN**

4. A 10 kg mass is dropped vertically on to a fixed horizontal plane and has an impact velocity of 15 m/s. The mass rebounds with a velocity of 5 m/s. If the contact time of mass and plane is 0.025 s, calculate (a) the impulse, and (b) the average value of the impulsive force on the plane.

(a) Impulse = change in momentum = $m(u_1 - v_1)$ where u_1 = impact velocity = 15 m/s and

v_1 = rebound velocity = - 5 m/s (v_1 is negative since it acts in the opposite direction to u_1)

Thus, **impulse** = $m(u_1 - v_1)$ = 10 kg (15 - - 5) m/s = 10 × 20 = **200 kg m/s**

(b) **Impulsive force** = $\frac{\text{impulse}}{\text{time}} = \frac{200\text{ kg m/s}}{0.025\text{ s}}$ = **8000 N** or **8 kN**

5. The hammer of a pile driver of mass 1.2 t falls 1.4 m on to a pile. The blow takes place in 20 ms and the hammer does not rebound. Determine the average applied force exerted on the pile by the hammer.

Initial velocity, u = 0, acceleration due to gravity, g = 9.81 m/s^2 and distance, s = 1.4 m.

Using the equation of motion: $v^2 = u^2 + 2gs$

gives: $v^2 = 0^2 + 2(9.81)(1.4)$

from which, impact velocity, $v = \sqrt{(2)(9.81)(1.4)}$ = 5.241 m/s

Neglecting the small distance moved by the pile and hammer after impact,

momentum lost by hammer = the change of momentum

= mv = 1200 kg × 5.241 m/s

Rate of change of momentum = $\frac{\text{change of momentum}}{\text{change of time}} = \frac{1200 \times 5.241}{20 \times 10^{-3}}$ = 314460 N

Since the impulsive force is the rate of change of momentum, **the average force exerted on the pile is 314.5 kN**

6. A tennis ball of mass 60 g is struck from rest with a racket. The contact time of ball on racket is 10 ms and the ball leaves the racket with a velocity of 25 m/s. Calculate (a) the impulse, and (b) the average force exerted by a racket on the ball.

(a) **Impulse** = change of momentum = mv

= (0.060 kg)(25 m/s) = **1.5 kg m/s**

(b) **Impulsive force** = $\frac{\text{impulse}}{\text{time}} = \frac{1.5\,\text{kg m/s}}{10 \times 10^{-3}}$ = **150 N**

7. In a press-tool operation, the tool is in contact with the work piece for 40 ms. If the average force exerted on the work piece is 90 kN, determine the change in momentum.

Change in momentum = applied force × time

= 90000 N × 40 × 10^{-3} = **3600 kg m/s**

EXERCISE 120, Page 267

Answers found from within the text of the chapter, pages 262 to 267.

EXERCISE 121, Page 267

1. (d) **2.** (b) **3.** (f) **4.** (c) **5.** (a) **6.** (c) **7.** (a) **8.** (g) **9.** (f) **10.** (f) **11.** (b) **12.** (e)

CHAPTER 27 TORQUE

EXERCISE 122, Page 271

1. Determine the torque developed when a force of 200 N is applied tangentially to a spanner at a distance of 350 mm from the centre of the nut.

Torque T = Fd, where force F = 200 N and distance, d = 350 mm = 0.35 m

Hence, **torque, T** = (200)(0.35) = **70 N m**

2. During a machining test on a lathe, the tangential force on the tool is 150 N. If the torque on the lathe spindle is 12 N m, determine the diameter of the work-piece.

Torque T = Fr, where torque T = 12 N m, force F = 150 N at radius r

Hence, $12 = (150)(r)$

from which, radius, $r = \frac{12}{150} = 0.08$ m = 80 mm

Hence, **diameter** = 2×80 = **160 mm**

EXERCISE 123, Page 273

1. A constant force of 4 kN is applied tangentially to the rim of a pulley wheel of diameter 1.8 m attached to a shaft. Determine the work done, in joules, in 15 revolutions of the pulley wheel.

Torque T = Fr, where F = 4000 N and radius r = $\frac{1.8}{2}$ = 0.9 m

Hence, torque T = (4000)(0.9) = 3600 N m

Work done = Tθ joules, where torque, T = 3600 N m and angular displacement,

θ = 15 revolutions = 15 × 2π rad = 30π rad.

Hence, **work done** = Tθ = (3600)(30π) = 339.3×10^3 = **339.3 kJ**

2. A motor connected to a shaft develops a torque of 3.5 kN m. Determine the number of revolutions made by the shaft if the work done is 11.52 MJ.

Work done = Tθ joules, where work done = 11.52×10^6 J and torque, T = 3500 N m

Hence, $11.52\times10^6 = 3500\times\theta$

from which, angular displacement, $\theta = \frac{11.52\times10^6}{3500}$ = 3291.43 rad

and **number of revolutions** = $\frac{3291.43}{2\pi}$ = **523.8 rev**

3. A wheel is turning with an angular velocity of 18 rad/s and develops a power of 810 W at this speed. Determine the torque developed by the wheel.

Power P = Tω, where P = 810 W and angular velocity, ω = 18 rad/s

Hence, $810 = T\times18$

from which, **torque, T** = $\frac{810}{18}$ = **45 N m**

4. Calculate the torque provided at the shaft of an electric motor that develops an output power of 3.2 hp at 1800 rev/min.

Power, P = 2πnT, where power P = 3.2 h.p. = 3.2 × 745.7 = 2386.24 W and $n = \frac{1800}{60} = 30$ rev/s

Hence, $2386.24 = 2\pi \times 30 \times T$

from which, **torque, T** = $\frac{2386.24}{2\pi \times 30}$ = **12.66 N m**

5. Determine the angular velocity of a shaft when the power available is 2.75 kW and the torque is 200 N m.

Power, P = 2πnT, where power P = 2750 W and torque T = 200 N m

Hence, $2750 = 2\pi \times n \times 200$

from which, $n = \frac{2750}{2\pi \times 200} = 2.1884$ rev/s

Angular velocity, ω = 2πn = 2π × 2.1884 = **13.75 rad/s**

6. The drive shaft of a ship supplies a torque of 400 kN m to its propeller at 400 rev/min. Determine the power delivered by the shaft.

Power, P = $\omega T = 2\pi nT = 2\pi \times \frac{400}{60} \times 400 \times 10^3$

$= 16.76 \times 10^6$ W = **16.76 MW**

7. A motor is running at 1460 rev/min and produces a torque of 180 N m. Determine the average power developed by the motor.

Power, P = $\omega T = 2\pi nT = 2\pi \times \frac{1460}{60} \times 180 = 27.52 \times 10^3$ W = **27.52 kW**

8. A wheel is rotating at 1720 rev/min and develops a power of 600 W at this speed. Calculate (a) the torque, (b) the work done, in joules, in a quarter of an hour.

(a) Power, P = $2\pi nT$ hence, $600 = 2\pi \times \frac{1720}{60} \times T$

from which, **torque, T** = $\frac{600 \times 60}{2\pi \times 1720}$ = **3.33 N m**

(b) Work done = $T\theta$, where torque T = 3.33 N m and

angular displacement in 15 minutes = (15×1720)rev = $(15 \times 1720 \times 2\pi)$ rad.

Hence, **work done** = $T\theta = (3.33)(15 \times 1720 \times 2\pi) = 540 \times 10^3$ J = **540 kJ**

EXERCISE 124, Page 275

1. A shaft system has a moment of inertia of 51.4 kg m^2. Determine the torque required to give it an angular acceleration of 5.3 rad/s^2.

Torque, T = Iα, where moment of inertia I = 51.4 kg m^2 and angular acceleration, α = 5.3 rad/s^2.

Hence, **torque, T** = Iα = (51.4)(5.3) = **272.4 N m**

2. A shaft has an angular acceleration of 20 rad/s^2 and produces an accelerating torque of 600 N m. Determine the moment of inertia of the shaft.

Torque, T = Iα, where torque T = 600 N m and angular acceleration, α = 20 rad/s^2.

Hence, $600 = I \times 20$

from which, **moment of inertia of the shaft, I** = $\frac{600}{20}$ = **30 kg $\mathbf{m^2}$**

3. A uniform torque of 3.2 kN m is applied to a shaft while it turns through 25 revolutions. Assuming no frictional or other resistance's, calculate the increase in kinetic energy of the shaft (i.e. the work done). If the shaft is initially at rest and its moment of inertia is 24.5 kg m^2, determine its rotational speed, in rev/min, at the end of the 25 revolutions.

Work done = Tθ = 3200 × (25 × 2π) = **502.65 kJ**

Increase in kinetic energy = 502650 J = $I\left(\frac{\omega_2^2 - \omega_1^2}{2}\right)$ where I = 24.5 kg m^2 and $\omega_1 = 0$

Hence, $502650 = 24.5\left(\frac{\omega_2^2 - 0}{2}\right)$

from which, $\omega_2^2 = \frac{502650 \times 2}{24.5}$ and $\omega_2 = \sqrt{\frac{502650 \times 2}{24.5}}$ = 202.565 rad/s

Hence, **rotational speed** = $202.565\,\text{rad/s} \times \dfrac{60\,\text{s/min}}{2\pi\,\text{rad/rev}}$ = **1934 rev/min**

4. An accelerating torque of 30 N m is applied to a motor, while it turns through 10 revolutions. Determine the increase in kinetic energy. If the moment of inertia of the rotor is 15 kg m^2 and its speed at the beginning of the 10 revolutions is 1200 rev/min, determine its speed at the end.

Increase in kinetic energy = work done = $T\theta = 30 \times (10 \times 2\pi)$ = **1885 J** or **1.885 kJ**

Increase in kinetic energy = 1885 J = $I\left(\dfrac{\omega_2^2 - \omega_1^2}{2}\right)$

where I = 15 kg m^2 and $\omega_1 = 1200 \times \dfrac{2\pi}{60} = 40\pi = 125.664$ rad/s

Hence, $$1885 = 15\left(\frac{\omega_2^2 - 125.664^2}{2}\right)$$

from which, $$\omega_2^2 - 125.664^2 = \frac{1885 \times 2}{15} = 251.333$$

Hence, $$\omega_2^2 = 251.333 + 125.664^2 = 16042.774$$

and $$\omega_2 = \sqrt{16042.774} = 126.66 \text{ rad/s}$$

Hence, **final speed** = $126.66\,\text{rad/s} \times \dfrac{60\,\text{s/min}}{2\pi\,\text{rad/rev}}$ = **1209.5 rev/min**

5. A shaft with its associated rotating parts has a moment of inertia of 48 kg m^2. Determine the uniform torque required to accelerate the shaft from rest to a speed of 1500 rev/min while it turns through 15 revolutions.

Work done = increase in kinetic energy = $T\theta = I\left(\dfrac{\omega_2^2 - \omega_1^2}{2}\right)$

where I = 48 kg m^2, $\omega_1 = 0$ and $\omega_2 = 1500 \times \dfrac{2\pi}{60} = 157.08$ rad/s

Hence, $T\theta = I\left(\frac{\omega_2^{\,2} - \omega_1^{\,2}}{2}\right)$ i.e. $T(15 \times 2\pi) = 48 \times \left(\frac{157.08^2 - 0}{2}\right) = 592179$

from which, **torque, T** $= \frac{592179}{15 \times 2\pi}$ = **6283 N m** or **6.283 kN m**

6. A small body, of mass 82 g, is fastened to a wheel and rotates in a circular path of 456 mm diameter. Calculate the increase in kinetic energy of the body when the speed of the wheel increases from 450 rev/min to 950 rev/min.

Increase in kinetic energy $= I\left(\frac{\omega_2^{\,2} - \omega_1^{\,2}}{2}\right) = m\,r^2\left(\frac{\omega_2^{\,2} - \omega_1^{\,2}}{2}\right)$

$$= (0.082)\left(\frac{0.456}{2}\right)^2\left(\frac{\left(\frac{950 \times 2\pi}{60}\right)^2 - \left(\frac{450 \times 2\pi}{60}\right)^2}{2}\right)$$

$$= (0.082)(0.051984)(\left(\frac{99.484^2 - 47.124^2}{2}\right)$$

= **16.36 J**

7. A system consists of three small masses rotating at the same speed about the same fixed axis. The masses and their radii of rotation are: 16 g at 256 mm, 23 g at 192 mm and 31 g at 176 mm. Determine (a) the moment of inertia of the system about the given axis, and (b) the kinetic energy in the system if the speed of rotation is 1250 rev/min.

(a) **Moment of inertia, I** $= \sum mr^2 = (0.016)(0.256)^2 + (0.023)(0.192)^2 + (0.031)(0.176)^2$

$$= 1.0486 \times 10^{-3} + 8.4787 \times 10^{-4} + 9.6026 \times 10^{-4}$$

$$= \mathbf{2.857 \times 10^{-3}\,kg\,m^2}$$

(b) **Kinetic energy in the system** = $I\frac{\omega^2}{2} = \left(2.857\times10^{-3}\right)\left(\frac{\left(\frac{1250\times2\pi}{60}\right)^2}{2}\right)$ = **24.48 J**

EXERCISE 125, Page 277

1. A motor has an efficiency of 72% when running at 2600 rev/min. If the output torque is 16 N m at this speed, determine the power supplied to the motor.

Power output, $P = 2\pi nT$

$= 2\pi\,(2600/60)(16) = 4356.34$

Efficiency = $\dfrac{\text{power output}}{\text{power input}} \times 100\%$ hence $72 = \dfrac{4356.34}{\text{power input}} \times 100$

from which, **power input** = $\dfrac{4356.34}{72} \times 100$ = **6050 W** or **6.05 kW**

2. The difference in tensions between the two sides of a belt round a driver pulley of radius 240 mm is 200 N. If the driver pulley wheel is on the shaft of an electric motor running at 700 rev/min and the power input to the motor is 5 kW, determine the efficiency of the motor. Determine also the diameter of the driven pulley wheel if its speed is to be 1200 rev/min.

Power output from motor = $(F_2 - F_1)r_x\,\omega_x$

$(F_2 - F_1) = 200$ N, radius $r_x = 240$ mm $= 0.24$ m and angular velocity, $\omega_x = \dfrac{700 \times 2\pi}{60}$ rad/s

Hence, power output from motor = $(F_2 - F_1)r_x\,\omega_x = (200)(0.24)\left(\dfrac{700 \times 2\pi}{60}\right) = 3518.58$ W

Power input = 5000 W

Hence, **efficiency of the motor** = $\dfrac{\text{power output}}{\text{power input}} = \dfrac{3518.58}{5000} \times 100$ = **70.37%**

$\dfrac{r_x}{r_y} = \dfrac{n_y}{n_x}$ from which, driven pulley wheel radius, $r_y = \dfrac{n_x r_x}{n_y} = \dfrac{700 \times 0.24}{1200} = 0.14$ m

from which, **diameter of driven pulley wheel** = 2 × radius = 2 × 0.14 = **0.28 m** or **280 mm**

3. A winch is driven by a 4 kW electric motor and is lifting a load of 400 kg to a height of 5.0 m. If the lifting operation takes 8.6 s, calculate the overall efficiency of the winch and motor.

The increase in potential energy is the work done and is given by mgh (see Chapter 20), where mass, m = 400 kg, g = 9.81 m/s^2 and height h = 5.0 m.

Hence, work done = mgh = (400)(9.81)(5.0) = 19.62 kJ.

Input power = 4 kW = 4000 W Output power = $\frac{\text{work done}}{\text{time taken}} = \frac{19620}{8.6} = 2281.4$ W

Efficiency = $\frac{\text{output power}}{\text{input power}} \times 100 = \frac{2281.4}{4000} \times 100 =$ **57.03%**

4. A belt and pulley system transmits a power of 5 kW from a driver to a driven shaft. The driver pulley wheel has a diameter of 200 mm and rotates at 600 rev/min. The diameter of the driven wheel is 400 mm. Determine the speed of the driven pulley and the tension in the slack side of the belt when the tension in the tight side of the belt is 1.2 kN.

r_x = 100 mm = 0.1 m, n_x = 600 rev/min, r_y = 200 mm = 0.2 m

$\frac{r_x}{r_y} = \frac{n_y}{n_x}$ from which, **speed of driven pulley**, $n_y = \frac{r_x n_x}{r_y} = \frac{0.1 \times 600}{0.2}$ = **300 rev/min**

Available power = $(F_2 - F_1) r_x \omega_x$

i.e. $5000 = (1200 - F_1)(0.1)\left(600 \times \frac{2\pi}{60}\right)$

i.e. $(1200 - F_1) = \frac{5000}{0.1\left(600 \times \frac{2\pi}{60}\right)} = 795.8$

Hence, **tension in slack side of belt**, $\mathbf{F_1}$ = 1200 – 795.8 = **404.2 N**

5. The average force on the cutting tool of a lathe is 750 N and the cutting speed is 400 mm/s. Determine the power input to the motor driving the lathe if the overall efficiency is 55%.

Force resisting motion = 750 N and velocity = 400 mm/s = 0.4 m/s

Output power from motor = resistive force × velocity of lathe (from Chapter 20)

$$= 750 \times 0.4 = 300 \text{ W}$$

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}} \times 100$$

hence $$55 = \frac{300}{\text{power input}} \times 100$$

from which, **power input** = $300 \times \frac{100}{55}$ = **545.5 W**

6. A ship's anchor has a mass of 5 t. Determine the work done in raising the anchor from a depth of 100 m. If the hauling gear is driven by a motor whose output is 80 kW and the efficiency of the haulage is 75%, determine how long the lifting operation takes.

The increase in potential energy is the work done and is given by mgh (see Chapter 20), where mass, m = 5 t = 5000 kg, g = 9.81 m/s^2 and height h = 100 m

Hence, **work done** = mgh = (5000)(9.81)(100) = **4.905 MJ**

Input power = 80 kW = 80000 W

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} \times 100$$

hence $$75 = \frac{\text{output power}}{80000} \times 100$$

from which, output power = $\frac{75}{100} \times 80000 = 60000 \text{ W} = \frac{\text{work done}}{\text{time taken}}$

Thus, **time taken for lifting operation** = $\frac{\text{work done}}{\text{output power}} = \frac{4.905 \times 10^6 \text{ J}}{60000 \text{ W}}$

= **81.75 s** = **1 min 22 s** to the nearest second.

EXERCISE 126, Page 278

Answers found from within the text of the chapter, pages 270 to 277.

EXERCISE 127, Page 278

1. (d) **2.** (b) **3.** (c) **4.** (a) **5.** (c) **6.** (d) **7.** (a) **8.** (b) **9.** (c) **10.** (d) **11.** (a) **12.** (c)

CHAPTER 28 PRESSURE IN FLUIDS

EXERCISE 128, Page 281

1. A force of 280 N is applied to a piston of a hydraulic system of cross-sectional area 0.010 m^2.

Determine the pressure produced by the piston in the hydraulic fluid.

Pressure, p = $\frac{\text{force}}{\text{area}} = \frac{280\,\text{N}}{0.010\,\text{m}^2} = 28000\,\text{Pa}$ = **28 kPa**

That is, **the pressure produced by the piston is 28 kPa**

2. Find the force on the piston of question 1 to produce a pressure of 450 kPa.

Pressure, p = 450 kPa = 450000 Pa

Pressure p = $\frac{\text{force}}{\text{area}}$ hence, **force** = pressure × area

$= 4540000 \times 0.010 = 4500$ N = **4.5 kN**

3. If the area of the piston in question 1 is halved and the force applied is 280 N, determine the new pressure in the hydraulic fluid.

New area = $\frac{0.010}{2} = 0.005\,\text{m}^2$

New pressure, p = $\frac{\text{force}}{\text{area}} = \frac{280\,\text{N}}{0.005\,\text{m}^2} = 56000\,\text{Pa}$ = **56 kPa**

EXERCISE 129, Page 283

1. Determine the pressure acting at the base of a dam, when the surface of the water is 35 m above base level. Take the density of water as 1000 kg/m^3. Take the gravitational acceleration as 9.8 m/s^2.

Pressure at base of dam, p = ρgh = 1000 $kg/m^3 \times 9.8\ m/s^2 \times 0.35$ m = **343000 Pa** = **343 kPa**

2. An uncorked bottle is full of sea water of density 1030 kg/m^3. Calculate, correct to 3 significant figures, the pressures on the side wall of the bottle at depths of (a) 30 mm, and (b) 70 mm below the top of the bottle. Take the gravitational acceleration as 9.8 m/s^2.

Pressure on the side wall of the bottle, p = ρgh

(a) When depth, h = 30 mm = 30×10^{-3} m,

pressure, p = 1030 $kg/m^3 \times 9.8\ m/s^2 \times 30 \times 10^{-3}\ m$ = **303 Pa**

(b) When depth, h = 70 mm = 70×10^{-3} m,

pressure, p = 1030 $kg/m^3 \times 9.8\ m/s^2 \times 70 \times 10^{-3}\ m$ = **707 Pa**

3. A U-tube manometer is used to determine the pressure at a depth of 500 mm below the free surface of a fluid. If the pressure at this depth is 6.86 kPa, calculate the density of the liquid used in the manometer. Take the gravitational acceleration as 9.8 m/s^2

Pressure, p = ρgh

hence, $6.86 \times 10^3\ Pa = \rho \times 9.8\ m/s^2 \times 500 \times 10^{-3}\ m$

from which, **density of liquid, ρ** = $\frac{6.86 \times 10^3}{9.8 \times 500 \times 10^{-3}}$ = **1400 kg/m³**

EXERCISE 130, Page 283

1. The height of a column of mercury in a barometer is 750 mm. Determine the atmospheric pressure, correct to 3 significant figures. Take the gravitational acceleration as 9.8 m/s^2 and the density of mercury as 13600 kg/m.

Atmospheric pressure, p = $\rho gh = 13600 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 750 \times 10^{-3} \text{m}$

$= 99960$ Pa = **100 kPa**

2. A U-tube manometer containing mercury gives a height reading of 250 mm of mercury when connected to a gas cylinder. If the barometer reading at the same time is 756 mm of mercury, calculate the absolute pressure of the gas in the cylinder, correct to 3 significant figures. Take the gravitational acceleration as 9.8 m/s^2 and the density of mercury as 13600 kg/m.

Pressure, $p_1 = \rho gh = 13600 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 250 \times 10^{-3} \text{m}$

$= 33320$ Pa = **33.32 kPa**

Pressure, $p_2 = \rho gh = 13600 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 756 \times 10^{-3} \text{m}$

$= 100760$ Pa = **100.76 kPa**

Absolute pressure = atmospheric pressure + gauge pressure

$= p_2 + p_1 = 100.76 + 33.32 =$ **134 kPa**

3. A water manometer connected to a condenser shows that the pressure in the condenser is 350 mm below atmospheric pressure. If the barometer is reading 760 mm of mercury, determine the absolute pressure in the condenser, correct to 3 significant figures. Take the gravitational acceleration as 9.8 m/s^2 and the density of water as 100 kg/m.

Pressure, $p_1 = -\rho_1 gh_1 = -1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 350 \times 10^{-3} \text{m}$

$= - 3430 \text{ Pa} = \mathbf{- 3.43 \text{ kPa}}$

Pressure, $p_2 = \rho_2 g h_2 = 13600 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 760 \times 10^{-3} \text{m}$

$= 101293 \text{ Pa} = \mathbf{101.3 \text{ kPa}}$

Absolute pressure = atmospheric pressure + gauge pressure

$= p_2 + p_1 = 101.3 - 3.43 = \mathbf{97.9 \text{ kPa}}$

4. A Bourdon pressure gauge shows a pressure of 1.151 MPa. If the absolute pressure is 1.25 MPa, find the atmospheric pressure in millimetres of mercury. Take the gravitational acceleration as 9.8 m/s ^{2} and the density of mercury as 13600 kg/m.

Atmospheric pressure = absolute pressure - gauge pressure

$= 1.25 \text{ MPa} - 1.151 \text{ MPa} = 0.099 \text{ MPa} = 0.099 \times 10^6 \text{ Pa}$

Atmospheric pressure, $p = \rho g h = 13600 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times h$

i.e. $0.099 \times 10^6 \text{ Pa} = 13600 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times h$

from which, height, $h = \dfrac{0.099 \times 10^6 \text{ Pa}}{13600 \text{kg/m}^3 \times 9.8 \text{m/s}^2} = 0.743 \text{ m}$

i.e. **atmospheric pressure in millimetres of mercury** $= 0.743 \text{ m} \times \dfrac{1000 \text{ mm}}{1 \text{ m}} = \mathbf{743 \text{ mm}}$

EXERCISE 131, Page 285

1. A body of volume 0.124 m^3 is completely immersed in water of density 1000 kg/m^3. What is the apparent loss of weight of the body? Take the gravitational acceleration as 9.8 m/s^2.

Mass, m = density, ρ × volume, V = 1000 kg/m^3 × 0.124 m^3

= 124 kg

Apparent loss of weight of the body, W = ρ × V × g

= 124 kg × 9.8 m/s^2 = 1215 N = **1.215 kN**

2. A body of weight 27.4 N and volume 1240 cm^3 is completely immersed in water of specific weight 9.81 kN/m^3. What is its apparent weight? Take the gravitational acceleration as 9.8 m/s^2 and the density of water as 1000 kg/m^3.

Body weight, W_1 = 27.4 N

Apparent weight, W_2 = 27.4 - ρ × V × g

$= 27.4 - (1000\ kg/m^3 \times 1240 \times 10^{-6} m^3 \times 9.8\ m/s^2)$

= 27.4 N – 12.152 N

= **15.25 N**

3. A body weighs 512.6 N in air and 256.8 N when completely immersed in oil of density 810 kg/m^3. What is the volume of the body? Take the gravitational acceleration as 9.8 m/s^2.

$W = \rho_{oil} \times V \times g$

i.e. $(512.6 - 256.8) = \rho_{oil} \times V \times g$

i.e. $255.8 = 810 \times V \times 9.8$

from which, **volume,** $\mathbf{V} = \frac{255.8}{810 \times 9.8} = \mathbf{0.03222\ m^3}$ or $\mathbf{32.22\ dm^3}$

4. A body weighs 243 N in air and 125 N when completely immersed in water. What will it weigh when completely immersed in oil of relative density 0.8? Take the gravitational acceleration as $9.8\ m/s^2$ and the density of water as $1000\ kg/m^3$.

$$W = \rho \times V \times g$$

i.e. $(243 - 125) = \rho_{water} \times V \times g$

i.e. $118 = 1000 \times V \times 9.8$

from which, volume, $V = \frac{118}{1000 \times 9.8} = 0.012041\ m^3$

Weight in oil $= 243 - \rho_{oil} \times V \times g = 243 - (0.8 \times 1000) \times 0.012041 \times 9.8$

$= 243 - 94.4 = \mathbf{148.6\ N}$

5. A watertight rectangular box, 1.2 m long and 0.75 m wide, floats with its sides and ends vertical in water of density $1000\ kg/m^3$. If the depth of the box in the water is 280 mm, what is its weight? Take the gravitational acceleration as $9.8\ m/s^2$.

Volume of box, $V = 1.2\ m \times 0.75\ m \times 280 \times 10^{-3} m = 0.252\ m^3$

Weight of box, $W = \rho \times V \times g$

$= 1000 \times 0.252 \times 9.8$

$= 2469.6\ N = \mathbf{2.47\ kN}$

6. A body weighs 18 N in air and 13.7 N when completely immersed in water of density $1000\ kg/m^3$. What is the density and relative density of the body? Take the gravitational acceleration as $9.8\ m/s^2$.

$$W = \rho \times V \times g$$

i.e. $(18 - 13.7) = \rho_{water} \times V \times g$

i.e. $4.3 = 1000 \times V \times 9.8$

from which, volume, $V = \dfrac{4.3}{1000 \times 9.8} = 4.388 \times 10^{-4}\,m^3$

Density of body, ρ = $\dfrac{mass}{volume} = \dfrac{18\,N}{4.388 \times 10^{-4}\,m^3} \times \dfrac{1}{9.8\,N/kg}$

= **4186 kg/ m^3** or **4.186 tonnes/ m^3**

Relative density = $\dfrac{density}{density\ of\ water} = \dfrac{4186\,kg/m^3}{1000\,kg/m^3}$ **= 4.186**

7. A watertight rectangular box is 660 mm long and 320 mm wide. Its weight is 336 N. If it floats with its sides and ends vertical in water of density 1020 kg/m^3, what will be its depth in the water? Take the gravitational acceleration as 9.8 m/s^2.

Volume of box, $V = 660 \times 10^{-3}\,m \times 320 \times 10^{-3}\,m \times D$ where D = depth of box

Weight, $W = \rho \times V \times g$

i.e. $336 = 1020\,kg/m^3 \times (660 \times 10^{-3}\,m \times 320 \times 10^{-3}\,m \times D) \times 9.8$

from which, **depth of box, D** = $\dfrac{336}{1020 \times 0.66 \times 0.32 \times 9.8}$ = 0.159 m = **159 mm**

8. A watertight drum has a volume of 0.165 m^3 and a weight of 115 N. It is completely submerged in water of density 1030 kg/m^3, held in position by a single vertical chain attached to the underside of the drum. What is the force in the chain? Take the gravitational acceleration as 9.8 m/s^2.

Weight of drum, W = 115 N

Upthrust = $\rho \times V \times g = 1030 \text{ kg/m}^3 \times 0.165 \text{ m}^3 \times 9.8 \text{ m/s}^2 = 1665.5 \text{ N}$

Hence, **the force in the chain** = 1665.5 – 115 = 1551 N = **1.551 kN**

EXERCISE 132, Page 291

Answers found from within the text of the chapter, pages 280 to 291.

EXERCISE 133, Page 291

1. (b) **2.** (d) **3.** (a) **4.** (a) **5.** (c) **6.** (d) **7.** (b) **8.** (c) **9.** (c) **10.** (d) **11.** (d) **12.** (d)

13. (c) **14.** (b) **15.** (c) **16.** (a) **17.** (b) **18.** (f) **19.** (a) **20.** (b) **21.** (c)

CHAPTER 29 HEAT ENERGY AND TRANSFER

EXERCISE 134, Page 295

1. Convert the following temperatures into the Kelvin scale: (a) 51°C (b) - 78°C (c) 183°C

Kelvin temperature, K = °C + 273

(a) When Celsius temperature = 51°C, **K** = 51 + 273 = **324 K**

(b) When Celsius temperature = - 78°C, **K** = - 78 + 273 = **195 K**

(c) When Celsius temperature = 183°C, **K** = 183 + 273 = **456 K**

2. Convert the following temperatures into the Celsius scale: (a) 307 K (b) 237 K (c) 415 K

If K = °C + 273 then °C = K - 273

(a) When Kelvin temperature = 307 K, **°C** = 307 – 273 = **34°C**

(a) When Kelvin temperature = 237 K, **°C** = 237 – 273 = **- 36°C**

(a) When Kelvin temperature = 415 K, **°C** = 415 – 273 = **142°C**

EXERCISE 135, Page 297

1. Determine the quantity of heat energy (in megajoules) required to raise the temperature of 10 kg of water from 0°C to 50°C. Assume the specific heat capacity of water is 4200 J/(kg °C).

Quantity of heat energy, **Q** = $mc(t_2 - t_1)$

$= 10 \text{ kg} \times 4200 \text{ J/(kg °C)} \times (50 - 0)\text{°C}$

= **2100000 J** or **2100 kJ** or **2.1 MJ**

2. Some copper, having a mass of 20 kg, cools from a temperature of 120°C to 70°C. If the specific heat capacity of copper is 390 J/(kg °C), how much heat energy is lost by the copper ?

Quantity of heat energy, **Q** = $mc(t_2 - t_1)$

$= 20 \text{ kg} \times 390 \text{ J/(kg °C)} \times (70 - 120)\text{°C}$

$= 20 \times 390 \times -50 = -390000$ J or 390 kJ

Hence, **the heat energy lost by the copper = 390 kJ**

3. A block of aluminium having a specific heat capacity of 950 J/(kg °C) is heated from 60°C to its melting point at 660°C. If the quantity of heat required is 2.85 MJ, determine the mass of the aluminium block.

Quantity of heat, $Q = mc(t_2 - t_1)$, hence,

$2.85 \times 10^6 \text{ J} = m \times 950 \text{ J/(kg °C)} \times (660 - 60)\text{°C}$

i.e. $2850000 = m \times 950 \times 600$

from which, **mass, m** = $\frac{2850000}{950 \times 600}$ kg = **5 kg**

---

4. 20.8 kJ of heat energy is required to raise the temperature of 2 kg of lead from 16°C to 96°C. Determine the specific heat capacity of lead.

Quantity of heat, $Q = mc(t_2 - t_1)$, hence:

20.8×10^3 J = 2 kg × c × (96 – 16)°C where c is the specific heat capacity,

i.e. $20800 = 2 \times c \times 80$

from which, **specific heat capacity of lead**, $c = \dfrac{20800}{2\times 80}$ = **130 J/(kg °C)**

5. 250 kJ of heat energy is supplied to 10 kg of iron which is initially at a temperature of 15°C. If the specific heat capacity of iron is 500 J/(kg °C) determine its final temperature.

Quantity of heat, $Q = mc(t_2 - t_1)$, hence,

250×10^3 J = 10 kg × 500 J/(kg °C) × (t_2 - 15)°C

from which, $(t_2 - 15) = \dfrac{250\times 10^3}{10\times 500} = 50$

Hence, the **final temperature, t_2** = 50 + 15 = **65°C**

EXERCISE 136, Page 299

1. Some ice, initially at - 40°C, has heat supplied to it at a constant rate until it becomes superheated steam at 150°C. Sketch a typical temperature/time graph expected and use it to explain the difference between sensible and latent heat.

See Section 29.5 and Figure 29.1 on page 298 of textbook. Just replace the - 30°C at A with - 40°C and replace 120°C at F with 150°C.

EXERCISE 137, Page 300

1. How much heat is needed to melt completely 25 kg of ice at 0°C. Assume the specific latent heat of fusion of ice is 335 kJ/kg.

Quantity of heat required, **Q**= mL = 25 kg × 335 kJ/kg

= **8375 kJ** or **8.375 MJ**

2. Determine the heat energy required to change 8 kg of water at 100°C to superheated steam at 100°C. Assume the specific latent heat of vaporisation of water is 2260 kJ/kg.

Quantity of heat required, **Q** = mL = 8 kg × 2260 kJ/kg

= **18080 kJ** or **18.08 MJ**

3. Calculate the heat energy required to convert 10 kg of ice initially at - 30°C completely into water at 0°C. Assume the specific heat capacity of ice is 2.1 kJ/(kg °C) and the specific latent heat of fusion of ice is 335 kJ/kg.

Quantity of heat energy needed, Q = sensible heat + latent heat.

The quantity of heat needed to raise the temperature of ice from - 30°C to 0°C

i.e. sensible heat, $Q_1 = mc(t_2 - t_1)$ = 10 kg × 2100 J/(kg°C) × (0 - - 30)°C

= (10 × 2100 × 30) J = **630 kJ**

The quantity of heat needed to melt 10 kg of ice at 0°C,

i.e. the latent heat, Q_2 = mL = 10 kg × 335 kJ/kg = **3350 kJ**

Total heat energy needed, Q = $Q_1 + Q_2$ = 630 + 3350 = **3980 kJ** = **3.98 MJ**

4. Determine the heat energy needed to convert completely 5 kg of water at 60°C to steam at 100°C, given that the specific heat capacity of water is 4.2 kJ/(kg °C) and the specific latent heat of vaporisation of water is 2260 kJ/kg.

Quantity of heat required = sensible heat + latent heat.

Sensible heat, $Q_1 = mc(t_2 - t_1) = 5 \text{ kg} \times 4.2 \text{ kJ/(kg °C)} \times (100 - 60)\text{°C}$

$= \mathbf{840 \text{ kJ}}$

Latent heat, $Q_2 = mL = 5 \text{ kg} \times 2260 \text{ kJ/kg} = \mathbf{11300 \text{ kJ}}$

Total heat energy required, $Q = Q_1 + Q_2 = (840 + 11300) \text{ kJ}$

$= \mathbf{12140 \text{ kJ}}$ or $\mathbf{12.14 \text{ MJ}}$

EXERCISE 138, Page 302

Answers found from within the text of the chapter, pages 294 to 302.

EXERCISE 139, Page 303

1. (d) **2.** (b) **3.** (a) **4.** (c) **5.** (b) **6.** (b) **7.** (b) **8.** (a) **9.** (c) **10.** (b) **11.** (d) **12.** (c) **13.** (d)

CHAPTER 3 INDICES, UNITS, PREFIXES AND ENGINEERING NOTATION

EXERCISE 13, Page 25

1. Evaluate 3^3 without the aid of a calculator

$3^3 = 3 \times 3 \times 3 = \mathbf{27}$

2. Evaluate 2^7 without the aid of a calculator

$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = \mathbf{128}$

3. Evaluate 10^5 without the aid of a calculator

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = \mathbf{100{,}000}$

4. Evaluate $2^4 \times 3^2 \times 2 \div 3$ without the aid of a calculator

$$2^4 \times 3^2 \times 2 \div 3 = \frac{2^4 \times 2 \times 3^2}{3} = \frac{16 \times 2 \times 9}{3} = \frac{16 \times 2 \times 3}{1} = 16 \times 2 \times 3 = 32 \times 3 = \mathbf{96}$$

5. Evaluate $25^{\frac{1}{2}}$ without the aid of a calculator

$25^{\frac{1}{2}} = \sqrt{25} = \pm\,\mathbf{5}$

6. Evaluate $\dfrac{10^5}{10^3}$ without the aid of a calculator

$$\frac{10^5}{10^3} = \frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10} = \frac{10 \times 10 \times 1 \times 1 \times 1}{1 \times 1 \times 1} \text{ by cancelling}$$

$$= 10 \times 10 = \mathbf{100}$$

7. Evaluate $\frac{10^2 \times 10^3}{10^5}$ without the aid of a calculator

$$\frac{10^2 \times 10^3}{10^5} = \frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{1} = \mathbf{1}$$

8. Evaluate $\frac{2^5 \times 64^{\frac{1}{2}} \times 3^2}{\sqrt{144} \times 3}$ taking positive square roots only

$$\frac{2^5 \times 64^{\frac{1}{2}} \times 3^2}{\sqrt{144} \times 3} = \frac{2^5 \times \sqrt{64} \times 3^2}{12 \times 3} = \frac{32 \times 8 \times 9}{36} = \frac{32 \times 8}{4} = \frac{32 \times 2}{1} = 32 \times 2 = \mathbf{64}$$

EXERCISE 14, Page 28

1. Evaluate $2^2 \times 2 \times 2^4$ without the aid of a calculator

$2^2 \times 2 \times 2^4 = 2^2 \times 2^1 \times 2^4 = 2^{2+1+4} = 2^7 = \mathbf{128}$

2. Evaluate $3^5 \times 3^3 \times 3$ in index form without the aid of a calculator

$3^5 \times 3^3 \times 3 = 3^5 \times 3^3 \times 3^1 = 3^{5+3+1} = \mathbf{3^9}$

3. Evaluate $\frac{2^7}{2^3}$ without the aid of a calculator

$\frac{2^7}{2^3} = 2^{7-3} = 2^4 = \mathbf{16}$

4. Evaluate $\frac{3^3}{3^5}$ without the aid of a calculator

$\frac{3^3}{3^5} = 3^{3-5} = 3^{-2} = \frac{1}{3^3} = \mathbf{\frac{1}{9}}$

5. Evaluate 7^0 without the aid of a calculator

$7^0 = \mathbf{1}$ (any real number raised to the power zero is 1)

6. Evaluate $\frac{2^3 \times 2 \times 2^6}{2^7}$ without the aid of a calculator

$\frac{2^3 \times 2 \times 2^6}{2^7} = 2^{3+1+6-7} = 2^3 = \mathbf{8}$

7. Evaluate $\dfrac{10\times10^6}{10^5}$ without the aid of a calculator

$$\frac{10\times10^6}{10^5}=10^{1+6-5}=10^2 = \mathbf{100}$$

8. Evaluate $10^4\div10$ without the aid of a calculator

$$10^4\div10=\frac{10^4}{10}=\frac{10^4}{10^1}=10^{4-1}=10^3 = \mathbf{1000}$$

9. Evaluate $\dfrac{10^3\times10^4}{10^9}$ without the aid of a calculator

$$\frac{10^3\times10^4}{10^9}=10^{3+4-9}=10^{-2}=\frac{1}{10^2} = \mathbf{\frac{1}{100}} \text{ or } 0.01$$

10. Evaluate $5^6\times5^2\div5^7$ without the aid of a calculator

$$5^6\times5^2\div5^7=\frac{5^6\times5^2}{5^7}=5^{6+2-7}=5^1 = \mathbf{5}$$

11. Evaluate $(7^2)^3$ in index form without the aid of a calculator

$$\left(7^2\right)^3=7^{2\times3} = \mathbf{7^6}$$

12. Evaluate $(3^3)^2$ without the aid of a calculator

$$\left(3^3\right)^2=3^{3\times2} = \mathbf{3^6} \text{ or } \mathbf{729}$$

---

13. Evaluate $\dfrac{3^2 \times 3^{-4}}{3^3}$ without the aid of a calculator

$$\frac{3^2 \times 3^{-4}}{3^3} = \frac{3^{2-4}}{3^3} = \frac{3^{-2}}{3^3} = 3^{-2-3} = 3^{-5} = \mathbf{\frac{1}{3^5}} \text{ or } \mathbf{\frac{1}{243}}$$

14. Evaluate $\dfrac{7^2 \times 7^{-3}}{7 \times 7^{-4}}$ without the aid of a calculator

$$\frac{7^2 \times 7^{-3}}{7 \times 7^{-4}} = \frac{7^{2-3}}{7^{1-4}} = \frac{7^{-1}}{7^{-3}} = \frac{7^3}{7^1} = 7^{3-1} = \mathbf{7^2 = 49}$$

15. Evaluate $\dfrac{2^3 \times 2^{-4} \times 2^5}{2 \times 2^{-2} \times 2^6}$ without the aid of a calculator

$$\frac{2^3 \times 2^{-4} \times 2^5}{2 \times 2^{-2} \times 2^6} = \frac{2^{3-4+5}}{2^{1-2+6}} = \frac{2^4}{2^5} = 2^{4-5} = 2^{-1} = \mathbf{\frac{1}{2}} \text{ or } \mathbf{0.5}$$

16. Evaluate $\dfrac{5^{-7} \times 5^2}{5^{-8} \times 5^3}$ without the aid of a calculator

$$\frac{5^{-7} \times 5^2}{5^{-8} \times 5^3} = 5^{-7+2-(-8)-3} = 5^{-7+2+8-3} = 5^0 = \mathbf{1}$$

17. Simplify $\dfrac{3^3 \times 5^2}{5^4 \times 3^4}$ expressing the answer in index form and with positive indices

$$\frac{3^3 \times 5^2}{5^4 \times 3^4} = 3^{3-4} \times 5^{2-4} = 3^{-1} \times 5^{-2} = \frac{1}{3^1} \times \frac{1}{5^2} = \mathbf{\frac{1}{3 \times 5^2}}$$

18. Simplify $\dfrac{7^{-2} \times 3^{-2}}{3^5 \times 7^4 \times 7^{-3}}$ expressing the answer in index form and with positive indices

$$\frac{7^{-2}\times 3^{-2}}{3^5\times 7^4\times 7^{-3}}=7^{-2-4-(-3)}\times 3^{-2-5}=7^{-3}\times 3^{-7}=\frac{1}{7^3}\times\frac{1}{3^7}\;=\;\mathbf{\frac{1}{7^3\times 3^7}}$$

19. Simplify $\frac{4^2\times 9^3}{8^3\times 3^4}$ expressing the answer in index form and with positive indices

$$\frac{4^2\times 9^3}{8^3\times 3^4}=\frac{(2^2)^2\times(3^2)^3}{(2^3)^3\times 3^4}=\frac{2^{2\times 2}\times 3^{2\times 3}}{2^{3\times 3}\times 3^4}=\frac{2^4\times 3^6}{2^9\times 3^4}=2^{4-9}\times 3^{6-4}=2^{-5}\times 3^2\;=\;\mathbf{\frac{3^2}{2^5}}$$

20. Evaluate $\left(\frac{1}{3^2}\right)^{-1}$

$$\left(\frac{1}{3^2}\right)^{-1}=\left(\frac{3^2}{1}\right)^{+1}=3^2\;=\mathbf{9}$$

21. Evaluate $81^{0.25}$

$$81^{0.25}=(3^4)^{\frac{1}{4}}=3^{4\times\frac{1}{4}}=3^1\;=\mathbf{3}$$

22. Evaluate $16^{-\frac{1}{4}}$

$$16^{(-1/4)}=(2^4)^{-\frac{1}{4}}=2^{4\times-\frac{1}{4}}=2^{-1}\;=\;\mathbf{\frac{1}{2}}$$

23. Evaluate $\left(\frac{4}{9}\right)^{1/2}$

$$\left(\frac{4}{9}\right)^{1/2}=\left(\frac{2^2}{3^2}\right)^{\frac{1}{2}}=\frac{2^{2\times\frac{1}{2}}}{3^{2\times\frac{1}{2}}}=\frac{2^1}{3^1}\;=\;\mathbf{\frac{2}{3}}$$

EXERCISE 15, Page 30

1. State the SI unit of volume?

The SI unit of volume is **cubic metres**, $\mathbf{m^3}$

2. State the SI unit of capacitance?

The SI unit of capacitance, is the **farad, F**

3. State the SI unit of area?

The SI unit of area is **square metres**, $\mathbf{m^2}$

4. State the SI unit of velocity?

The SI unit of velocity is **metres per second, m/s**

5. State the SI unit of density?

The SI unit of density is **kilograms per cubic metre**, $\mathbf{kg/m^3}$

6. State the SI unit of energy?

The SI unit of energy is the **joule, J**

7. State the SI unit of charge?

The SI unit of charge is the **coulomb, C**

8. State the SI unit of power?

The SI unit of power is the **watt, W**

9. State the SI unit of electric potential?

The SI unit of electric potential is the **volt, V**

10. State which quantity has the unit kg ?

The quantity which has the unit kg is **mass, m**

11. State which quantity has the unit symbol Ω ?

The quantity which has the unit symbol Ω is **electrical resistance, R**

12. State which quantity has the unit Hz ?

The quantity which has the unit Hz is **frequency, f**

13. State which quantity has the unit m/s^2 ?

The quantity which has the unit m/s^2 is **acceleration, a**

14. State which quantity has the unit symbol A ?

The quantity which has the unit symbol A is **electric current**

15. State which quantity has the unit symbol H ?

The quantity which has the unit symbol H is **inductance**

16. State which quantity has the unit symbol m ?

The quantity which has the unit symbol m is **length**

17. State which quantity has the unit symbol K ?

The quantity which has the unit symbol K is **thermodynamic temperature**

18. State which quantity has the unit rad/s ?

The quantity which has the unit rad/s is **angular velocity**

19. What does the prefix G mean?

The prefix G means **multiply by 1000,000,000** i.e. **multiply by** $\mathbf{10^{9}}$

20. What is the symbol and meaning of the prefix milli?

The symbol for milli is **m** and the prefix milli means **divide by 1000** i.e. **multiply by** $\mathbf{10^{-3}}$

21. What does the prefix 'p' mean?

The prefix 'p' means **divide by 1000,000,000,000** i.e. **multiply by** $\mathbf{10^{-12}}$

22. What is the symbol and meaning of the prefix 'mega'?

The symbol for mega is **M** and the prefix mega means **multiply by 1000,000** i.e. **multiply by** $\mathbf{10^{6}}$

EXERCISE 16, Page 32

1. Express in standard form: (a) 73.9 (b) 28.4 (c) 197.62

(a) 73.9 = $\mathbf{7.39 \times 10}$

(b) 28.4 = $\mathbf{2.84 \times 10}$

(c) 197.62 = $\mathbf{1.9762 \times 10^2}$

2. Express in standard form: (a) 2748 (b) 33,170 (c) 274,218

(a) 2748 = $\mathbf{2.748 \times 10^3}$

(b) 33170 = $\mathbf{3.317 \times 10^4}$

(c) 274218 = $\mathbf{2.74218 \times 10^5}$

3. Express in standard form: (a) 0.2401 (b) 0.0174 (c) 0.00923

(a) 0.2401 = $\mathbf{2.401 \times 10^{-1}}$

(b) 0.0174 = $\mathbf{1.74 \times 10^{-2}}$

(c) 0.00923 = $\mathbf{9.23 \times 10^{-3}}$

4. Express in standard form: (a) 1702.3 (b) 10.04 (c) 0.0109

(a) 1702.3 = $\mathbf{1.7023 \times 10^3}$

(b) 10.04 = $\mathbf{1.004 \times 10}$

(c) 0.0109 = $\mathbf{1.09 \times 10^{-2}}$

5. Express in standard form: (a) $\frac{1}{2}$ (b) $11\frac{7}{8}$ (c) $130\frac{3}{5}$ (d) $\frac{1}{32}$

(a) $\frac{1}{2}$ = 0.5 = $\mathbf{5 \times 10^{-1}}$

(b) $11\frac{7}{8}$ = 11.875 = $\mathbf{1.1875 \times 10}$

(c) $130\frac{3}{5}$ = 130.6 = $\mathbf{1.306 \times 10^2}$

(d) $\frac{1}{32}$ = 0.03125 = $\mathbf{3.125 \times 10^{-2}}$

6. Express the following numbers as integers or decimal fractions:

(a) 1.01×10^3 (b) 9.327×10^2 (c) 5.41×10^4 (d) 7×10^0

(a) 1.01×10^3 = **1010**

(b) 9.327×10^2 = **932.7**

(c) 5.41×10^4 = **54100**

(d) 7×10^0 = **7**

7. Express the following numbers as integers or decimal fractions:

(a) 3.89×10^{-2} (b) 6.741×10^{-1} (c) 8×10^{-3}

(a) 3.89×10^{-2} = **0.0389**

(b) 6.741×10^{-1} = **0.6741**

(c) 8×10^{-3} = **0.008**

8. Evaluate, stating the answers in standard form: (a) $(4.5 \times 10^{-2})(3 \times 10^3)$ (b) $2 \times (5.5 \times 10^4)$

(a) $(4.5 \times 10^{-2})(3 \times 10^3) = 13.5 \times 10^{3-2} = 13.5 \times 10 = 135 = \mathbf{1.35 \times 10^2}$

(b) $2 \times (5.5 \times 10^4) = 11 \times 10^4 = 110000 = \mathbf{1.1 \times 10^5}$

9. Evaluate, stating the answers in standard form: (a) $\dfrac{6\times10^{-3}}{3\times10^{-5}}$ (b) $\dfrac{(2.4\times10^{3})(3\times10^{-2})}{(4.8\times10^{4})}$

(a) $\dfrac{6\times10^{-3}}{3\times10^{-5}} = 2\times10^{-3-(-5)} = 2\times10^{-3+5} = 2\times10^{2} = 200 = \mathbf{2\times10^{2}}$

(b) $\dfrac{(2.4\times10^{3})(3\times10^{-2})}{(4.8\times10^{4})} = \dfrac{2.4\times3}{4.8}\times10^{3-2-4} = \mathbf{1.5\times10^{-3}}$

10. Write the following statements in standard form.

(a) The density of aluminium is 2710 kg m^{-3}

(b) Poisson's ratio for gold is 0.44

(c) The impedance of free space is 376.73 Ω

(d) The electron rest energy is 0.511 MeV

(e) Proton charge-mass ratio is 95,789,700 C kg^{-1}

(f) The normal volume of a perfect gas is 0.02241 m^3 mol^{-1}

(a) The density of aluminium is 2710 kg m^{-3} = $\mathbf{2.71\times10^{3}\ kg\ m^{-3}}$

(b) Poisson's ratio for gold is 0.44 = $\mathbf{4.4\times10^{-1}}$

(c) The impedance of free space is 376.73 Ω = $\mathbf{3.7673\times10^{2}\ \Omega}$

(d) The electron rest energy is 0.511 MeV = $\mathbf{5.11\times10^{-1}}$ **MeV**

(e) Proton charge-mass ratio is 95789700 C kg^{-1} = $\mathbf{9.57897\times10^{7}\ C\ kg^{-1}}$

(f) The normal volume of a perfect gas is 0.02241 m^3 mol^{-1} = $\mathbf{2.241\times10^{-2}\ m^{3}\ mol^{-1}}$

EXERCISE 17, Page 33

1. Express 60,000 Pa in engineering notation in prefix form

60,000 Pa = $\mathbf{60 \times 10^{2}}$ **Pa** = **60 kPa**

2. Express 0.00015 W in engineering notation in prefix form

0.00015 W = **0.15 mW** or **150 μW**

3. Express 5×10^{7} V in engineering notation in prefix form

5×10^{7} V = 50000000 V = $\mathbf{50 \times 10^{6}}$ = **50 MV**

4. Express 5.5×10^{-8} F in engineering notation in prefix form

$5.5\times10^{-8}\,\text{F} = \frac{5.5}{10^{8}} = \frac{55}{10^{9}} = 55\times10^{-9}$ F = **55 nF**

5. Express 100,000 W in engineering notation in prefix form

100,000 W = $\mathbf{100 \times 10^{3}}$ **W** = **100 kW**

6. Express 0.00054 A in engineering notation in prefix form

0.00054 A = $\mathbf{0.54 \times 10^{-3}}$ **A** = **0.54 mA** or $\mathbf{540 \times 10^{-6}}$ **A** = **540 μA**

7. Express 15×10^{5} Ω in engineering notation in prefix form

15×10^{5} Ω = 1500000 Ω = $\mathbf{1.5 \times 10^{6}}$ **Ω** = **1.5 MΩ**

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8. Express 225×10^{-4} V in engineering notation in prefix form

225×10^{-4} V = 0.0225 V = $\mathbf{22.5 \times 10^{-3}}$ **V = 22.5 mV**

9. Express 35,000,000,000 Hz in engineering notation in prefix form

35,000,000,000 Hz = $\mathbf{35 \times 10^{9}}$ **Hz = 35 GHz**

10. Express 1.5×10^{-11} F in engineering notation in prefix form

1.5×10^{-11} F = $\mathbf{15 \times 10^{-12}}$ **F = 15 pF**

11. Express 0.000017 A in engineering notation in prefix form

0.000017 A = $\mathbf{17 \times 10^{-6}}$ **A = 17 μA**

12. Express 46200 Ω in engineering notation in prefix form

46200 Ω = $\mathbf{46.2 \times 10^{3}}$ **Ω = 46.2 kΩ**

13. Rewrite 0.003 mA in μA

$$0.003 \text{ mA} = \frac{0.003}{1000} = \frac{0.003}{10^3} = \frac{0.003 \times 10^3}{10^3 \times 10^3} = \frac{3}{10^6} = 3 \times 10^{-6} = \mathbf{3\ \mu A}$$

14. Rewrite 2025 kHz as MHz

2025 kHz = 2025000 Hz = 2.025×10^{6} hz = **2.025 MHz**

15. Rewrite 6250 cm in metres

$$6250 \text{ cm} = \frac{6250}{100} \text{ m} = \mathbf{62.50 \text{ m}}$$

16. Rewrite 34.6 g in kg

$$34.6 \text{ g} = \frac{34.6}{1000} \text{ kg} = \mathbf{0.0346 \text{ kg}}$$

17. Use a calculator to evaluate in engineering notation: $4.5 \times 10^{-7} \times 3 \times 10^{4}$

$$4.5 \times 10^{-7} \times 3 \times 10^{4} = 4.5 \times 3 \times 10^{-7+4} = \mathbf{13.5 \times 10^{-3}}$$

18. Use a calculator to evaluate in engineering notation: $\dfrac{(1.6\times10^{-5})(25\times10^{3})}{(100\times10^{-6})}$

$$\frac{(1.6\times10^{-5})(25\times10^{3})}{(100\times10^{-6})} = \frac{1.6\times25}{100}\times10^{-5+3-(-6)} = \frac{1.6}{4}\times10^{4} = 0.4\times10000 = 4000 = \mathbf{4 \times 10^{3}}$$

19. The distance from Earth to the moon is around 3.8×10^{8} m. State the distance in kilometres.

$$\text{Dis tan ce} = 3.8\times10^{8} \text{ m} = \frac{3.8\times10^{8}}{10^{3=}} = \mathbf{3.8\times10^{5}\,km}$$

20. The radius of a hydrogen atom is 0.53×10^{-10} m. State the radius in nanometres.

$$\text{Radius} = 0.53\times10^{-10} \text{ m} = \frac{0.53}{10^{10}} \text{ m} = \frac{0.53 \text{ m}}{10^{10}}\times10^{9} \text{ nm/m} = \mathbf{0.053 \text{ nm}}$$

21. The tensile stress acting on a rod is 5600000 Pa. Write this value in engineering notation.

Tensile stress = 5600000 Pa = 5.6×10^{6} = **5.6 MPa**

22. The expansion of a rod is 0.0043 m. Write this in engineering notation.

Expansion = 0.0043 m = $\mathbf{4.3 \times 10^{-3}}$ **m** = **4.3 mm**

CHAPTER 30 THERMAL EXPANSION

EXERCISE 140, Page 309

1. A length of lead piping is 50.0 m long at a temperature of 16°C. When hot water flows through it the temperature of the pipe rises to 80°C. Determine the length of the hot pipe if the coefficient of linear expansion of lead is $29 \times 10^{-6} K^{-1}$.

Length $L_1 = 50.0$ m, temperature $t_1 = 16°C$, $t_2 = 80°C$ and $\alpha = 29 \times 10^{-6} K^{-1}$

Length of pipe at 80°C is given by:

$$\mathbf{L_2} = L_1[1 + \alpha(t_2 - t_1)] = 50.0[1 + (29 \times 10^{-6})(80 - 16)]$$

$$= 50.0[1 + 0.001856]$$

$$= 50.0[1.001856] = \mathbf{50.0928\ m}$$

i.e. an increase in length of 0.0928 m or 92.28 mm

2. A rod of metal is measured at 285 K and is 3.521 m long. At 373 K the rod is 3.523 m long. Determine the value of the coefficient of linear expansion for the metal.

Length $L_1 = 3.521$ m, $L_2 = 3.523$ m, temperature $t_1 = 285$ K and temperature $t_2 = 373$ K

Length $L_2 = L_1[1 + \alpha(t_2 - t_1)]$

i.e. $3.523 = 3.521[1 + \alpha(373 - 285)]$

$3.523 = 3.521 + (3.521)(\alpha)(88)$

i.e. $3.523 – 3.521 = (3.521)(\alpha)(88)$

Hence, the coefficient of linear expansion, $\alpha = \dfrac{0.002}{(3.521)(88)} = 0.00000645$

i.e. **coefficient of linear expansion,** $\boldsymbol{\alpha = 6.45 \times 10^{-6} K^{-1}}$

3. A copper overhead transmission line has a length of 40.0 m between its supports at 20°C.

Determine the increase in length at 50°C if the coefficient of linear expansion of copper is $17 \times 10^{-6}\,K^{-1}$.

$$\text{Length } L_2 = L_1[1 + \alpha(t_2 - t_1)]$$

$$= L_1 + L_1\alpha(t_2 - t_1)$$

Hence, **increase in length** $= L_1\alpha(t_2 - t_1)$

$$= (40.0\text{ m})(17 \times 10^{-6}\,K^{-1})(50 - 20)°C$$

$$= (40.0)(17 \times 10^{-6})(30)$$

$$= \mathbf{0.0204\ m} \quad \text{or} \quad \mathbf{20.4\ mm}$$

4. A brass measuring tape measures 2.10 m at a temperature of 15°C. Determine

(a) the increase in length when the temperature has increased to 40°C

(b) the percentage error in measurement at 40°C.

Assume the coefficient of linear expansion of brass to be $18 \times 10^{-6}\,K^{-1}$.

Length $L_1 = 2.10$ m, temperature $t_1 = 15°C$, $t_2 = 40°C$ and $\alpha = 18 \times 10^{-6}\,K^{-1}$

(a) Length $L_2 = L_1[1 + \alpha(t_2 - t_1)] = L_1 + L_1\alpha(t_2 - t_1)$

Hence, **increase in length** $= L_1\alpha(t_2 - t_1)$

$$= (2.10\text{ m})(18 \times 10^{-6}\,K^{-1})(40 - 15)°C$$

$$= (2.10)(18 \times 10^{-6})(25)$$

$$= \mathbf{0.000945\ m} \quad \text{or} \quad \mathbf{0.945\ mm}$$

(b) **Percentage error in measurement at 40°C** $= \dfrac{\text{increase in length}}{\text{original length}} = \dfrac{0.000945}{2.10} \times 100\%$

$$= \mathbf{0.045\%}$$

5. A pendulum of a 'grandfather' clock is 2.0 m long and made of steel. Determine the change in length of the pendulum if the temperature rises by 15 K. Assume the coefficient of linear expansion of steel to be $15 \times 10^{-6} K^{-1}$.

$$\text{Length } L_2 = L_1[1 + \alpha(t_2 - t_1)] = L_1 + L_1\alpha(t_2 - t_1)$$

Hence, **increase in length** $= L_1\alpha(t_2 - t_1)$

$$= (2.0 \text{ m})(15 \times 10^{-6} K^{-1})(15 \text{ K})$$

$$= (2.0)(15 \times 10^{-6})(15)$$

$$= \mathbf{0.00045 \ m} \quad \text{or} \quad \mathbf{0.45 \ mm}$$

6. A temperature control system is operated by the expansion of a zinc rod which is 200 mm long at 15°C. If the system is set so that the source of heat supply is cut off when the rod has expanded by 0.20 mm, determine the temperature to which the system is limited. Assume the coefficient of linear expansion of zinc to be $31 \times 10^{-6} K^{-1}$.

Length L_1 = 200 mm = 0.20 m, L_2 = 200 + 0.20 mm = 200.2 mm = 0.2002, temperature t_1 = 15°C

$$\text{Length } L_2 = L_1[1 + \alpha(t_2 - t_1)] = L_1 + L_1\alpha(t_2 - t_1)$$

Hence, **increase in length** $= L_1\alpha(t_2 - t_1)$

i.e. $$0.2002 - 0.20 = (0.20)(31 \times 10^{-6})(t_2 - 15)$$

$$0.0002 = (0.20)(31 \times 10^{-6})(t_2 - 15)$$

i.e. $$(t_2 - 15) = \frac{0.0002}{(0.20)(31\times10^{-6})} = 32.26°C$$

i.e. **the temperature to which the system is limited, t_2** = 32.26 + 15 **= 47.26°C**

7. A length of steel railway line is 30.0 m long when the temperature is 288 K. Determine the increase in length of the line when the temperature is raised to 303 K. Assume the coefficient of linear expansion of steel to be $15 \times 10^{-6} K^{-1}$.

$$\text{Length } L_2 = L_1[1 + \alpha(t_2 - t_1)] = L_1 + L_1\alpha(t_2 - t_1)$$

Hence, **increase in length** $= L_1\alpha(t_2 - t_1)$

$$= (30.0 \text{ m})(15 \times 10^{-6} K^{-1})(303 - 288)K$$

$$= (30.0)(15 \times 10^{-6})(15)$$

$$= \mathbf{0.00675 \text{ m}} \quad \text{or} \quad \mathbf{6.75 \text{ mm}}$$

8. A brass shaft is 15.02 mm in diameter and has to be inserted in a hole of diameter 15.0 mm. Determine by how much the shaft must be cooled to make this possible, without using force. Take the coefficient of linear expansion of brass as $18 \times 10^{-6} K^{-1}$.

Length L_1 = 15.02 mm = 0.01502 m, L_2 = 15 mm = 0.015 m

$$\text{Length } L_2 = L_1[1 + \alpha(t_2 - t_1)]$$

i.e. $$0.015 = 0.01502[1 + (18 \times 10^{-6} K^{-1})(t_2 - t_1)]$$

$$0.015 = 0.01502 + (0.01502)(18 \times 10^{-6})(t_2 - t_1)$$

i.e. $$0.015 - 0.01502 = (0.01502)(18 \times 10^{-6})(t_2 - t_1)$$

Hence, $$(t_2 - t_1) = \frac{-0.00002}{(0.01502)(18 \times 10^{-6})} = -73.98 \text{ K}$$

i.e. **the shaft must be cooled by 74 K**

EXERCISE 141, Page 312

1. A silver plate has an area of 800 mm^2 at 15°C. Determine the increase in the area of the plate when the temperature is raised to 100°C. Assume the coefficient of linear expansion of silver to be $19 \times 10^{-6} K^{-1}$.

$$A_2 = A_1[1 + \beta(t_2 - t_1)]$$

i.e. $A_2 = A_1[1 + 2\alpha(t_2 - t_1)]$ since $\beta = 2\alpha$, to a very close approximation

i.e. $A_2 = A_1 + A_1 2\alpha(t_2 - t_1)$

Hence, area increase $= A_1 2\alpha(t_2 - t_1)$

$$= \left(800 \times 10^{-6} m^2\right)\left(2 \times 19 \times 10^{-6} K^{-1}\right)(100 - 15)°C$$

$$= 800 \times 10^{-6} \times 2 \times 19 \times 10^{-6} \times 85$$

$$= 2.584 \times 10^{-6}\ m^2 \text{ or } \mathbf{2.584\ mm^2}$$

2. At 283 K a thermometer contains 440 mm^3 of alcohol. Determine the temperature at which the volume is 480 mm^3 assuming that the coefficient of cubic expansion of the alcohol is $12 \times 10^{-4} K^{-1}$.

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

i.e. $480 \times 10^{-9} = 440 \times 10^{-9}[1 + (12 \times 10^{-4})(t_2 - 283)]$

from which, $480 = 440 + 440(12 \times 10^{-4})(t_2 - 283)$

and $480 - 440 = 440(12 \times 10^{-4})(t_2 - 283)$

from which, $(t_2 - 283) = \dfrac{40}{440 \times 12 \times 10^{-4}} = 75.76\ K$

and **temperature, t_2** $= 75.76 + 283 =$ **358.8 K**

3. A zinc sphere has a radius of 30.0 mm at a temperature of 20°C. If the temperature of the sphere is raised to 420°C, determine the increase in: (a) the radius, (b) the surface area, (c) the volume of the sphere. Assume the coefficient of linear expansion for zinc to be $31 \times 10^{-6}\,K^{-1}$.

(a) Initial radius, $L_1 = 30.0$ mm, initial temperature, $t_1 = 20 + 273 = 293$ K,

final temperature, $t_2 = 420 + 273 = 693$ K and $\alpha = 31 \times 10^{-6}\,K^{-1}$.

New radius at 693 K is given by:

$$L_2 = L_1[1 + \alpha(t_2 - t_1)]$$

i.e. $$L_2 = 30.0[1 + (31 \times 10^{-6})(693 - 293)]$$

$$= 30.0[1 + 0.0124] = 30.372 \text{ mm}$$

Hence the increase in the radius is 0.372 mm

(b) Initial surface area of sphere, $A_1 = 4\pi r^2 = 4\pi(30.0)^2 = 3600\pi \text{ mm}^2$

New surface area at 693 K is given by:

$$A_2 = A_1[1 + \beta(t_2 - t_1)]$$

i.e. $$A_2 = A_1[1 + 2\alpha(t_2 - t_1)]$$ since $\beta = 2\alpha$, to a very close approximation

Thus $$A_2 = 3600\pi[1 + 2(31 \times 10^{-6})(400)]$$

$$= 3600\pi[1 + 0.0248] = 3600\pi + 3600\pi(0.0248)$$

Hence increase in surface area $= 3600\pi(0.0248) =$ **280.5 mm²**

(c) Initial volume of sphere, $V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(30.0)^3 \text{ mm}^3$

New volume at 693 K is given by:

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

i.e. $$V_2 = V_1[1 + 3\alpha(t_2 - t_1)]$$ since $\gamma = 3\alpha$, to a very close approximation

Thus $$V_2 = \frac{4}{3}\pi(30.0)^3[1 + 3(31 \times 10^{-6})(400)]$$

$$= \frac{4}{3}\pi(30.0)^3[1 + 0.0372] = \frac{4}{3}\pi(30.0)^3 + \frac{4}{3}\pi(30.0)^3(0.0372)$$

Hence, the increase in volume $= \frac{4}{3}\pi(30.0)^3(0.0372) = \mathbf{4207\ mm^3}$

4. A block of cast iron has dimensions of 50 mm by 30 mm by 10 mm at 15°C. Determine the increase in volume when the temperature of the block is raised to 75°C. Assume the coefficient of linear expansion of cast iron to be $11 \times 10^{-6}\ K^{-1}$.

Initial volume of sphere, $V_1 = 50 \times 30 \times 10 = 15000\ mm^3$

New volume at 75°C is given by:

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

i.e. $$V_2 = V_1[1 + 3\alpha(t_2 - t_1)]$$ since $\gamma = 3\alpha$, to a very close approximation

Thus $$V_2 = 15000\,[1 + 3(11 \times 10^{-6})(75 - 15)]$$

$$= 15000[1 + 0.00198] = 15000 + 15000\,(0.00198)$$

Hence, the increase in volume $= 15000\,(0.00198) = \mathbf{29.7\ mm^3}$

5. Two litres of water, initially at 20°C, is heated to 40°C. Determine the volume of water at 40°C if the coefficient of volumetric expansion of water within this range is $30 \times 10^{-5}\ K^{-1}$.

New volume at 40°C is given by:

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

$$= 2[1 + (30 \times 10^{-5})(40 - 20)]$$

$$= 2[1 + 0.006] = 2[1.006]$$

$$= \mathbf{2.012\ litres}$$

6. Determine the increase in volume, in litres, of 3 m^3 of water when heated from 293 K to boiling point if the coefficient of cubic expansion is $2.1 \times 10^{-4} K^{-1}$ (1 litre $\approx 10^{-3} m^3$).

Initial volume of sphere, $V_1 = 3 \times 10^3 = 3000$ litres

New volume at boiling point (i.e. 373 K) is given by:

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

Thus $$V_2 = 3000\,[1 + (2.1 \times 10^{-4})(373 - 293)]$$

$$= 3000[1 + 0.0168] = 3000 + 3000\,(0.0168)$$

Hence, the increase in volume = 3000 (0.0168) = **50.4 litres**

7. Determine the reduction in volume when the temperature of 0.5 litre of ethyl alcohol is reduced from 40°C to - 15°C. Take the coefficient of cubic expansion for ethyl alcohol as $1.1 \times 10^{-3} K^{-1}$.

New volume at - 15°C is given by:

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

Thus $$V_2 = 0.5\,[1 + (1.1 \times 10^{-3})(- 15 - 40)]$$

$$= 0.5\,[1 + (1.1 \times 10^{-3})(- 55)]$$

$$= 0.5 + (0.5)(1.1 \times 10^{-3})(- 55)$$

Hence, the reduction in volume = $(0.5)(1.1 \times 10^{-3})(55)$= **0.03025 litres**

EXERCISE 142, Page 313

Answers found from within the text of the chapter, pages 306 to 312.

EXERCISE 143, Page 313

1. (b) **2.** (c) **3.** (a) **4.** (d) **5.** (b) **6.** (c) **7.** (c) **8.** (a) **9.** (c) **10.** (b)

CHAPTER 31 IDEAL GAS LAWS

EXERCISE 144, Page 317

1. The pressure of a mass of gas is increased from 150 kPa to 750 kPa at constant temperature. Determine the final volume of the gas, if its initial volume is 1.5 m^3.

Since the change occurs at constant temperature (i.e. an isothermal change), Boyle's law applies,

i.e. $p_1V_1 = p_2V_2$ where $p_1 = 150$ kPa, $p_2 = 750$ kPa and $V_1 = 1.5\ m^3$

Hence, $(150)(1.5) = (750)V_2$

from which, **volume** $\mathbf{V_2} = \frac{150 \times 1.5}{750} = \mathbf{0.3\ m^3}$

2. In an isothermal process, a mass of gas has its volume reduced from 50 cm^3 to 32 cm^3. If the initial pressure of the gas is 80 kPa, determine its final pressure.

Since the change occurs at constant temperature (i.e. an isothermal change), Boyle's law applies,

i.e. $p_1V_1 = p_2V_2$ where $p_1 = 80$ kPa, $V_1 = 50\ cm^3$ and $V_2 = 32\ cm^3$

Hence, $(80)(50) = (p_2)(32)$

from which, **pressure** $\mathbf{p_2} = \frac{80 \times 50}{32} = \mathbf{125\ kPa}$

3. The piston of an air compressor compresses air to $\frac{1}{4}$ of its original volume during its stroke. Determine the final pressure of the air if the original pressure is 100 kPa, assuming an isothermal change.

Since the change occurs at constant temperature (i.e. an isothermal change), Boyle's law applies, i.e. $p_1V_1 = p_2V_2$ where $p_1 = 100$ kPa, $V_2 = \frac{1}{4}V_1$

Hence, $(100)(V_1) = (p_2)(\frac{1}{4}V_1)$

from which, **pressure** $\mathbf{p_2} = \dfrac{100 \times V_1}{\frac{1}{4}V_1} = \dfrac{100}{\frac{1}{4}}$ = **400 kPa**

4. A quantity of gas in a cylinder occupies a volume of 2 m^3 at a pressure of 300 kPa. A piston slides in the cylinder and compresses the gas, according to Boyle's law, until the volume is 0.5 m^3. If the area of the piston is 0.02 m^2, calculate the force on the piston when the gas is compressed.

An isothermal process means constant temperature and thus Boyle's law applies, i.e. $p_1V_1 = p_2V_2$ where $V_1 = 2\ m^3$, $V_2 = 0.5\ m^3$ and $p_1 = 300$ kPa.

Hence, $(300)(2) = p_2(0.5)$

from which, pressure, $p_2 = \dfrac{300 \times 2}{0.5} = 1200$ kPa

Pressure $= \dfrac{\text{force}}{\text{area}}$, from which, force = pressure × area.

Hence, **force on the piston** $= (1200 \times 10^3\ \text{Pa})(0.02\ m^2) = 24000$ N = **24 kN**

5. The gas in a simple pump has a pressure of 400 mm of mercury (Hg) and a volume of 10 mL. If the pump is compressed to a volume of 2 mL, calculate the pressure of the gas, assuming that its temperature does not change?

Since $P_1V_1 = P_2V_2$

then (400 mm Hg)(10 mL) = (P_2)(2 mL)

from which, **new pressure,** $\mathbf{P_2} = \frac{400\,\text{mm Hg} \times 10\,\text{mL}}{2\,\text{mL}}$ = **2000 mm of mercury**

EXERCISE 145, Page 319

1. Some gas initially at 16°C is heated to 96°C at constant pressure. If the initial volume of the gas is 0.8 m^3, determine the final volume of the gas.

Since the change occurs at constant pressure (i.e. an isobaric process), Charles' law applies,

i.e. $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where $V_1 = 0.8\ m^3$, $T_1 = 16°C = (16 + 273)K = 289\ K$ and $T_2 = (96 + 273)K = 369\ K$.

Hence, $$\frac{0.8}{289} = \frac{V_2}{369}$$

from which, **volume at 96°C,** $\mathbf{V_2} = \frac{(0.8)(369)}{289} = \mathbf{1.02\ m^3}$

2. A gas is contained in a vessel of volume 0.02 m^3 at a pressure of 300 kPa and a temperature of 15°C. The gas is passed into a vessel of volume 0.015 m^3. Determine to what temperature the gas must be cooled for the pressure to remain the same.

Since the process is isobaric it takes place at constant pressure and hence Charles' law applies,

i.e. $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where $T_1 = (15 + 273)K = 288\ K$ and $V_1 = 0.02\ m^3$ and $V_2 = 0.015\ m^3$

Hence $$\frac{0.02}{288} = \frac{0.015}{T_2}$$

from which, **final temperature,** $\mathbf{T_2} = \frac{(0.015)(288)}{0.02} = \mathbf{216\ K}$ or $(216 - 273)°C$ i.e. **- 57°C**

3. In an isobaric process gas at a temperature of 120°C has its volume reduced by a sixth. Determine the final temperature of the gas.

Since the process is isobaric it takes place at constant pressure and hence Charles' law applies, i.e. $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where $T_1 = (120 + 273)K = 393$ K and $V_2 = \frac{5}{6}V_1$

Hence $$\frac{V_1}{393} = \frac{\frac{5}{6}V_1}{T_2}$$

from which, **final temperature,** $\mathbf{T_2} = \dfrac{\left(\frac{5}{6}V_1\right)(393)}{V_1} = \frac{5}{6}(393)$ = **327.5 K**

= (327.5 - 273)°C = **54.5°C**

4. The volume of a balloon is 30 litres at a temperature of 27ºC. If the balloon is under a constant internal pressure, calculate its volume at a temperature of 12ºC.

Since $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where $T_1 = (27 + 273)K = 300$ K

and $T_2 = (12 + 273)K = 285$ K

Hence, $$\frac{30\,\text{litres}}{300\,\text{K}} = \frac{V_2}{285\,\text{K}}$$

and **new volume,** $\mathbf{V_2} = \dfrac{30\,\text{litres} \times 285\,\text{K}}{300\,\text{K}}$ = **28.5 litres**

EXERCISE 146, Page 320

1. Gas, initially at a temperature of 27°C and pressure 100 kPa, is heated at constant volume until its temperature is 150°C. Assuming no loss of gas, determine the final pressure of the gas.

Since the gas is at constant volume, the pressure law applies, i.e. $\frac{p_1}{T_1} = \frac{p_2}{T_2}$

where $T_1 = (27 + 273)K = 300$ K, $T_2 = (150 + 273)K = 423$ K and $p_1 = 100$ kPa

Hence, $$\frac{100}{300} = \frac{p_2}{423}$$

from which, **final pressure,** $\mathbf{p_2} = \frac{(100)(423)}{300} = \mathbf{141\ kPa}$

2. A pressure vessel is subjected to a gas pressure of 8 atmospheres at a temperature of 15ºC. The vessel can withstand a maximum pressure of 28 atmospheres. Calculate the gas temperature increase the vessel can withstand.

Since $$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

where $T_1 = (15 + 273)K = 288$ K

Hence, $$\frac{8\,\text{atmospheres}}{288\,\text{K}} = \frac{28\,\text{atmospheres}}{T_2}$$

from which, new temperature, $T_2 = \frac{28\,\text{atmospheres} \times 288\,\text{K}}{8\,\text{atmospheres}}$ = **1008 K** or (1008 – 273)ºC = **735ºC**

Hence, **temperature rise =** (1008 – 288)K **= 720 K**

or **temperature rise =** (735 – 15)ºC **= 720ºC**

Note that a temperature **change** of 720 K = a temperature **change** of 720ºC

EXERCISE 147, Page 321

1. A gas A in a container exerts a pressure of 120 kPa at a temperature of 20°C. Gas B is added to the container and the pressure increases to 300 kPa at the same temperature. Determine the pressure that gas B alone exerts at the same temperature.

Initial pressure, p_A = 120 kPa, and the pressure of gases A and B together, $p = p_A + p_B$ = 300 kPa

By Dalton's law of partial pressure, **the pressure of gas B alone is**

$$\mathbf{p_B} = p - p_A = 300 - 120 = \mathbf{180\ kPa}$$

EXERCISE 148, Page 322

1. A gas occupies a volume of 1.20 m^3 when at a pressure of 120 kPa and a temperature of 90°C. Determine the volume of the gas at 20°C if the pressure is increased to 320 kPa.

Using the combined gas law: $$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

where V_1 = 1.20 m^3, p_1 = 120 kPa, p_2 = 320 kPa, T_1 = (90 + 273)K = 363 K and

T_2 = (20 + 273)K = 293 K, gives: $$\frac{(120)(1.20)}{363} = \frac{(320)\,V_2}{293}$$

from which, **volume at 20°C, V_2** = $\frac{(120)(1.20)(293)}{(363)(320)}$ = **0.363 m^3**

2. A given mass of air occupies a volume of 0.5 m^3 at a pressure of 500 kPa and a temperature of 20°C. Find the volume of the air at STP.

Using the combined gas law: $$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

where V_1 = 0.5 m^3, p_1 = 500 kPa, T_1 = (20 + 273)K = 293 K , p_2 = 101.325 kPa, and

T_2 = 0°C = 273 K, gives: $$\frac{(500)(0.5)}{293} = \frac{(101.325)\,V_2}{273}$$

from which, **volume at STP, V_2** = $\frac{(500)(0.5)(273)}{(293)(101.325)}$ = **2.30 m^3**

3. A balloon is under an internal pressure of 110 kPa with a volume of 16 litres at a temperature of 22ºC. If the balloon's internal pressure decreases to 50 kPa, what will be its volume if the temperature also decreases to 12ºC.

Using the combined gas law: $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

where V_1 = 16 litres, p_1 = 110 kPa, p_2 = 50 kPa, T_1 = (22 + 273)K = 295 K and

T_2 = (12 + 273)K = 285 K, gives: $\frac{(110)(16)}{295} = \frac{(50)\,V_2}{285}$

from which, **volume at 12°C, V_2** = $\frac{(110)(16)(285)}{(295)(50)}$ = **34.0 litres**

4. A spherical vessel has a diameter of 2.0 m and contains hydrogen at a pressure of 300 kPa and a temperature of - 30°C. Determine the mass of hydrogen in the vessel. Assume the characteristic gas constant R for hydrogen is 4160 J/(kg K).

From the characteristic gas equation, pV = mRT

where p = 300 kPa , V = $\frac{4}{3}\pi(1.0)^3$ = 4.1888 m^3, T = (- 30 + 273)K = 243 K, and

R = 4160 J/(kg K).

Hence $(300\times 10^3)(4.1888) = m(4160)(243)$

from which, **mass of air, m** = $\frac{(300\times10^3)(4.1888)}{(4160)(243)}$ = **1.24 kg**

5. A cylinder 200 mm in diameter and 1.5 m long contains oxygen at a pressure of 2 MPa and a temperature of 20°C. Determine the mass of oxygen in the cylinder. Assume the characteristic gas constant for oxygen is 260 J/(kg K).

From the characteristic gas equation, pV = mRT

where p = 2 MPa , V = $\pi(0.1)^2(1.5) = 0.0471$ m^3, T = (20 + 273)K = 293 K, and R = 260 J/(kg K).

Hence $(2\times 10^6)(0.0471) = m(260)(293)$

from which, **mass of air, m** = $\frac{(2\times10^6)(0.0471)}{(260)(293)}$ = **1.24 kg**

6. A gas is pumped into an empty cylinder of volume 0.1 m^3 until the pressure is 5 MPa. The temperature of the gas is 40°C. If the cylinder mass increases by 5.32 kg when the gas has been added, determine the value of the characteristic gas constant.

From the characteristic gas equation, pV = mRT

from which, R = $\frac{pV}{mT}$ where p = 5 $\times10^6$ Pa, V = 0.1 m^3, T = (40 + 273)K = 313 K and m = 5.32 kg

Hence, the **characteristic gas constant**, R = $\frac{pV}{mT} = \frac{(5\times10^6)(0.1)}{(5.32)(313)}$

= **300 J/(kg K)**

7. The mass of a gas is 1.2 kg and it occupies a volume of 13.45 m^3 at STP. Determine its characteristic gas constant.

From the characteristic gas equation, pV = mRT

from which, R = $\frac{pV}{mT}$ where m = 1.2 kg, V = 13.45 m^3, p = 101.325 kPa, , T = 0°C = 273 K

Hence, the **characteristic gas constant**, R = $\frac{pV}{mT} = \frac{(101.325\times10^3)(13.45)}{(1.2)(273)}$

= **4160 J/(kg K)**

8. 30 cm^3 of air initially at a pressure of 500 kPa and temperature 150°C is expanded to a volume of 100 cm^3 at a pressure of 200 kPa. Determine the final temperature of the air, assuming no losses during the process.

Using the combined gas law: $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

where $V_1 = 30\ cm^3$, $V_2 = 100\ cm^3$, $p_1 = 500$ kPa, $p_2 = 200$ kPa, and $T_1 = (150 + 273)K = 423$ K,

gives: $$\frac{(500)(30)}{423} = \frac{(200)(100)}{T_2}$$

from which, **final temperature,** $\mathbf{T_2}$ = $\frac{(200)(100)(423)}{(500)(30)}$ = **564 K** or (564 – 273)°C = **291°C**

---

9. A quantity of gas in a cylinder occupies a volume of 0.05 m^3 at a pressure of 400 kPa and a temperature of 27°C. It is compressed according to Boyle's law until its pressure is 1 MPa, and then expanded according to Charles' law until its volume is 0.03 m^3. Determine the final temperature of the gas.

Using the combined gas law: $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

where $V_1 = 0.05\ m^3$, $V_2 = 0.03\ m^3$, $p_1 = 400$ kPa, $p_2 = 1$ MPa = 1000 kPa,

and $T_1 = (27 + 273)K = 300$ K, gives: $$\frac{(0.05)(400)}{300} = \frac{(0.03)(1000)}{T_2}$$

from which, **final temperature,** $\mathbf{T_2}$ = $\frac{(0.03)(1000)(300)}{(0.05)(400)}$ = **450 K** or **177°C**

---

10. Some air at a temperature of 35°C and pressure 2 bar occupies a volume of 0.08 m^3. Determine the mass of the air assuming the characteristic gas constant for air to be 287 J/(kg K). (1 bar = 10^5 Pa)

From the characteristic gas equation, pV = mRT

where T = (35 + 273)K = 308 K, p = 2 bar = 2×10^5 Pa, V = 0.08 m^3 and R = 287 J/(kg K).

Hence $$(2\times10^5)(0.08) = m(287)(308)$$

from which, **mass of air, m** = $\frac{(2\times10^5)(0.08)}{(287)(308)}$ = **0.181 kg**

11. Determine the characteristic gas constant R of a gas that has a specific volume of 0.267 m^3/kg at a temperature of 17°C and pressure 200 kPa.

From the characteristic gas equation, pV = mRT

from which, R = $\frac{pV}{mT}$ where p = 200 $\times10^3$ Pa, T = (17 + 273)K = 290 K and

specific volume, V/m = 0.267 m^3/kg.

Hence the **characteristic gas constant**, R = $\left(\frac{p}{T}\right)\left(\frac{V}{m}\right) = \left(\frac{200\times10^3}{290}\right)(0.267)$

= **184 J/(kg K)**

EXERCISE 149, Page 325

1. A vessel P contains gas at a pressure of 800 kPa at a temperature of 25°C. It is connected via a valve to vessel Q that is filled with similar gas at a pressure of 1.5 MPa and a temperature of 25°C. The volume of vessel P is 1.5 m^3 and that of vessel Q is 2.5 m^3. Determine the final pressure at 25°C when the valve is opened and the gases are allowed to mix. Assume R for the gas to be 297 J/(kg K).

For vessel P:

$p_P = 800 \times 10^3$ Pa, $T_P = (25 + 273)$K = 298 K, $V_P = 1.5\ m^3$ and R = 297 J/(kg K)

From the characteristic gas equation, $p_P V_P = m_P R T_P$

Hence $$(800 \times 10^3)(1.5) = m_P(297)(298)$$

from which, mass of gas in vessel P, $m_P = \dfrac{(800\times10^3)(1.5)}{(297)(298)} = 13.558$ kg

For vessel Q:

$p_Q = 1.5 \times 10^6$ Pa, $T_Q = (25 + 273)$K = 298 K, $V_Q = 2.5\ m^3$ and R = 297 J/(kg K)

From the characteristic gas equation, $p_Q V_Q = m_Q R T_Q$

Hence $$(1.5 \times 10^6)(2.5) = m_Q(297)(298)$$

from which, mass of gas in vessel Q, $m_Q = \dfrac{(1.5\times10^6)(2.5)}{(297)(298)} = 42.370$ kg

When the valve is opened, mass of mixture, $m = m_P + m_Q = 13.558 + 42.370 = 55.928$ kg.

Total volume, $V = V_P + V_Q = 1.5 + 2.5 = 4.0\ m^3$, R = 297 J/(kg K), T = 298 K.

From the characteristic gas equation, pV = mRT

$$p(4.0) = (55.928)(297)(298)$$

from which, **final pressure, p** = $\dfrac{(55.928)(297)(298)}{4.0}$ = **1.24 MPa**

2. A vessel contains 4 kg of air at a pressure of 600 kPa and a temperature of 40°C. The vessel is connected to another by a short pipe and the air exhausts into it. The final pressure in both vessels is 250 kPa and the temperature in both is 15°C. If the pressure in the second vessel before the air entered was zero, determine the volume of each vessel. Assume R for air is 287 J/(kg K).

For vessel 1: $m_1 = 4$ kg, $p_1 = 600 \times 10^3$ Pa, $T_1 = (40 + 273)$K = 313 K and R = 287 J/(kg K)

From the characteristic gas equation, $p_1 V_1 = m_1 R T_1$

Hence $(600 \times 10^3)\, V_1 = (4)(287)(313)$

from which, **volume of vessel 1, V_1** = $\dfrac{(4)(287)(313)}{(600\times 10^3)}$ = **0.60 m^3**

From the characteristic gas equation, pV = mRT

$$(250 \times 10^3)\, V_{Total} = (4)(287)(15 + 273)$$

from which, total volume, $V_{Total} = \dfrac{(4)(287)(288)}{250\times 10^3} = 1.32\ m^3$

Hence, **volume of vessel 2, V_2** = 1.32 – 0.60 = **0.72 m^3**

3. A vessel has a volume of 0.75 m^3 and contains a mixture of air and carbon dioxide at a pressure of 200 kPa and a temperature of 27°C. If the mass of air present is 0.5 kg determine (a) the partial pressure of each gas, and (b) the mass of carbon dioxide. Assume the characteristic gas constant for air to be 287 J/(kg K) and for carbon dioxide 184 J/(kg K).

(a) V = 0.75 m^3, p = 200 kPa, T = (27 + 273)K = 300 K, m_{air} = 0.50 kg, R_{air} = 287 J/(kg K).

If p_{air} is the partial pressure of the air, then using the characteristic gas equation,

$$p_{air} V = m_{air} R_{air} T \quad \text{gives:} \quad (p_{air})(0.75) = (0.50)(287)(300)$$

from which, **the partial pressure of the air,** $\boldsymbol{p_{air}} = \frac{(0.50)(287)(300)}{(0.75)} = \mathbf{57.4\ kPa}$

By Dalton's law of partial pressure the total pressure p is given by the sum of the partial pressures, i.e. $p = p_{air} + p_{CO_2}$, from which,

the partial pressure of the carbon dioxide, $\boldsymbol{p_{CO_2}} = p - p_{air} = 200 - 57.4 = \mathbf{142.6\ kPa}$

(b) From the characteristic gas equation, $p_{CO_2} V = m_{CO_2} R_{CO_2} T$

Hence, $(142.6 \times 10^3)(0.75) = m_{CO_2}(184)(300)$

from which, **mass of hydrogen,** $\boldsymbol{m_{CO_2}} = \frac{(142.6 \times 10^3)(0.75)}{(184)(300)} = \mathbf{1.94\ kg}$

4. A mass of gas occupies a volume of 0.02 m^3 when its pressure is 150 kPa and its temperature is 17°C. If the gas is compressed until its pressure is 500 kPa and its temperature is 57°C, determine (a) the volume it will occupy and (b) its mass, if the characteristic gas constant for the gas is 205 J/(kg K).

(a) Using the combined gas law: $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

where $V_1 = 0.02\ m^3$, $p_1 = 150$ kPa, $T_1 = (17 + 273)$K = 290 K, $p_2 = 500$ kPa, and

$T_2 = (57 + 273)$K = 330 K, gives: $\frac{(150)(0.02)}{290} = \frac{(500)V_2}{330}$

from which, **volume at STP,** $\boldsymbol{V_2} = \frac{(150)(0.02)(330)}{(290)(500)} = \mathbf{0.0068\ m^3}$

(b) From the characteristic gas equation, $pV = mRT$

where T = 290 K, p = 150 kPa, V = 0.02 m^3 and R = 205 J/(kg K).

Hence $(150000)(0.02) = m(205)(290)$

from which, **mass of air, m** = $\frac{(150000)(0.02)}{(205)(290)}$ = **0.050 kg**

5. A compressed air cylinder has a volume of 0.6 m^3 and contains air at a pressure of 1.2 MPa absolute and a temperature of 37°C. After use the pressure is 800 kPa absolute and the temperature is 17°C. Calculate (a) the mass of air removed from the cylinder, and (b) the volume the mass of air removed would occupy at STP conditions. Take R for air as 287 J/(kg K) and atmospheric pressure as 100 kPa.

(a) From the characteristic gas equation, $p_1V_1 = mRT_1$

from which, mass of air, m = $\frac{p_1V_1}{R\,T_1} = \frac{(1.2\times10^6)(0.6)}{(287)(37+273)}$ = 8.0926 kg

Also, $p_2V_2 = mRT_2$

from which, mass of air, m = $\frac{p_2V_2}{R\,T_2} = \frac{(800\times10^3)(0.6)}{(287)(17+273)}$ = 5.767 kg

Hence, **the mass of air removed from the cylinder** = 8.093 **–** 5.767 = **2.33 kg**

(b) From the characteristic gas equation, $pV = mRT$

where p = 100 kPa, m = 2.33 kg, R = 287 J/(kg K) and T = (0 + 273)K = 273 K

Hence, $(100000)(V) = (2.33)(287)(273)$

from which, **the volume the mass of air removed would occupy at STP conditions,**

$$\mathbf{V} = \frac{(2.33)(287)(273)}{(100000)} = \mathbf{1.83\ m^3}$$

EXERCISE 150, Page 325

Answers found from within the text of the chapter, pages 315 to 325.

EXERCISE 151, Page 325

1. (a) **2.** (d) **3.** (b) **4.** (b) **5.** (c) **6.** (d) **7.** (b) **8.** (c) **9.** (c) **10.** (b)

CHAPTER 32 THE MEASUREMENT OF TEMPERATURE

EXERCISE 152, Page 331

1. A platinum-platinum/rhodium thermocouple generates an e.m.f. of 7.5 mV. If the cold junction is at a temperature of 20°C, determine the temperature of the hot junction. Assume the sensitivity of the thermocouple to be 6 μV/°C

Temperature difference for 7.5 mV = $\frac{7.5\times 10^{-3}\,\text{V}}{6\times 10^{-6}\,\text{V}/°\text{C}}$ = 1250°C

Temperature at hot junction = temperature of cold junction + temperature difference

= 20°C + 1250°C = **1270°C**

EXERCISE 153, Page 333

1. A platinum resistance thermometer has a resistance of 100 Ω at 0°C. When measuring the temperature of a heat process a resistance value of 177 Ω is measured using a Wheatstone bridge. Given that the temperature coefficient of resistance of platinum is 0.0038/°C, determine the temperature of the heat process, correct to the nearest degree.

$R_\theta = R_0(1 + \alpha\theta)$, where $R_0 = 100\ \Omega$, $R_\theta = 177\ \Omega$ and $\alpha = 0.0038$/°C.

Rearranging gives: $R_\theta = R_0(1 + \alpha\theta) = R_0 + R_0\alpha\theta$

i.e. $R_\theta - R_0 = R_0\alpha\theta$

and **temperature,** $\theta = \dfrac{R_\theta - R_0}{\alpha R_0} = \dfrac{177 - 100}{(0.0038)(100)} = \mathbf{203°C}$

EXERCISE 154, Page 336

Answers found from within the text of the chapter, pages 327 to 336.

EXERCISE 155, Page 336

1. (c) **2.** (b) **3.** (d) **4.** (b) **5.** (i) **6.** (a) **7.** (e) **8.** (d) **9.** (e) or (f) **10.** (k) **11.** (b) **12.** (g)

CHAPTER 33 AN INTRODUCTION TO ELECTRIC CIRCUITS

EXERCISE 156, Page 344

1. In what time would a current of 10 A transfer a charge of 50 C?

Q = I × t hence, **time, t** = $\frac{Q}{I} = \frac{50}{10}$ = **5 s**

2. A current of 6 A flows for 10 minutes. What charge is transferred?

Charge, Q = I × t = 6 × (10 × 60) = **3600 C**

3. How long must a current of 100 mA flow so as to transfer a charge of 80 C?

Q = I × t hence, **time, t** = $\frac{Q}{I} = \frac{80}{100\times10^{-3}}$ = 800 s = $\frac{800}{60}$ min = **13 min 20 s**

EXERCISE 157, Page 346

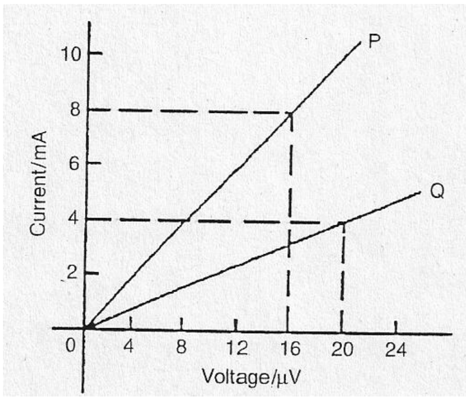
1. The current flowing through a heating element is 5 A when a p.d. of 35 V is applied across it. Find the resistance of the element.

Resistance, R = $\frac{V}{I} = \frac{35}{5}$ = **7 Ω**

2. An electric light bulb of resistance 960 Ω is connected to a 240 V supply. Determine the current flowing in the bulb.

Current, I = $\frac{V}{R} = \frac{240}{960} = \frac{1}{4}$ = **0.25 A**

3. Graphs of current against voltage for two resistors P and Q are shown below. Determine the value of each resistor.



For resistor P, **R** = $\frac{V}{I} = \frac{16\,\mu V}{8\,mA} = \frac{16\times10^{-6}\,V}{8\times10^{-3}\,A}$ = $\mathbf{2\times10^{-3}\,\Omega}$ = **2 mΩ**

For resistor Q, **R** = $\frac{V}{I} = \frac{20\,\mu V}{4\,mA} = \frac{20\times10^{-6}\,V}{4\times10^{-3}\,A}$ = $\mathbf{5\times10^{-3}\,\Omega}$ = **5 mΩ**

4. Determine the p.d. which must be applied to a 5 kΩ resistor such that a current of 6 mA may flow.

P.d., V = I × R = $6 \times 10^{-3} \times 5 \times 10^{3}$ = **30 V**

EXERCISE 158, Page 349

1. The hot resistance of a 250 V filament lamp is 625 Ω. Determine the current taken by the lamp and its power rating.

Current, I = $\frac{V}{I} = \frac{250}{625}$ = **0.4 A**

Power rating, P = V × I = 250 × 0.4 = **100 W**

(or **P** = $\frac{V^2}{R} = \frac{250^2}{625}$ = **100 W** or **P** = $I^2R = (0.4)^2 \times 625$ = **100 W**)

2. Determine the resistance of an electric fire which takes a current of 12 A from a 240 V supply. Find also the power rating of the fire and the energy used in 20 h.

Resistance, R = $\frac{V}{I} = \frac{240}{12}$ = **20 Ω**

Power rating, P = V × I = 240 × 12 = 2880 W or **2.88 kW**

Energy = power × time = 2.88 kW × 20 h = **57.6 kWh**

3. Determine the power dissipated when a current of 10 mA flows through an appliance having a resistance of 8 kΩ.

Power, P = $I^2R = (10\times10^{-3})^2(8\times10^3)$ = **0.8 W**

4. 85.5 J of energy are converted into heat in nine seconds. What power is dissipated?

Power, P = $\frac{\text{energy}}{\text{time}} = \frac{85.5\,\text{J}}{9\,\text{s}}$ = **9.5 W**

5. A current of 4 A flows through a conductor and 10 W is dissipated. What p.d. exists across the ends of the conductor?

Power, P = V × I   from which, **p.d., V** = $\frac{P}{I} = \frac{10}{4}$ = **2.5 V**

6. Find the power dissipated when: (a) a current of 5 mA flows through a resistance of 20 kΩ

(b) a voltage of 400 V is applied across a 120 kΩ resistor

(c) a voltage applied to a resistor is 10 kV and the current flow is 4 mA

(a) **Power, P** = $I^2R = \left(5\times10^{-3}\right)^2\left(20\times10^3\right)$ = **0.5 W**

(b) **Power, P** = $\frac{V^2}{R} = \frac{400^2}{120\times10^3}$ = **1.33 W**

(c) **Power, P** = V × I = $\left(10\times10^3\right)\left(4\times10^{-3}\right)$ = **40 W**

7. A d.c. electric motor consumes 72 MJ when connected to 400 V supply for 2 h 30 min. Find the power rating of the motor and the current taken from the supply.

Power = $\frac{\text{energy}}{\text{time}} = \frac{72\times10^6\text{ J}}{2.5\times60\times60}$ = 8000 W = **8 kW = power rating of motor**

Power, P = V × I,   hence, **current, I** = $\frac{P}{V} = \frac{8\times10^3}{400}$ = **20 A**

8. A p.d. of 500 V is applied across the winding of an electric motor and the resistance of the winding is 50 Ω. Determine the power dissipated by the coil.

Power, P = $\frac{V^2}{R} = \frac{500^2}{50}$ = 5000 W = **5 kW**

9. In a household during a particular week three 2 kW fires are used on average 25 h each and eight 100 W light bulbs are used on average 35 h each. Determine the cost of electricity for the week if 1 unit of electricity costs 15p

Energy in week = $3(2 \text{ kW} \times 25 \text{ h}) + 8(100\times10^{-3} \text{ kW} \times 35 \text{ h}) = 150 + 28 = 178$ kWh

Cost = $178 \times 15 = 2670$p = **£26.70**

10. Calculate the power dissipated by the element of an electric fire of resistance 30 Ω when a current of 10 A flows in it. If the fire is on for 30 hours in a week determine the energy used. Determine also the weekly cost of energy if electricity costs 13.50p per unit.

Power, P = $I^2R = (10)^2 \times (30) = 3000$ W or **3 kW**

Energy = power × time = 3 kW × 30 h = **90 kWh**

Cost = 90×13.50p = 1215p = **£12.15**

EXERCISE 159, Page 350

1. A television set having a power rating of 120 W and electric lawnmower of power rating 1 kW are both connected to a 240 V supply. If 3 A, 5 A and 10 A fuses are available state which is the most appropriate for each appliance.

Power, $P = V \times I$ hence, current, $I = \frac{P}{V}$

For the television, $I = \frac{P}{V} = \frac{120}{240} = 0.5$ A, hence **the 3 A fuse is the most appropriate**

For the lawnmower, $I = \frac{P}{V} = \frac{1000}{240} = 4.17$ A, hence **the 5 A fuse is the most appropriate**

EXERCISE 160, Page 350

Answers found from within the text of the chapter, pages 341 to 350.

EXERCISE 161, Page 351

1. (d) **2.** (a) **3.** (c) **4.** (b) **5.** (d) **6.** (d) **7.** (b) **8.** (c) **9.** (a) **10.** (a) **11.** (c) **12.** (c)

13. (b) **14.** (a) **15.** (c) **16.** (b) **17.** (d) **18.** (d)

CHAPTER 34 RESISTANCE VARIATION

EXERCISE 162, Page 356

1. The resistance of a 2 m length of cable is 2.5 Ω. Determine (a) the resistance of a 7 m length of the same cable and (b) the length of the same wire when the resistance is 6.25 Ω.

(a) If the resistance of a 2 m length of cable is 2.5 Ω, then a 1 m length of cable is 1.25 Ω

Thus, the resistance of a 7 m length of cable is 7 × 1.25 = **8.75 Ω**

(b) If the resistance of a 2 m length of cable is 2.5 Ω, then a $\frac{2}{2.5}$ m length of cable is 1 Ω

Thus, a resistance of 6.25 Ω corresponds to a length of $6.25 \times \frac{2}{2.5}$ m = **5 m**

2. Some wire of cross-sectional area 1 mm^2 has a resistance of 20 Ω. Determine (a) the resistance of a wire of the same length and material if the cross-sectional area is $4\,mm^2$, and (b) the cross-sectional area of a wire of the same length and material if the resistance is 32 Ω.

(a) $R \propto \frac{1}{a}$ thus a wire of cross-sectional area $4\,mm^2$ has a resistance of $\frac{20}{4}$ = **5 Ω**

(b) Since wire of cross-sectional area $1\,mm^2$ has a resistance of 20 Ω,

then a c.s.a. of 20 mm^2 has a resistance of 1 Ω.

Hence, a resistance of 32 Ω corresponds to a c.s.a. of $\frac{20}{32}$ = $\mathbf{0.625\,mm^2}$

3. Some wire of length 5 m and cross-sectional area $2\,mm^2$ has a resistance of 0.08 Ω. If the wire is drawn out until its cross-sectional area is $1\,mm^2$, determine the resistance of the wire.

$R = \frac{\rho l}{a}$ i.e. $0.08 = \frac{\rho(5)}{2\times 10^{-6}}$ from which, resistivity, $\rho = \frac{0.08 \times 2\times 10^{-6}}{5} = 0.032\times 10^{-6}$

If c.s.a. = $1\,mm^2$ (i.e. half the original c.s.a.) then the length will double, i.e. l = 2 × 5 = 10 m

Hence, **resistance**, $R = \frac{\rho l}{a} = \frac{(0.032 \times 10^{-6})(10)}{1 \times 10^{-6}}$ = **0.32 Ω**

4. Find the resistance of 800 m of copper cable of cross-sectional area 20 mm^2. Take the resistivity of copper as 0.02 μΩm

Resistance, $R = \frac{\rho l}{a} = \frac{(0.02 \times 10^{-6}\,\Omega m)(800\,m)}{20 \times 10^{-6}\,m^2}$ = **0.8 Ω**

5. Calculate the cross-sectional area, in mm^2, of a piece of aluminium wire 100 m long and having a resistance of 2 Ω. Take the resistivity of aluminium as $0.03 \times 10^{-6}\,\Omega m$

Since $R = \frac{\rho l}{a}$ then **c.s.a., a** = $\frac{\rho l}{R} = \frac{(0.03 \times 10^{-6}\,\Omega m)(100\,m)}{2\,\Omega} = 1.5 \times 10^{-6}\,m^2$ = $\mathbf{1.5\,mm^2}$

6. The resistance of 500 m of wire of cross-sectional area 2.6 mm^2 is 5 Ω. Determine the resistivity of the wire in μΩm

Since $R = \frac{\rho l}{a}$ then **resistivity, ρ** = $\frac{R\,a}{l} = \frac{(5\,\Omega)(2.6 \times 10^{-6}\,m^2)}{500\,m}$ = $\mathbf{2.6 \times 10^{8}\,\Omega m}$ or **0.026 μΩm**

7. Find the resistance of 1 km of copper cable having a diameter of 10 mm if the resistivity of copper is $0.017 \times 10^{-6}\,\Omega m$.

Resistance, $R = \frac{\rho l}{a} = \frac{\rho l}{\pi r^2} = \frac{(0.017 \times 10^{-6}\,\Omega m)(1 \times 10^{3}\,m)}{\pi (5)^2 \times 10^{-6}\,m^2}$ = **0.216 Ω**

EXERCISE 163, Page 358

1. A coil of aluminium wire has a resistance of 50 Ω when its temperature is 0°C. Determine its resistance at 100°C if the temperature coefficient of resistance of aluminium at 0°C is 0.0038/°C

Resistance at 100°C, $R_{100} = R_0[1+\alpha_0(100)] = 50[1+100(0.0038)] = 50[1+0.38]$ = **69 Ω**

2. A copper cable has a resistance of 30 Ω at a temperature of 50°C. Determine its resistance at 0°C. Take the temperature coefficient of resistance of copper at 0°C as 0.0043/°C.

$R_{50} = R_0[1+\alpha_0(50)]$ from which,

$$\textbf{resistance at 0°C, } R_0 = \frac{R_{50}}{1+50\alpha_0} = \frac{30}{1+50(0.0043)} = \frac{30}{1.215} = \mathbf{24.69\ \Omega}$$

3. The temperature coefficient of resistance for carbon at 0°C is - 0.00048/°C. What is the significance of the minus sign? A carbon resistor has a resistance of 500 Ω at 0°C. Determine its resistance at 50°C.

For carbon, resistance falls with increase of temperature, hence the minus sign.

$R_{50} = R_0[1+\alpha_0(50)] = 500[1+50(-0.00048)] = 500[1-0.024]$ = **488 Ω**

4. A coil of copper wire has a resistance of 20 Ω at 18°C. If the temperature coefficient of resistance of copper at 18°C is 0.004/°C, determine the resistance of the coil when the temperature rises to 98°C

Resistance at θ°C, $R_\theta = R_{18}[1 + \alpha_{18}(\theta - 18)]$

Hence, resistance at 98°C, $R_{98} = 20[1 + (0.004)(98 - 18)]$

$= 20[1 + (0.004)(80)]$

$= 20\,[1 + 0.32]$

$= 20(1.32) = \mathbf{26.4\ \Omega}$

5. The resistance of a coil of nickel wire at 20°C is 100 Ω. The temperature of the wire is increased and the resistance rises to 130 Ω. If the temperature coefficient of resistance of nickel is 0.006/°C at 20°C, determine the temperature to which the coil has risen.

$$R_\theta = R_{20}\left[1+\alpha_{20}(\theta-20)\right]$$

i.e. $130 = 100\left[1+0.006(\theta-20)\right] = 100+0.6(\theta-20)$

i.e. $130 - 100 = 0.6\,(\theta - 20)$

and $(\theta - 20) = \dfrac{130-100}{0.6} = \dfrac{30}{0.6} = 50$

Hence, **temperature to which the coil has risen,** $\theta = 50 + 20 =$ **70°C**

6. Some aluminium wire has a resistance of 50 Ω at 20°C. The wire is heated to a temperature of 100°C. Determine the resistance of the wire at 100°C, assuming that the temperature coefficient of resistance at 0°C is 0.004/°C

$R_{20} = 50\ \Omega$, $\alpha_0 = 0.004$/°C and $\dfrac{R_{20}}{R_{100}} = \dfrac{\left[1+\alpha_0(20)\right]}{\left[1+\alpha_0(100)\right]}$

Hence, $R_{100} = \dfrac{R_{20}\left[1+100\alpha_0\right]}{\left[1+20\alpha_0\right]} = \dfrac{50[1+100(0.004)]}{[1+20(0.004)]} = \dfrac{50[1+0.40]}{[1+0.08]} = \dfrac{50(1.40)}{(1.08)} = \mathbf{64.8\ \Omega}$

i.e. **the resistance of the wire at 100°C is 64.8 Ω**

7. A copper cable is 1.2 km long and has a cross-sectional area of $5\,\text{mm}^2$. Find its resistance at 80°C if at 20°C the resistivity of copper is $0.02\times10^{-6}\,\Omega\text{m}$ and its temperature coefficient of resistance is 0.004/°C.

Resistance at 20°C, $R_{20} = \frac{\rho l}{a} = \frac{\left(0.02 \times 10^{-6}\,\Omega\text{m}\right)\left(1.2 \times 10^{3}\,\text{m}\right)}{5 \times 10^{-6}\,\text{m}^2}$ = 4.8 Ω

Resistance at 80°C, $R_{80} = R_{20}\left[1 + \alpha_{20}(80 - 20)\right] = 4.8\left[1 + 0.004(60)\right] = 4.8[1.24]$ = **5.95 Ω**

EXERCISE 164, Page 358

Answers found from within the text of the chapter, pages 353 to 358.

EXERCISE 165, Page 358

1. (c) **2.** (d) **3.** (b) **4.** (d) **5.** (d) **6.** (c) **7.** (b)

CHAPTER 35 BATTERIES AND ALTERNATIVE SOURCES OF ENERGY

EXERCISE 166, Page 365

1. Twelve cells, each with an internal resistance of 0.24 Ω and an e.m.f. of 1.5 V are connected (a) in series, (b) in parallel. Determine the e.m.f. and internal resistance of the batteries so formed.

(a) Total e.m.f. in series = 12×1.5 = **18 V**

Total internal resistance in series = 12×0.24 = **2.88 Ω**

(b) Total e.m.f. in parallel = **1.5 V**

Total internal resistance in parallel = $\frac{1}{12} \times 0.24$ = **0.02 Ω**

2. A cell has an internal resistance of 0.03 Ω and an e.m.f. of 2.2 V. Calculate its terminal p.d. if it delivers (a) 1 A, (b) 20 A, (c) 50 A.

(a) Terminal p.d., $V = E - Ir = 2.2 - (1)(0.03)$ = **2.17 V**

(b) Terminal p.d., $V = E - Ir = 2.2 - (20)(0.03) = 2.2 - 0.6$ = **1.6 V**

(c) Terminal p.d., $V = E - Ir = 2.2 - (50)(0.03) = 2.2 - 1.5$ = **0.7 V**

3. The p.d. at the terminals of a battery is 16 V when no load is connected and 14 V when a load taking 8 A is connected. Determine the internal resistance of the battery.

When no load is connected the e.m.f. of the battery, E, is equal to the terminal p.d., V, i.e. E = 16 V

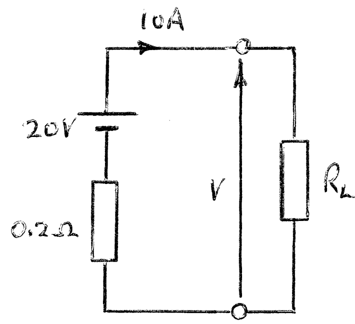
When current I = 8 A and terminal p.d. V = 14 V, then $V = E - Ir$

i.e. $14 = 16 - (8)r$

Hence, rearranging, gives $8r = 16 - 14 = 2$

and the **internal resistance**, **r** = $\frac{2}{8} = \frac{1}{4}$ = **0.25 Ω**

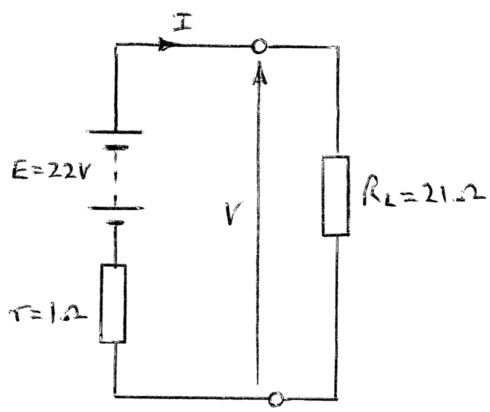
4. A battery of e.m.f. 20 V and internal resistance 0.2 Ω supplies a load taking 10 A. Determine the p.d. at the battery terminals and the resistance of the load.



P.d. at battery terminals, V = E – Ir = 20 – 10(0.2) = **18 V**

Load resistance, $R_L = \frac{V}{I} = \frac{18}{10}$ = **1.8 Ω**

5. Ten 2.2 V cells, each having an internal resistance of 0.1 Ω are connected in series to a load of 21 Ω. Determine (a) the current flowing in the circuit, and (b) the p.d. at the battery terminals.



E.m.f., E = 10 × 2.2 = 22V, and internal resistance, r = 10 × 0.1 = 1 Ω

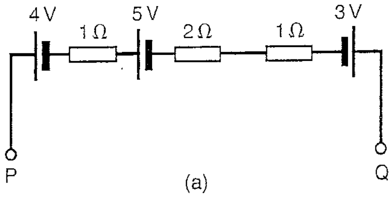
(a) **Current**, I = $\frac{E}{r+R_L} = \frac{22}{1+21}$ = **1 A**

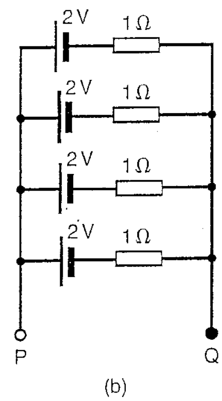
(b) **P.d. at the battery terminals**, V = E – Ir = 22 – (1)(1) = **21 V** (or $V = I R_L = (1)(21)$ = **21 V**)

6. For the circuits shown below the resistors represent the internal resistance of the batteries.

Find, in each case: (i) the total e.m.f. across PQ

(ii) the total equivalent internal resistances of the batteries.





(a)(i) Total e.m.f. across PQ , **E** = 4 + 5 – 3 = **6 V**

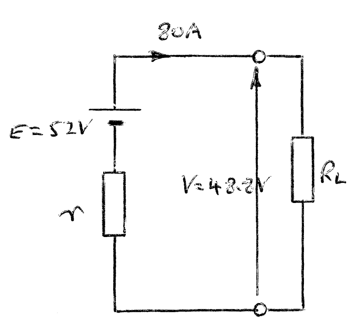
(ii) Total internal resistance, **r** = 1 + 1 + 2 = **4 Ω**

(b)(i) Total e.m.f. across PQ , **E = 2 V**

(ii) Total internal resistance, r is given by $\frac{1}{r} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 4 \;= 1$

Hence, **r** = $\frac{1}{4}$ = **0.25 Ω**

7. The voltage at the terminals of a battery is 52 V when no load is connected and 48.8 V when a load taking 80 A is connected. Find the internal resistance of the battery. What would be the terminal voltage when a load taking 20 A is connected?



$V = E - Ir$, hence $48.8 = 52 - 80\ r$

from which, $80\ r = 52 - 48.8 = 3.2$

and **internal resistance**, $\mathbf{r} = \dfrac{3.2}{80} = \mathbf{0.04\ \Omega}$

When I = 20 A, **terminal voltage, V** = 52 – 20(0.04) = 52 – 0.8 = **51.2 V**

EXERCISE 167, Page 371

Answers found from within the text of the chapter, pages 360 to 371.

EXERCISE 168, Page 372

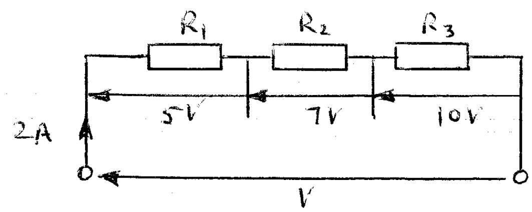
1. (d) **2.** (a) **3.** (b) **4.** (c) **5.** (b) **6.** (d) **7.** (d) **8.** (b) **9.** (c) **10.** (d) **11.** (c) **12.** (a)

13. (c)

CHAPTER 36 SERIES AND PARALLEL NETWORKS

EXERCISE 169, Page 377

1. The p.d.'s measured across three resistors connected in series are 5 V, 7 V and 10 V, and the supply current is 2 A. Determine (a) the supply voltage, (b) the total circuit resistance and (c) the values of the three resistors.

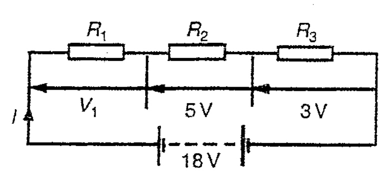


(a) Supply voltage, **V** = 5 + 7 + 10 = **22 V**

(b) Total circuit resistance, $R_T = \frac{V}{I} = \frac{22}{2} = \mathbf{11\ \Omega}$

(c) $R_1 = \frac{V_1}{I} = \frac{5}{2} = \mathbf{2.5\ \Omega}$, $R_2 = \frac{V_2}{I} = \frac{7}{2} = \mathbf{3.5\ \Omega}$ and $R_3 = \frac{V_3}{I} = \frac{10}{2} = \mathbf{5\ \Omega}$

2. For the circuit shown below, determine the value of V_1. If the total circuit resistance is 36 Ω determine the supply current and the value of resistors R_1, R_2 and R_3



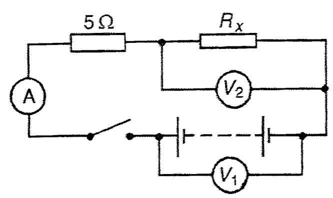
Supply voltage, $18 = V_1 + 5 + 3$

Hence, **voltage,** $\mathbf{V_1} = 18 - 5 - 3 = \mathbf{10\ V}$

Supply current, $\mathbf{I} = \frac{V}{R_T} = \frac{18}{36} = \mathbf{0.5\ A}$

$\mathbf{R_1} = \frac{V_1}{I} = \frac{10}{0.5} = \mathbf{20\ \Omega}$ $\mathbf{R_2} = \frac{V_2}{I} = \frac{5}{0.5} = \mathbf{10\ \Omega}$ $\mathbf{R_3} = \frac{V_3}{I} = \frac{3}{0.5} = \mathbf{6\ \Omega}$

3. When the switch in the circuit shown is closed the reading on voltmeter 1 is 30 V and that on voltmeter 2 is 10 V. Determine the reading on the ammeter and the value of resistor R_X

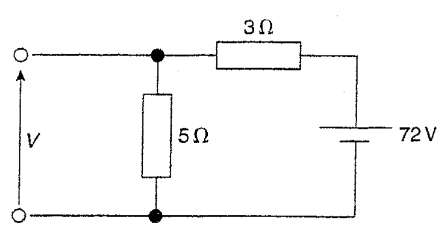


Voltage across 5 Ω resistor = $V_1 - V_2 = 30 - 10 = 20$ V

Hence, current in 5 Ω resistor, i.e. **reading on the ammeter** = $\frac{V_{5\Omega}}{5} = \frac{20}{5} = \mathbf{4\ A}$

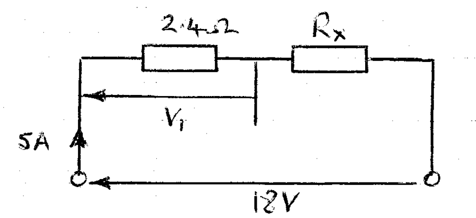
Total resistance, $R_T = \frac{V_T}{I} = \frac{30}{4} = 7.5\,\Omega$, hence $R_X = 7.5 - 5 = \mathbf{2.5\ \Omega}$

4. Calculate the value of voltage V in the diagram below.



Voltage, V = $\left(\frac{5}{5+3}\right)(72) = \mathbf{45\ V}$

5. Two resistors are connected in series across an 18 V supply and a current of 5 A flows. If one of the resistors has a value of 2.4 Ω determine (a) the value of the other resistor and (b) the p.d. across the 2.4 Ω resistor.



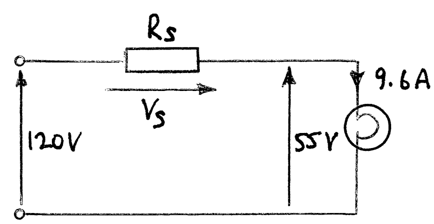
The circuit is shown above.

(a) Total resistance, $R_T = \frac{18}{5} = 3.6\ \Omega$, hence $R_X = 3.6 - 2.4 = \mathbf{1.2\ \Omega}$

(b) $V_1 = 5 \times 2.4 = \mathbf{12\ V}$

6. An arc lamp takes 9.6 A at 55 V. It is operated from a 120 V supply. Find the value of the stabilising resistor to be connected in series.

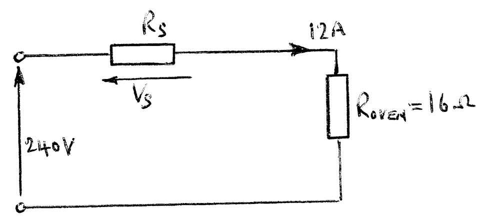
A circuit diagram is shown below.



The purpose of the stabilising resistor R_S is to cause a volt drop V_S – in this case equal to 120 – 55, i.e. 65 V. Hence, $R_S = \frac{V_S}{I} = \frac{65}{9.6} = \mathbf{6.77\ \Omega}$

7. An oven takes 15 A at 240 V. It is required to reduce the current to 12 A. Find (a) the resistor which must be connected in series, and (b) the voltage across the resistor.

(a) If the oven takes 15 A at 240 V, then resistance of oven, $R_{oven} = \frac{240}{15} = 16$ A



A circuit diagram is shown above.

If the current is reduced to 12 A then the total resistance of the circuit, $R_T = \frac{V}{I} = \frac{240}{12} = 20\ \Omega$

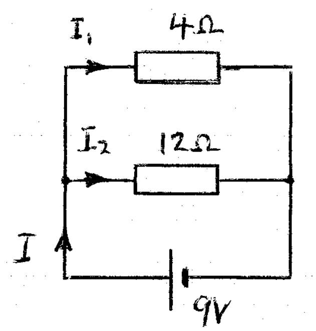
and $\quad R_T = R_S + R_{oven}$

i.e. $\quad 20 = R_S + 16 \quad$ from which, **series resistor**, $R_S = 20 - 16 = \mathbf{4\ \Omega}$

(b) **Voltage across series resistor**, $V_S = I \times R_S = 12 \times 4 = \mathbf{48\ \Omega}$

EXERCISE 170, Page 383

1. Resistances of 4 Ω and 12 Ω are connected in parallel across a 9 V battery. Determine (a) the equivalent circuit resistance, (b) the supply current, and (c) the current in each resistor.



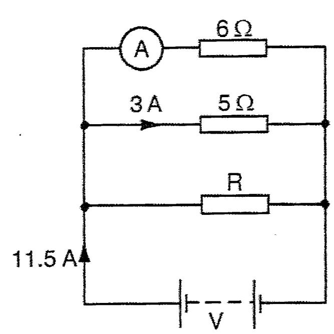
(a) Equivalent circuit resistance, $R_T = \frac{4 \times 12}{4+12} = \frac{48}{16} = \mathbf{3\ \Omega}$ (or use $\frac{1}{R_T} = \frac{1}{4} + \frac{1}{12}$)

(b) Supply current, $I = \frac{V}{R_T} = \frac{9}{3} = \mathbf{3\ A}$

(c) $I_1 = \frac{9}{4} = \mathbf{2.25\ A}$, $I_2 = \frac{9}{12} = \mathbf{0.75\ A}$ (or, by current division, $I_1 = \left(\frac{12}{4+12}\right)(3) = \mathbf{2.25\ A}$

and $I_2 = \left(\frac{4}{4+12}\right)(3) = \mathbf{0.75\ A}$)

2. For the circuit shown determine (a) the reading on the ammeter, and (b) the value of resistor R.



(a) V = 3 × 5 = 15 V. Hence, **ammeter reading**, $I_{6\,\Omega} = \frac{V}{6} = \frac{15}{6} = \mathbf{2.5\ A}$

(b) $I_R = 11.5 - 3 - 2.5 = 6$ A hence, $R = \frac{V}{I} = \frac{15}{6} = \mathbf{2.5\ \Omega}$

3. Find the equivalent resistance when the following resistance's are connected (a) in series (b) in Parallel (i) 3 Ω and 2 Ω (ii) 20 kΩ and 40 kΩ (iii) 4 Ω, 8 Ω and 16 Ω (iv) 800 Ω, 4 kΩ and 1500 Ω

(a)(i) Total resistance, $\mathbf{R_T} = 3 + 2 = \mathbf{5\ \Omega}$

(ii) Total resistance, $\mathbf{R_T} = 20 + 40 = \mathbf{60\ k\Omega}$

(iii) Total resistance, $\mathbf{R_T} = 4 + 8 + 16 = \mathbf{28\ \Omega}$

(iv) Total resistance, $\mathbf{R_T} = 800 + 4000 + 1500 = \mathbf{6300\ \Omega}$ or $\mathbf{6.3\ k\Omega}$

(b)(i) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ from which, $\mathbf{R_T} = \frac{6}{5} = \mathbf{1.2\ \Omega}$

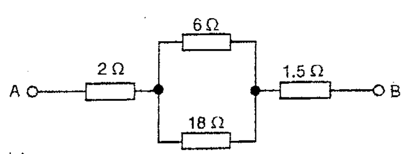
(ii) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{20} + \frac{1}{40} = \frac{3}{40}$ from which, $\mathbf{R_T} = \frac{40}{3} = \mathbf{13.33\ k\Omega}$

(iii) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{7}{16}$ from which, $\mathbf{R_T} = \frac{16}{7} = \mathbf{2.29\ \Omega}$

(iv) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{800} + \frac{1}{4000} + \frac{1}{1500} = \frac{13}{6000}$

from which, $\mathbf{R_T} = \frac{6000}{13} = \mathbf{461.54\ \Omega}$

4. Find the total resistance between terminals A and B of circuit (a) shown below.

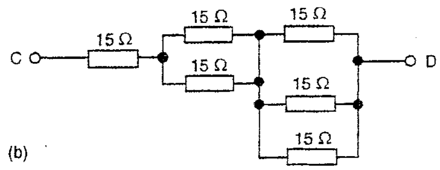


(a)

Resistance of parallel branches, $R_p = \frac{6 \times 18}{6 + 18} = 4.5\ \Omega$

Total circuit resistance, $\mathbf{R_T} = 2 + 4.5 + 1.5 = \mathbf{8\ \Omega}$

5. Find the equivalent resistance between terminals C and D of circuit (b) shown below.



Resistance of first parallel branches, $R_{P_1} = \dfrac{15 \times 15}{15 + 15} = 7.5\ \Omega$

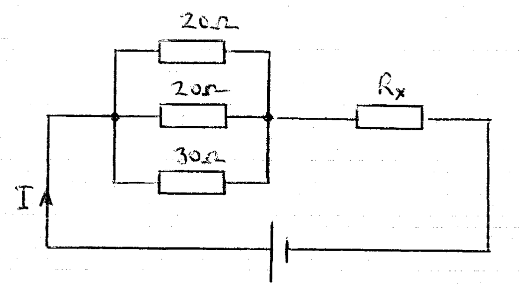
Resistance of second parallel branches, R_{P_2} is given by: $\dfrac{1}{R_{P_2}} = \dfrac{1}{15} + \dfrac{1}{15} + \dfrac{1}{15} = \dfrac{3}{15} = \dfrac{1}{5}$

i.e. $R_{P_2} = 5\ \Omega$

Total circuit resistance, $\mathbf{R_T} = 15 + 7.5 + 5 = \mathbf{27.5\ \Omega}$

6. Resistors of 20 Ω, 20 Ω and 30 Ω are connected in parallel. What resistance must be added in series with the combination to obtain a total resistance of 10 Ω. If the complete circuit expends a power of 0.36 kW, find the total current flowing.

The circuit is shown below.



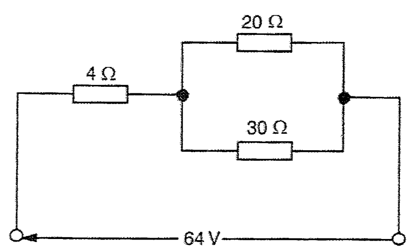
For the parallel branch, $\dfrac{1}{R_P} = \dfrac{1}{20} + \dfrac{1}{20} + \dfrac{1}{30}$ from which, $R_P = 7.5\ \Omega$

Hence, **resistance to be added in series**, $R_X = R_T - R_P = 10 - 7.5 = \mathbf{2.5\ \Omega}$

Power, $P = I^2 R$ hence $0.36 \times 10^3 = I^2 (10)$

from which, **total current flowing, I** = $\sqrt{\frac{360}{10}} = \sqrt{36} = \mathbf{6\ A}$

7. (a) Calculate the current flowing in the 30 Ω resistor shown in the circuit below



(b) What additional value of resistance would have to be placed in parallel with the 20 Ω and 30 Ω resistors, to change the supply current to 8 A, the supply voltage remaining constant.

(a) Total resistance, $R_T = 4 + \frac{20 \times 30}{20 + 30} = 4 + 12 = 16\ \Omega$

Hence, total current, $I = \frac{V}{R_T} = \frac{64}{16} = 4\ A$

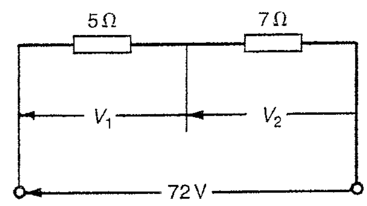
and, by current division, $I_{30\,\Omega} = \left(\frac{20}{20+30}\right)(4) = \mathbf{1.6\ A}$

(b) If I = 8 A then new total resistance, $R_{T_2} = \frac{64}{8} = 8\ \Omega$ and the resistance of the parallel branch will be: $8 - 4 = 4\ \Omega$

i.e. $\frac{1}{4} = \frac{1}{20} + \frac{1}{30} + \frac{1}{R_X}$ where R_X is the additional resistance to be placed in parallel

from which, $\frac{1}{R_X} = \frac{1}{4} - \frac{1}{20} - \frac{1}{30}$ from which, $\mathbf{R_X = 6\ \Omega}$

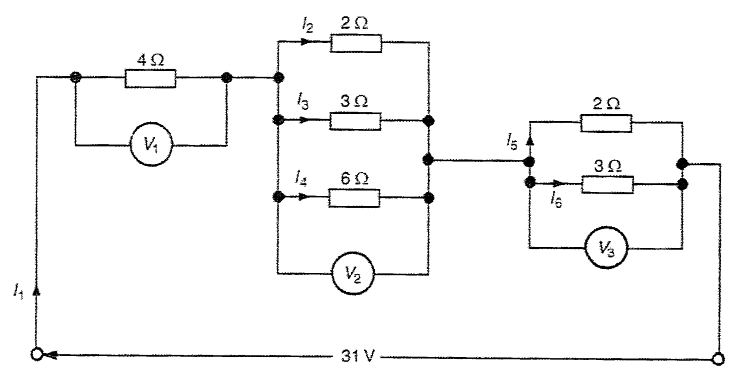
8. For the circuit shown below, find (a) V_1, (b) V_2, without calculating the current flowing



(a) **Voltage,** $\mathbf{V_1} = \left(\frac{5}{5+7}\right)(72) = \mathbf{30\ V}$ by voltage division

(b) **Voltage,** $\mathbf{V_2} = \left(\frac{7}{5+7}\right)(72) = \mathbf{42\ V}$

9. Determine the currents and voltages indicated in the circuit below.



$$\frac{1}{R_{P_1}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \quad \text{from which,} \quad R_{P_1} = 1\ \Omega$$

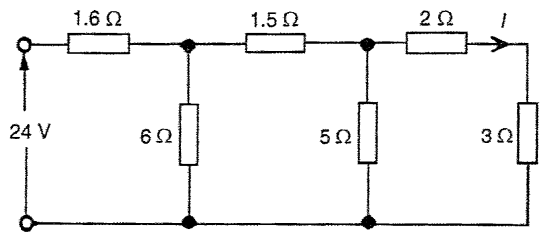
$$R_{P_2} = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

Hence, total resistance, $R_T = 4 + 1 + 1.2 = 6.2\ \Omega$

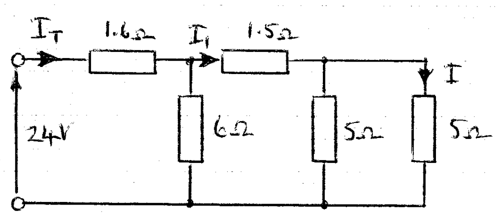
$\mathbf{I_1} = \frac{31}{6.2} = \mathbf{5\ A}$, $\mathbf{V_1} = I_1(4) = 5 \times 4 = \mathbf{20\ V}$, $\mathbf{V_2} = 5 \times 1 = \mathbf{5\ V}$ and $\mathbf{V_3} = 5 \times 1.2 = \mathbf{6\ V}$

$\mathbf{I_2} = \frac{V_2}{2} = \frac{5}{2} = \mathbf{2.5\ A}$, $\mathbf{I_3} = \frac{5}{3} = \mathbf{1.67\ A}$, $\mathbf{I_4} = \mathbf{0.83\ A}$, $\mathbf{I_5} = \frac{V_3}{2} = \frac{6}{2} = \mathbf{3\ A}$ and $\mathbf{I_6} = \frac{6}{3} = \mathbf{2\ A}$

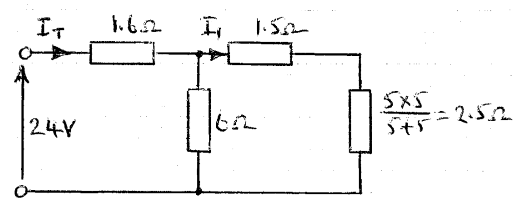
10. Find the current I in the circuit below.



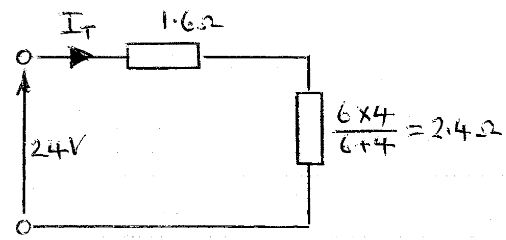
The circuit is reduced step by step as shown in diagrams (a) to (d) below.



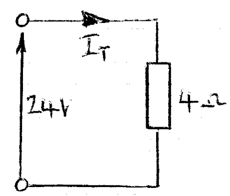
(a)



(b)



(c)



(d)

From (d), $I_T = \frac{24}{4} = 6$ A

From (b), $I_1 = \left(\frac{6}{6+4}\right)(6) = 3.6$ A

and from (a), $I = \left(\frac{5}{5+5}\right)(3.6) = \mathbf{1.8\ A}$

EXERCISE 171, Page 385

1. If four identical lamps are connected in parallel and the combined resistance is 100 Ω, find the resistance of one lamp.

If each lamp has a resistance of R then:

$$\frac{1}{100} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{4}{R} \quad \text{and} \quad R = 4 \times 100 = \mathbf{400\ \Omega = resistance\ of\ a\ lamp}$$

2. Three identical filament lamps are connected (a) in series, (b) in parallel across a 210 V supply. State for each connection the p.d. across each lamp.

(a) In **series**, p.d. across each lamp $= \frac{210}{3} = \mathbf{70\ V}$

(b) In **parallel**, p.d. across each lamp = **210 V**

EXERCISE 172, Page 385

Answers found from within the text of the chapter, pages 3374to 385.

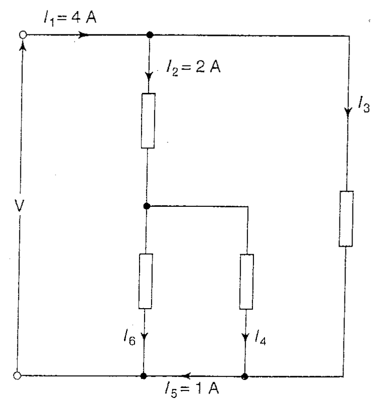
EXERCISE 173, Page 386

1. (a) **2.** (c) **3.** (c) **4.** (c) **5.** (a) **6.** (d) **7.** (b) **8.** (c) **9.** (d) **10.** (d)

CHAPTER 37 KIRCHHOFF'S LAWS

EXERCISE 174, Page 392

1. Find currents I_3, I_4 and I_6 in the circuit below.

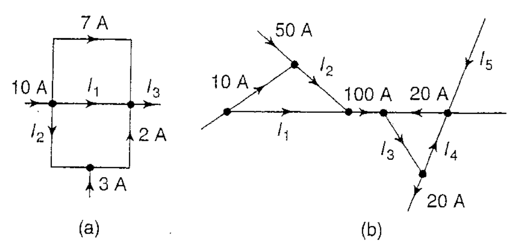


By Kirchhoff's law, $I_1 = I_2 + I_3$ i.e. $4 = 2 + I_3$ from which, $\mathbf{I_3} = 4 - 2 = \mathbf{2\ A}$

Also, $I_3 + I_4 = I_5$ i.e. $I_3 + I_4 = I_5$ i.e. $2 + I_4 = 1$ from which, $\mathbf{I_4} = 1 - 2 = \mathbf{-1\ A}$

And $I_5 + I_6 = I_1$ i.e. $1 + I_6 = 4$ from which, $\mathbf{I_6} = 4 - 1 = \mathbf{3\ A}$

2. For the networks shown below, find the values of the currents marked.



(a) $3 + I_2 = 2$ from which, $\mathbf{I_2} = 2 - 3 = \mathbf{-1\ A}$

Also, $10 = 7 + I_1 + I_2$ i.e. $10 = 7 + I_1 + (-1)$ from which, $\mathbf{I_1} = 10 + 1 - 7 = \mathbf{4\ A}$

And $7 + I_1 + 2 = I_3$ i.e. $7 + 4 + 2 = I_3$ i.e. $\mathbf{I_3 = 13\ A}$

(b) $50 + 10 = I_2$ i.e. $\mathbf{I_2 = 60\ A}$

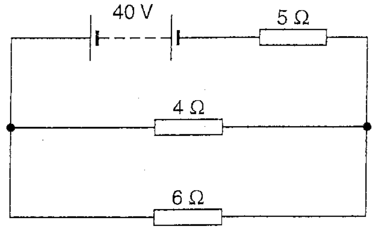
Also, $I_1 + I_2 = 100$ from which, $I_2 = 100 - I_1 = 100 - 60$ i.e. $\mathbf{I_2 = 40\ A}$

And $100 + 20 = I_3$ i.e. $\mathbf{I_3 = 120\ A}$

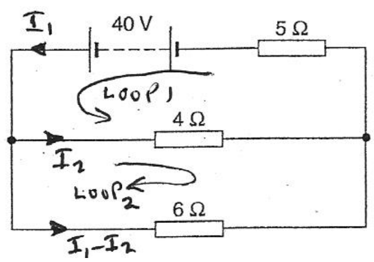
And $I_3 = 20 + I_4$ i.e. $I_4 = I_3 - 20 = 120 - 20$ i.e. $\mathbf{I_4 = 100\ A}$

And $I_4 + I_5 = 20$ i.e. $100 + I_5 = 20$ from which, $\mathbf{I_5} = 20 - 100 = \mathbf{-80\ A}$

3. Use Kirchhoff's laws to find the current flowing in the 6 Ω resistor of the circuit below and the power dissipated in the 4 Ω resistor.



The currents are labelled as shown in the diagram below.



Kirchhoff's voltage law is now applied to each loop in turn:

For loop 1: $40 = 5I_1 + 4I_2$ (1)

For loop 2: $0 = 4I_2 - 6(I_1 - I_2)$ (2)

Equation (2) simplifies to:

$$0 = -6I_1 + 10I_2 \quad (3)$$

5 × equation (1) gives:

$$200 = 25I_1 + 20I_2 \quad (4)$$

$2 \times$ equation (3) gives:

$$0 = -12I_1 + 20I_2 \qquad (5)$$

Equation (4) - equation (5) gives:

$$200 = (25I_1 - -12I_1)$$

i.e. $$200 = 37I_1$$

Hence, current, $I_1 = \frac{200}{37} = 5.4054$ A

Substituting $I_1 = 5.405$ into equation (1) gives:

$$40 = 5(5.405) + 4I_2$$

$$40 = 27.025 + 4I_2$$

and $$40 - 27.025 = 4I_1$$

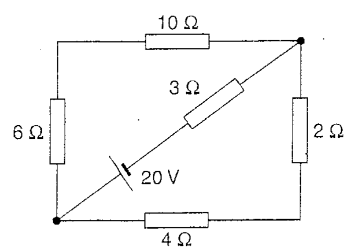
from which, $$I_2 = \frac{40 - 27.025}{4} = \frac{12.975}{4} = 3.2438 \text{ A}$$

Hence, **the current flowing in the 6 Ω resistance** is

i.e. $I_1 - I_2 = (5.4054 - 3.2438) =$ **2.162 A**

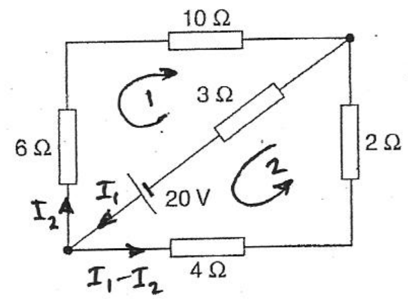
Power dissipated in the 4 Ω resistor = $(I_2)^2 R = (3.2438)^2 \times 4 =$ **42.09 W**

4. Find the current flowing in the 3 Ω resistor for the network shown in circuit (a) below. Find also the p.d. across the 10 Ω and 2 Ω resistors.



(a)

The currents are labelled as shown in the diagram below.



Loop 1: $20 = 3I_1 + (6 + 10)I_2$

i.e. $3I_1 + 16I_2 = 20$ (1)

Loop 2: $20 = 3I_1 + (I_1 - I_2)(4 + 2)$

i.e. $9I_1 - 6I_2 = 20$ (2)

3 × equation (1) gives: $9I_1 + 48I_2 = 60$ (3)

Equation (3) – (2) gives: $54I_2 = 40$

from which, $I_2 = \dfrac{40}{54} = 0.7407$ A

Substituting in (1) gives: $3I_1 + 16(0.7407) = 20$

i.e. $3I_1 = 20 - 11.851 = 8.149$

from which, $I_1 = \dfrac{8.149}{3} = 2.716$ A

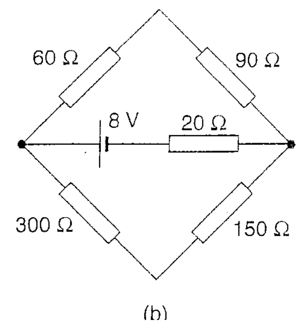
i.e. **the current flowing in the 3 Ω resistor = 2.716 A**

P.d. across the 10 Ω resistor = $I_2 \times 10 = 0.7407 \times 10 =$ **7.407 V**

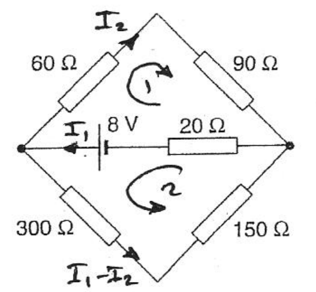
P.d. across the 2 Ω resistor = $(I_1 - I_2) \times 2 = (2.716 - 0.7407) \times 2$

$= 1.9753 \times 2 =$ **3.951 V**

5. For the network shown in circuit (b) below, find: (a) the current in the battery, (b) the current in the 300 Ω resistor, (c) the current in the 90 Ω resistor, and (d) the power dissipated in the 150 Ω resistor.



The currents are labelled as shown in the diagram below.



(a)

Loop 1: $8 = 20I_1 + (60 + 90)I_2$

i.e. $20I_1 + 150I_2 = 8$ (1)

Loop 2: $8 = 20I_1 + (I_1 - I_2)(300 + 150)$

i.e. $470I_1 - 450I_2 = 8$ (2)

3 × equation (1) gives: $60I_1 + 450I_2 = 24$ (3)

Equation (2) + (3) gives: $530I_1 = 32$

from which, $I_1 = \frac{32}{530} = 0.06038$ A

i.e. **the current in the battery = 60.38 mA**

(b)

Substituting in (1) gives: $20(0.06038) + 150I_2 = 8$

i.e. $150I_2 = 8 – 1.2076 = 6.7924$

from which, $I_2 = \dfrac{6.7924}{150} = 0.04528 \text{ A} = 45.28 \text{ mA}$

Hence, **current in 300 Ω resistor** = $I_1 - I_2$ = 60.38 – 45.28 = **15.10 mA**

(c)

The current in the 90 Ω resistor = I_2 = **45.28 mA**

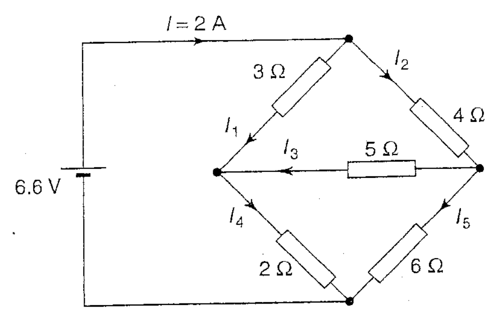
(d)

The power dissipated in the 150 Ω resistor = $(I_1 - I_2)^2 \times 150$

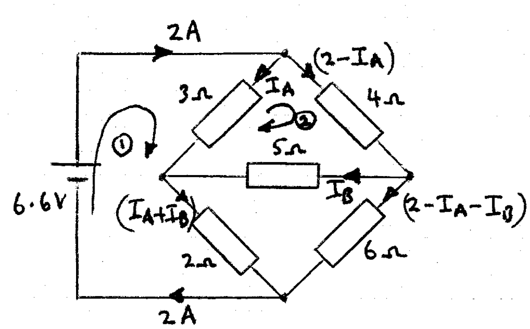
$$= \left(15.10\times10^{-3}\right)^2 \times 150$$

$$= \mathbf{0.0342\ W} \text{ or } \mathbf{34.20\ mW}$$

6. For the bridge network shown in circuit (c) below, find the currents I_1 to I_5



(c)



From loop 1: $6.6 = 3I_A + 2(I_A + I_B)$

From loop 2: $0 = 4(2 - I_A) + 5I_B - 3I_A$

i.e. $5I_A + 2I_B = 6.6$ (1)

and $\qquad -7I_A + 5I_B = -8 \qquad (2)$

$5 \times (1)$ gives: $\qquad 25I_A + 10I_B = 33 \qquad (3)$

$2 \times (2)$ gives: $\qquad -14I_A + 10I_B = -16 \qquad (4)$

(3) – (4) gives: $\qquad 39I_A \qquad = 49$

and $\qquad I_A = \frac{49}{39} = 1.256$ A

Substituting in (1) gives: $\qquad 5(1.256) + 2I_B = 6.6$

from which, $\qquad I_B = \frac{6.6 - 5(1.256)}{2} = 0.160$ A

Hence, correct to 2 decimal places, $\qquad \mathbf{I_1} = I_A = \mathbf{1.26\ A}$

$\mathbf{I_2} = 2 - I_A = 2 - 1.256 = \mathbf{0.74\ A}$

$\mathbf{I_3} = I_B = \mathbf{0.16\ A}$

$\mathbf{I_4} = I_A + I_B = 1.256 + 0.160 = \mathbf{1.42\ A}$

$\mathbf{I_5} = 2 - I_A - I_B = 2 - 1.26 - 0.16 = \mathbf{0.58\ A}$

EXERCISE 175, Page 393

Answers found from within the text of the chapter, pages 388 to 393.

EXERCISE 176, Page 393

1. (a) **2.** (d) **3.** (c) **4.** (b) **5.** (c)

CHAPTER 38 MAGNETISM AND ELECTROMAGNETISM

EXERCISE 177, Page 401

1. What is the flux density in a magnetic field of cross-sectional area 20 cm^2 having a flux of 3 mWb?

Flux density, B = $\dfrac{\Phi}{A} = \dfrac{3 \times 10^{-3}\,\text{Wb}}{20 \times 10^{-4}\,\text{m}^2}$ = **1.5 T**

2. Determine the total flux emerging from a magnetic pole face having dimensions 5 cm by 6 cm, if the flux density is 0.9 T.

$B = \dfrac{\Phi}{A}$ from which, **flux**, $\Phi = B \times A = 0.9 \times 5 \times 6 \times 10^{-4}$ = **2.7 mWb**

3. The maximum working flux density of a lifting electromagnet is 1.9 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 611 mWb determine the radius of the pole face.

$B = \dfrac{\Phi}{A} = \dfrac{\Phi}{\pi r^2}$ from which, $r^2 = \dfrac{\Phi}{B\pi}$ and **radius, r** = $\sqrt{\left(\dfrac{\Phi}{B\pi}\right)} = \sqrt{\dfrac{611 \times 10^{-3}}{1.9 \times \pi}}$ = 0.32 m or **32 cm**

4. An electromagnet of square cross-section produces a flux density of 0.45 T. If the magnetic flux is 720 μWb find the dimensions of the electromagnet cross-section.

$B = \dfrac{\Phi}{A}$ from which, area, $A = \dfrac{\Phi}{B} = \dfrac{720 \times 10^{-6}}{0.45} = 1.6 \times 10^{-3}\,\text{m}^2$

Let the side of the square section = x, then $x^2 = 1.6 \times 10^{-3}\,\text{m}^2$

and side, x = $\sqrt{1.6 \times 10^{-3}}\,\text{m}$ = 0.04 m = 4 cm i.e. **the dimensions of the electromagnet cross-section = 4 cm by 4 cm**

EXERCISE 178, Page 404

1. A conductor carries a current of 70 A at right-angles to a magnetic field having a a flux density of 1.5 T. if the length of the conductor in the field is 200 mm calculate the force acting on the conductor. What is the force when the conductor and field are at an angle of 45°?

Force, F = B I l sin θ = $1.5\times 70\times 200\times 10^{-3}\sin 90°$ = **21.0 N**

When θ = 45°, **F** = 21.0 sin 45° = **14.8 N**

2. Calculate the current required in a 240 mm length of conductor of a d.c. motor when the conductor is situated at right-angles to the magnetic field of flux density 1.25 T, if a force of 1.20 N is to be exerted on the conductor.

Force, F = B I l sin θ   i.e.   1.20 = 1.25 × I × 240×10^{-3} × sin 90º

from which, **current, I** = $\dfrac{1.20}{1.25\times 240\times 10^{-3}\times\sin 90°}$ = **4.0 A**

3. A conductor 30 cm long is situated at right-angles to a magnetic field. Calculate the flux density of the magnetic field if a current of 15 A in the conductor produces a force on it of 3.6 N

Force, F = B I l sin θ   from which, **flux density, B** = $\dfrac{F}{I\,l\sin\theta} = \dfrac{3.6}{15\times 0.30\times\sin 90°}$ = **0.80 T**

4. A conductor 300 mm long carries a current of 13 A and is at right-angles to a magnetic field between two circular pole faces, each of diameter 80 mm. If the total flux between the pole faces is 0.75 mWb calculate the force exerted on the conductor.

When conductor and field are at right angles, force, F = B I l where B = $\dfrac{\Phi}{A}$

Hence, **force, F** = $\dfrac{\Phi}{A} \times I \times l = \dfrac{0.75 \times 10^{-3}}{\pi(40)^2 10^{-6}} \times 13 \times 300 \times 10^{-3}$ = **0.582 N**

5. (a) A 400 mm length of conductor carrying a current of 25 A is situated at right-angles to a magnetic field between two poles of an electric motor. The poles have a circular cross-section. If the force exerted on the conductor is 80 N and the total flux between the pole faces is 1.27 mWb, determine the diameter of a pole face.

(b) If the conductor in part (a) is vertical, the current flowing downwards and the direction of the magnetic field is from left to right, what is the direction of the 80 N force?

(a) Force, F = B I l = $\dfrac{\Phi}{A} \times I \times l = \dfrac{\Phi}{\pi r^2} \times I \times l$

from which, **radius, r** = $\sqrt{\dfrac{\Phi \times I \times l}{F \times \pi}} = \sqrt{\dfrac{1.27 \times 10^{-3} \times 25 \times 400 \times 10^{-3}}{80 \times \pi}} = 7.1 \times 10^{-3}\,\text{m}$ = 7.1 mm

Hence, **diameter** = 2 × r = 2 × 7.1 = **14.2 mm**

(b) By Fleming's left hand rule, the direction of the force is **towards the viewer**.

EXERCISE 179, Page 405

1. Calculate the force exerted on a charge of 2×10^{-18}C travelling at 2×10^{6} m/s perpendicular to a field of density 2×10^{-7}T.

Force, F = Q v B = $2\times10^{-18}\times2\times10^{6}\times2\times10^{-7}$ = $\mathbf{8\times10^{-19}\,N}$

2. Determine the speed of a 10^{-19} C charge travelling perpendicular to a field of flux density 10^{-7} T, if the force on the charge is 10^{-20} N.

Force, F = Q v B from which, **speed, v** = $\frac{F}{QB} = \frac{10^{-20}}{10^{-19}\times10^{-7}}$ = $\mathbf{10^{6}\,m/s}$

EXERCISE 180, Page 405

Answers found from within the text of the chapter, pages 395 to 405.

EXERCISE 181, Page 405

1. (d) **2.** (d) **3.** (a) **4.** (a) **5.** (b) **6.** (b) **7.** (d) **8.** (c) **9.** (d) **10.** (a) **11.** (c) **12.** (c)

CHAPTER 39 ELECTROMAGNETIC INDUCTION

EXERCISE 182, Page 412

1. A conductor of length 15 cm is moved at 750 mm/s at right-angles to a uniform flux density of 1.2 T. Determine the e.m.f. induced in the conductor.

Length, ℓ = 15 cm = 0.15 m and velocity, v = 750 mm/s = 0.75 m/s

Induced e.m.f., E = B ℓ l v sin θ = 1.2 × 0.15 × 0.75 × sin 90º **= 0.135 V**

2. Find the speed that a conductor of length 120 mm must be moved at right angles to a magnetic field of flux density 0.6 T to induce in it an e.m.f. of 1.8 V.

Induced e.m.f., E = B ℓ v from which, **speed, v** = $\frac{E}{B\ell} = \frac{1.8}{0.6 \times 0.12}$ = **25 m/s**

3. A 25 cm long conductor moves at a uniform speed of 8 m/s through a uniform magnetic field of flux density 1.2 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 15 ohms resistance.

Induced e.m.f., E = B ℓ v = 1.2 × 0.25 × 8 = 2.4 V

(a) If the conductor is open circuited, then **no current will flow**.

(b) **Current, I** = $\frac{E}{R} = \frac{2.4}{15}$ = **0.16 A**

4. A straight conductor 500 mm long is moved with constant velocity at right angles both to its length and to a uniform magnetic field. Given that the e.m.f. induced in the conductor is 2.5 V and the velocity is 5 m/s, calculate the flux density of the magnetic field. If the conductor forms part of a closed circuit of total resistance 5 ohms, calculate the force on the conductor.

Induced e.m.f., E = B ℓ v i.e. $2.5 = B \times 0.500 \times 5 \times \sin 90°$

from which, **flux density, B** = $\dfrac{2.5}{0.500 \times 5 \times \sin 90°}$ = **1 T**

Force on conductor, F = B I ℓ sin θ = $B \times \left(\dfrac{E}{R}\right) \times \ell \times \sin\theta = (1)\left(\dfrac{2.5}{5}\right)(0.500)(\sin 00°)$

= **0.25 N**

5. A car is travelling at 80 km/h. Assuming the back axle of the car is 1.76 m in length and the vertical component of the earth's magnetic field is 40 μT, find the e.m.f. generated in the axle due to motion.

Generated e.m.f, E = B ℓ v = $40 \times 10^{-6} \times 1.76 \times \dfrac{80 \times 10^3}{60 \times 60}$ = **1.56 mV**

6. A conductor moves with a velocity of 20 m/s at an angle of (a) 90° (b) 45° (c) 30°, to a magnetic field produced between two square-faced poles of side length 2.5 cm. If the flux on the pole face is 60 mWb, find the magnitude of the induced e.m.f. in each case.

Induced e.m.f., E = B ℓ v sin θ

(a) When θ = 90°, E = B ℓ v sin 90° = $\dfrac{\Phi}{A} \times l \times v \times \sin 90° = \dfrac{60 \times 10^{-3}}{(2.5)^2 10^{-4}} \times 2.5 \times 10^{-2} \times 20 \times \sin 90°$ = **48 V**

(b) When θ = 45°, E = B ℓ v sin 45° = 48 sin 45° = **33.9 V**

(c) When θ = 30°, E = B ℓ v sin 30° = 48 sin 30° = **24 V**

7. A conductor 400 mm long is moved at 70° to a 0.85 T magnetic field. If it has a velocity of 115 km/h, calculate (a) the induced voltage, and (b) force acting on the conductor if connected to a 8 Ω resistor.

(a) **Induced voltage, E** = Bℓ v sin θ = $(0.85)(0.4)\left(\frac{115\times 1000}{60\times 60}\right)$(sin 70°) = 10.206 V   or   **10.21 V**

(b) **Force on conductor, F** = B I ℓ sin θ = $B\times\left(\frac{E}{R}\right)\times\ell\times\sin\theta = (0.85)\left(\frac{10.206}{8}\right)$(0.4)(sin 70°)

= **0.408 N**

EXERCISE 183, Page 414

1. The mutual inductance between two coils is 150 mH. Find the magnitude of the e.m.f. induced in one coil when the current in the other is increasing at a rate of 30 A/s.

The magnitude of the e.m.f. induced, $|E_2| = M\frac{dI_1}{dt} = 150\times10^{-3}\left(\frac{30}{1}\right)$ = **4.5 V**

2. Determine the mutual inductance between two coils when a current changing at 50 A/s in one coil induces an e.m.f. of 80 mV in the other.

$|E_2| = M\frac{dI_1}{dt}$ hence, **mutual inductance, M** = $\frac{|E_2|}{\frac{dI_1}{dt}} = \frac{80\times10^{-3}}{50}$ = **1.6 mH**

3. Two coils have a mutual inductance of 0.75 H. Calculate the magnitude of the e.m.f. induced in one coil when a current of 2.5 A in the other coil is reversed in 15 ms.

Induced e.m.f., $|E_2| = M\frac{dI_1}{dt} = (0.75)\left(\frac{2.5 - -2.5}{15\times10^{-3}}\right)$ = **250 V**

4. The mutual inductance between two coils is 240 mH. If the current in one coil changes from 15 A to 6 A in 12 ms, calculate (a) the average e.m.f. induced in the other, (b) the change of flux linked with the other if it is wound with 400 turns.

(a) **Induced e.m.f.**, $E_2 = -M\frac{dI_1}{dt} = -\left(240\times10^{-3}\right)\left(\frac{15-6}{12\times10^{-3}}\right)$ = **- 180 V**

(b) $E = N\frac{d\Phi}{dt}$ from which, **change of flux**, $d\Phi = \frac{Edt}{N} = \frac{180\times12\times10^{-3}}{400}$ = **5.4 mWb**

EXERCISE 184, Page 416

1. A transformer has 600 primary turns connected to a 1.5 kV supply. Determine the number of secondary turns for a 240 V output voltage, assuming no losses.

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **secondary turns**, $N_2 = N_1\left(\frac{V_2}{V_1}\right) = (600)\left(\frac{240}{1500}\right)$

= **96 turns**

2. An ideal transformer with a turns ratio 2:9 is fed from a 220 V supply. Determine its output voltage.

$\frac{N_1}{N_2} = \frac{2}{9}$ and $V_1 = 220\,V$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **output voltage**, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (220)\left(\frac{9}{2}\right)$ = **990 V**

3. An ideal transformer has a turns ratio of 12:1 and is supplied at 192 V. Calculate the secondary voltage.

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **secondary voltage**, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (192)\left(\frac{1}{12}\right)$

= **16 V**

4. A transformer primary winding connected across a 415 V supply has 750 turns. Determine how many turns must be wound on the secondary side if an output of 1.66 kV is required.

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **secondary turns**, $N_2 = N_1\left(\frac{V_2}{V_1}\right) = (750)\left(\frac{1660}{415}\right)$ = **3000 turns**

5. An ideal transformer has a turns ratio of 15:1 and is supplied at 180 V when the primary current is 4 A. Calculate the secondary voltage and current.

$\frac{N_1}{N_2} = \frac{12}{1}$, $V_1 = 220\,V$ and $I_1 = 4\,A$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **output voltage**, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (180)\left(\frac{1}{15}\right)$ = **12 V**

$\frac{N_1}{N_2} = \frac{I_2}{I_1}$ from which, **secondary current**, $I_2 = I_1\left(\frac{N_1}{N_2}\right) = (4)\left(\frac{15}{1}\right)$ = **60 A**

6. A step-down transformer having a turns ratio of 20:1 has a primary voltage of 4 kV and a load of 10 kW. Neglecting losses, calculate the value of the secondary current.

$\frac{N_1}{N_2} = \frac{20}{1}$ and $V_1 = 4000\,V$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, output voltage, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (4000)\left(\frac{1}{20}\right)$ = 200 V

Secondary power = $V_2\, I_2$ = 10000 i.e. 200 I_2 = 10000

from which, **secondary current**, $I_2 = \frac{10000}{200}$ = **50 A**

7. A transformer has a primary to secondary turns ratio of 1:15. Calculate the primary voltage necessary to supply a 240 V load. If the load current is 3 A determine the primary current. Neglect any losses.

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **primary voltage**, $V_1 = V_2\left(\frac{N_1}{N_2}\right) = (240)\left(\frac{1}{15}\right)$ = **16 V**

$\frac{N_1}{N_2} = \frac{I_2}{I_1}$ from which, **primary current**, $I_1 = I_2\left(\frac{N_2}{N_1}\right) = (3)\left(\frac{15}{1}\right)$ = **45 A**

8. A 20 Ω resistance is connected across the secondary winding of a single-phase power transformer whose secondary voltage is 150 V. Calculate the primary voltage and the turns ratio if the supply current is 5 A, neglecting losses.

Secondary current, $I_2 = \frac{V_2}{R_2} = \frac{150}{20} = 7.5 \text{ A}$, $I_1 = 5\text{A}$ and $V_2 = 150 \text{ V}$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **primary voltage**, $V_1 = V_2\left(\frac{N_1}{N_2}\right) = V_2\left(\frac{I_2}{I_1}\right) = (150)\left(\frac{7.5}{5}\right)$ = **225 V**

Turns ratio, $\frac{N_1}{N_2} = \frac{I_2}{I_1} = \frac{7.5}{5} = 1.5$ or $\frac{3}{2}$ or **3:2**

EXERCISE 185, Page 416

Answers found from within the text of the chapter, pages 408 to 416.

EXERCISE 186, Page 417

1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (d) **6.** (a) **7.** (b) **8.** (c) **9.** (d) **10.** (a) **11.** (b) **12.** (a) **13.** (d) **14.** (b) and (c)

CHAPTER 4 CALCULATIONS AND EVALUATION OF FORMULAE

EXERCISE 18, Page 37

1. Evaluate $\dfrac{17.35\times 34.27}{41.53\div 3.76}$ correct to 3 decimal places

Using a calculator, $\dfrac{17.35\times 34.27}{41.53\div 3.76}$ = **53.832** correct to 3 decimal places

2. Evaluate $\dfrac{(4.527+3.63)}{(452.51\div 34.75)}+0.468$ correct to 5 significant figures

Using a calculator, $\dfrac{(4.527+3.63)}{(452.51\div 34.75)}+0.468$ = **1.0944** correct to 5 significant figures

3. Evaluate $52.34-\dfrac{(912.5\div 41.46)}{(24.6-13.652)}$ correct to 3 decimal places

Using a calculator, $52.34-\dfrac{(912.5\div 41.46)}{(24.6-13.652)}$ = **50.330** correct to 3 decimal places

4. Evaluate 3.5^2

Using a calculator, 3.5^2 = **12.25**

5. Evaluate $(0.036)^2$ in engineering form

Using a calculator, $(0.036)^2$ = **0.001296** = $\mathbf{1.296 \times 10^{-3}}$

6. Evaluate 1.563^2 correct to 5 significant figures

Using a calculator, $1.563^2 = \mathbf{2.4430}$ correct to 5 significant figures

7. Evaluate 3.14^3 correct to 4 significant figures

Using a calculator, $3.14^3 = \mathbf{30.96}$ correct to 4 significant figures

8. Evaluate $(0.38)^3$ correct to 4 decimal places

Using a calculator, $(0.38)^3 = \mathbf{0.0549}$ correct to 4 decimal places

9. Evaluate $\dfrac{1}{1.75}$ correct to 3 decimal places

Using a calculator, $\dfrac{1}{1.75} = \mathbf{0.571}$ correct to 3 decimal places

10. Evaluate $\dfrac{1}{0.0250}$

Using a calculator, $\dfrac{1}{0.0250} = \mathbf{40}$

11. Evaluate $\dfrac{1}{0.00725}$ correct to 1 decimal place

Using a calculator, $\dfrac{1}{0.00725} = \mathbf{137.9}$ correct to 1 decimal place

12. Evaluate $\dfrac{1}{0.065} - \dfrac{1}{2.341}$ correct to 4 significant figures

Using a calculator, $\dfrac{1}{0.065} - \dfrac{1}{2.341} = \mathbf{14.96}$ correct to 4 significant figures

13. Evaluate 2.1^4

Using a calculator, 2.1^4 = **19.4481**

14. Evaluate $(0.22)^5$ correct to 5 significant figures in engineering form

Using a calculator, $(0.22)^5 = 0.00051536 = \mathbf{515.36 \times 10^{-6}}$ correct to 5 significant figures

15. Evaluate $(1.012)^7$ correct to 4 decimal places

Using a calculator, $(1.012)^7$ = **1.0871** correct to 4 decimal places

16. Evaluate $1.1^3 + 2.9^4 - 4.4^2$ correct to 4 significant figures

Using a calculator, $1.1^3 + 2.9^4 - 4.4^2$ = **52.70** correct to 4 significant figures

17. Evaluate $\sqrt{123.7}$ correct to 5 significant figures

Using a calculator, $\sqrt{123.7}$ = **11.122** correct to 5 significant figures

18. Evaluate $\sqrt{0.69}$ correct to 4 significant figures

Using a calculator, $\sqrt{0.69}$ = **0.8307** correct to 4 significant figures

19. Evaluate $\sqrt[3]{17}$ correct to 3 decimal places

Using a calculator, $\sqrt[3]{17}$ = **2.571** correct to 3 decimal places

20. Evaluate $\sqrt[5]{3.12}$ correct to 4 decimal places

Using a calculator, $\sqrt[5]{3.12}$ = **1.256** correct to 4 decimal places

21. Evaluate $\sqrt[6]{2451} - \sqrt[4]{46}$ correct to 3 decimal places

Using a calculator, $\sqrt[6]{2451} - \sqrt[4]{46}$ = **1.068** correct to 3 decimal places

22. Evaluate $5\times10^{-3}\times7\times10^{8}$ in engineering form

Using a calculator, $5\times10^{-3}\times7\times10^{8}$ = $\mathbf{3.5\times10^{6}}$ in engineering form

23. Evaluate $\dfrac{3\times10^{-4}}{8\times10^{-9}}$ in engineering form

Using a calculator, $\dfrac{3\times10^{-4}}{8\times10^{-9}}$ = $\mathbf{37.5\times10^{3}}$ in engineering form

24. Evaluate $\dfrac{6\times10^{3}\times14\times10^{-4}}{2\times10^{6}}$ in engineering form

Using a calculator, $\dfrac{6\times10^{3}\times14\times10^{-4}}{2\times10^{6}}$ = $\mathbf{4.2\times10^{-6}}$ in engineering form

25. Evaluate $\dfrac{99\times10^{5}\times6.7\times10^{-3}}{36.2\times10^{-4}}$ correct to 4 significant figures

Using a calculator, $\dfrac{99\times10^{5}\times6.7\times10^{-3}}{36.2\times10^{-4}}$ = $\mathbf{18.32\times10^{6}}$ correct to 4 significant figures

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EXERCISE 19, Page 38

1. Evaluate $\frac{2}{3} - \frac{1}{6} + \frac{3}{7}$ as a fraction

Using a calculator, $\frac{2}{3} - \frac{1}{6} + \frac{3}{7} = \mathbf{\frac{13}{14}}$

2. Evaluate $2\frac{5}{6} + 1\frac{5}{8}$ as a decimal, correct to 4 significant figures

Using a calculator, $2\frac{5}{6} + 1\frac{5}{8} = \frac{107}{24}$ = **4.458** correct to 4 significant figures

3. Evaluate $\frac{1}{3} - \frac{3}{4} \times \frac{8}{21}$ as a fraction

Using a calculator, $\frac{1}{3} - \frac{3}{4} \times \frac{8}{21} = \mathbf{\frac{1}{21}}$

4. Evaluate $8\frac{8}{9} \div 2\frac{2}{3}$ as a mixed number

Using a calculator, $8\frac{8}{9} \div 2\frac{2}{3} = \frac{10}{3} = \mathbf{3\frac{1}{3}}$

5. Evaluate $\frac{\left(4\frac{1}{5} - 1\frac{2}{3}\right)}{\left(3\frac{1}{4} \times 2\frac{3}{5}\right)} - \frac{2}{9}$ as a decimal, correct to 3 significant figures

Using a calculator, $\frac{\left(4\frac{1}{5} - 1\frac{2}{3}\right)}{\left(3\frac{1}{4} \times 2\frac{3}{5}\right)} - \frac{2}{9} = \frac{118}{15210}$ = **0.0776** correct to 3 significant figures

6. Evaluate sin 15.78° correct to 4 decimal places

Using a calculator, sin 15.78° = **0.2719** correct to 4 decimal places

7. Evaluate cos 63.74° correct to 4 decimal places

Using a calculator, cos 63.74° = **0.4424** correct to 4 decimal places

8. Evaluate tan 39.55° - sin 52.53° correct to 4 decimal places

Using a calculator, tan 39.55° - sin 52.53° = 0.82580 – 0.79367 = **0.0321** correct to 4 decimal places

9. Evaluate sin(0.437 rad) correct to 4 decimal places

Using a calculator, sin(0.437 rad) = **0.4232** correct to 4 decimal places

12. Evaluate cos(1.42 rad) correct to 4 decimal places

Using a calculator, cos(1.42 rad) = **0.1502** correct to 4 decimal places

11. Evaluate tan(5.673 rad) correct to 4 decimal places

Using a calculator, tan(5.673 rad) = **- 0.6992** correct to 4 decimal places

12. Evaluate $\frac{(\sin 42.6°)(\tan 83.2°)}{\cos 13.8°}$ correct to 4 decimal places

Using a calculator, $\frac{(\sin 42.6°)(\tan 83.2°)}{\cos 13.8°}$ = **5.8452** correct to 4 decimal places

13. Evaluate 1.59π correct to 4 significant figures

Using a calculator, 1.59π = **4.995** correct to 4 significant figures

14. Evaluate $\pi^2\left(\sqrt{13}-1\right)$ correct to 4 significant figures

Using a calculator, $\pi^2\left(\sqrt{13}-1\right)$ = **25.72** correct to 4 significant figures

15. Evaluate $3e^{(2\pi-1)}$ correct to 4 significant figures

Using a calculator, $3e^{(2\pi-1)}$ = 590.989 = **591.0** correct to 4 significant figures

16. Evaluate $2\pi e^{\frac{\pi}{3}}$ correct to 4 significant figures

Using a calculator, $2\pi e^{\frac{\pi}{3}}$ = **17.90** correct to 4 significant figures

17. Evaluate $\sqrt{\left[\dfrac{5.52\pi}{2e^{-2}\times\sqrt{26.73}}\right]}$ correct to 4 significant figures

Using a calculator, $\sqrt{\left[\dfrac{5.52\pi}{2e^{-2}\times\sqrt{26.73}}\right]}$ = **3.520** correct to 4 significant figures

18. Evaluate $\sqrt{\left[\dfrac{e^{\left(2-\sqrt{3}\right)}}{\pi\times\sqrt{8.57}}\right]}$ correct to 4 significant figures

Using a calculator, $\sqrt{\left[\dfrac{e^{\left(2-\sqrt{3}\right)}}{\pi\times\sqrt{8.57}}\right]}$ = **0.3770** correct to 4 significant figures

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EXERCISE 20, Page 40

1. The circumference C of a circle is given by the formula C = 2πr. Determine the circumference given r = 8.40 mm

Using a calculator, **circumference, C** = 2π × 8.40 = **52.78 mm**

2. A formula used in connection with gases is $R = \frac{PV}{T}$. Evaluate R when P = 1500, V = 5 and T = 200

Using a calculator, $\mathbf{R} = \frac{PV}{T} = \frac{1500 \times 5}{200} = \mathbf{37.5}$

3. The velocity of a body is given by v = u + at. The initial velocity u is measured when time t is 15 seconds and found to be 12 m/s. If the acceleration a is 9.81 m/s^2 calculate the final velocity v.

Using a calculator, **final velocity, v** = u + at = 12 + (9.81)(15) = 12 + 9.81 × 15 = **159 m/s**

4. Find the distance s, given that $s = \frac{1}{2}gt^2$. Time t = 0.032 seconds and acceleration due to gravity g = 9.81 m/s^2. Give the answer in millimetres.

Using a calculator, **distance, s** $= \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81 \times (0.032)^2 = 5.02 \times 10^{-3}$ m

$= 5.02 \times 10^{-3} \times 10^3$ mm = **5.02 mm**

5. The energy stored in a capacitor is given by $E = \frac{1}{2}CV^2$ joules. Determine the energy when capacitance $C = 5 \times 10^{-6}$ farads and voltage V = 240 V.

Using a calculator, **energy, E** $= \frac{1}{2}CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times 240^2 =$ **0.144 J**

6. Resistance R_2 is given by $R_2 = R_1(1 + \alpha t)$. Find R_2, correct to 4 significant figures, when $R_1 = 220$, $\alpha = 0.00027$ and $t = 75.6$

Using a calculator, **resistance, R_2** $= R_1(1 + \alpha t) = 220(1 + 0.00027 \times 75.6)$

$= 220(1 + 0.020412) = 220(1.020412) = \mathbf{224.5}$

7. Density $= \dfrac{\text{mass}}{\text{volume}}$. Find the density when the mass is 2.462 kg and the volume is 173 cm^3. Give the answer in units of kg/m^3. (Note that $1\ cm^3 = 10^{-6}\ m^3$)

Using a calculator, **density** $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{2.462\text{ kg}}{173 \times 10^{-6}\text{ m}^3} = \mathbf{14230\ kg/m^3}$

8. Velocity = frequency × wavelength. Find the velocity when the frequency is 1825 Hz and the wavelength is 0.154 m

Velocity = frequency × wavelength = 1825 × 0.154 = **281.1 m/s**

9. Evaluate resistance R_T, given $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ when $R_1 = 5.5\ \Omega$, $R_2 = 7.42\ \Omega$ and $R_3 = 12.6\ \Omega$

Using a calculator, $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{5.5} + \dfrac{1}{7.42} + \dfrac{1}{12.6} = 0.395954\ldots$

from which, **resistance, R_T** $= \dfrac{1}{0.395954} = \mathbf{2.526\ \Omega}$

10. Power $= \dfrac{\text{force} \times \text{distance}}{\text{time}}$. Find the power when a force of 3760 N raises an object a distance of 4.73 m in 35 s

Using a calculator, **power** $= \dfrac{\text{force} \times \text{distance}}{\text{time}} = \dfrac{3760 \times 4.73}{35}$ N m/s = **508.1 W** since 1 W = 1 N m/s

11. The potential difference, V volts, available at battery terminals is given by V = E - Ir. Evaluate V when E = 5.62, I = 0.70 and R = 4.30

Using a calculator, **potential difference, V** = E – Ir = 5.62 – (0.70)(4.30) = 5.62 – 3.01 = **2.61 V**

12. Given force F = $\frac{1}{2}m(v^2 - u^2)$, find F when m = 18.3, v = 12.7 and u = 8.24

Using a calculator, **force F** = $\frac{1}{2}m(v^2 - u^2) = \frac{1}{2}\times 18.3\times\left(12.7^2 - 8.24^2\right)$ = **854.5**

13. Energy, E joules, is given by the formula E = $\frac{1}{2}LI^2$. Evaluate the energy when L = 5.5 and I = 1.2

Using a calculator, **energy, E** = $\frac{1}{2}LI^2 = \frac{1}{2}\times 5.5\times 1.2^2$ = **3.96 J**

14. The current I amperes in an a.c. circuit is given by I = $\frac{V}{\sqrt{(R^2 + X^2)}}$. Evaluate the current when V = 250, R = 11.0 and X = 16.2

Using a calculator, **current, I** = $\frac{V}{\sqrt{(R^2 + X^2)}} = \frac{250}{\sqrt{\left(11.0^2 + 16.2^2\right)}}$ = **12.77 A**

15. Distance s metres is given by the formula s = ut + $\frac{1}{2}at^2$. If u = 9.50, t = 4.60 and a = - 2.50, evaluate the distance.

Using a calculator, **distance, s** = ut + $\frac{1}{2}at^2 = 9.50\times 4.60 + \frac{1}{2}\times(-2.50)\times 4.60^2$

= 43.7 – 26.45 = **17.25**

16. The area, A, of any triangle is given by A = $\sqrt{[s(s-a)(s-b)(s-c)]}$ where $s=\dfrac{a+b+c}{2}$.

Evaluate the area, given a = 3.60 cm, b = 4.00 cm and c = 5.20 cm

Using a calculator, $s=\dfrac{a+b+c}{2}=\dfrac{3.60+4.00+5.20}{2}=6.40$

Hence, **area, A** = $\sqrt{[s(s-a)(s-b)(s-c)]}=\sqrt{[6.40(6.40-3.60)(6.40-4.00)(6.40-5.20)]}$

$= \mathbf{7.184\ cm^2}$

CHAPTER 40 ALTERNATING VOLTAGES AND CURRENTS

EXERCISE 187, Page 423

1. Determine the periodic time for the following frequencies: (a) 2.5 Hz (b) 100 Hz (c) 40 kHz

(a) Periodic time, **T** = $\frac{1}{f} = \frac{1}{2.5}$ = **0.4 s**

(b) Periodic time, **T** = $\frac{1}{f} = \frac{1}{100}$ = **0.01 s** or **10 ms**

(c) Periodic time, **T** = $\frac{1}{f} = \frac{1}{40 \times 10^3}$ = **25 μs**

2. Calculate the frequency for the following periodic times: (a) 5 ms (b) 50 μs (c) 0.2 s

(a) Frequency, **f** = $\frac{1}{T} = \frac{1}{5 \times 10^{-3}}$ = **200 Hz** or **0.2 kHz**

(b) Frequency, **f** = $\frac{1}{T} = \frac{1}{50 \times 10^{-6}}$ = **20 kHz**

(c) Frequency, **f** = $\frac{1}{T} = \frac{1}{0.2}$ = **5 Hz**

3. An alternating current completes 4 cycles in 5 ms. What is its frequency?

Time for one cycle, T = $\frac{5}{4}$ ms = 1.25 ms

Hence, frequency, **f** = $\frac{1}{T} = \frac{1}{1.25 \times 10^{-3}}$ = **800 Hz**

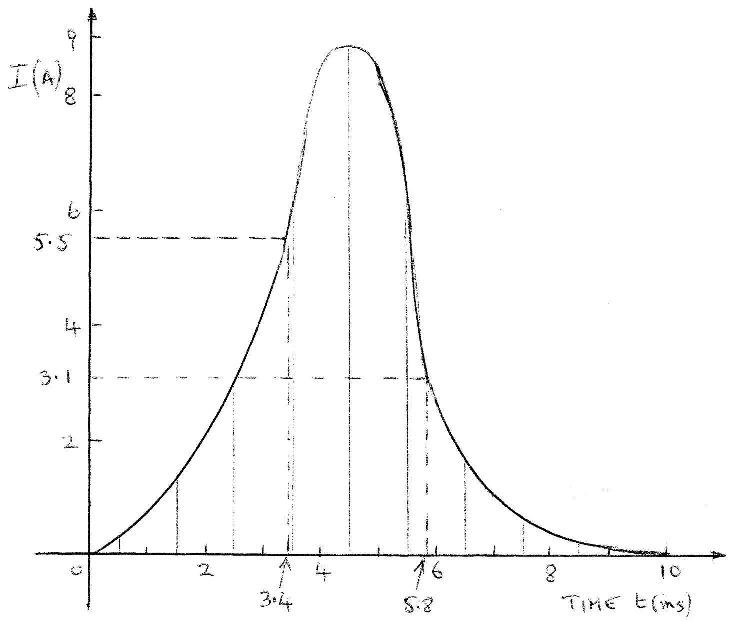
EXERCISE 188, Page 426

1. An alternating current varies with time over half a cycle as follows:

Current (A)	0	0.7	2.0	4.2	8.4	8.2	2.5	1.0	0.4	0.2	0
time (ms)	0	1	2	3	4	5	6	7	8	9	10

The negative half cycle is similar. Plot the curve and determine: (a) the frequency (b) the instantaneous values at 3.4 ms and 5.8 ms (c) its mean value, and (d) its r.m.s. value.

The graph is shown plotted below.



(a) Periodic time, T = 2 × 10 ms = 20 ms, hence, **frequency, f** = $\frac{1}{T} = \frac{1}{20 \times 10^{-3}}$ = **50 Hz**

(b) At 3.4 ms, current, i = **5.5 A**

and at 5.8 ms, i = **3.1 A**

(c) Mean value = $\frac{\text{area under curve}}{\text{length of base}}$ Using the mid-ordinate rule,

$$\text{area under curve} = (1\times10^{-3})(0.3+1.4+3.1+6.0+8.8+5.5+1.6+0.8+0.3+0.2)$$

$$= (1\times10^{-3})(28) = 28\times10^{-3}$$

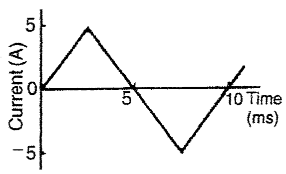
Hence, **mean value** = $\dfrac{28\times10^{-3}}{10\times10^{-3}}$ = **2.8 A**

(d) r.m.s. value = $\sqrt{\left(\dfrac{0.3^2+1.4^2+3.1^2+6.0^2+8.8^2+5.5^2+1.6^2+0.8^2+0.3^2+0.2^2}{10}\right)}$

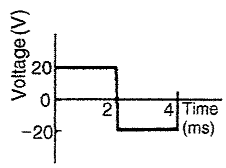
$= \sqrt{\dfrac{158.68}{10}}$ = 3.98 A or **4.0 A**, correct to 2 significant figures.

2. For the waveforms shown below, determine for each (i) the frequency (ii) the average value over half a cycle (iii) the r.m.s. value (iv) the form factor (v) the peak factor.

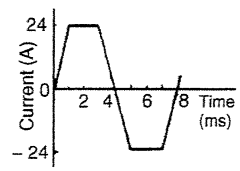
(a)

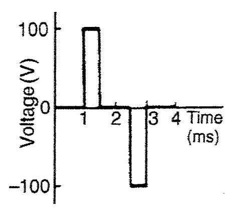


(b)



(c)

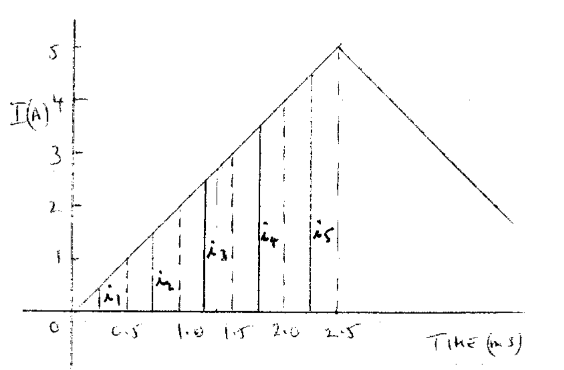




(d)

(a) (i) T = 10 ms, hence, **frequency, f** = $\dfrac{1}{T}=\dfrac{1}{10\times10^{-3}}$ = **100 Hz**

(ii) Average value = $\dfrac{\text{area under curve}}{\text{length of base}}=\dfrac{\frac{1}{2}\left(5\times10^{-3}\right)(5)}{5\times10^{-3}}$ = **2.50 A**



(iii) R.m.s. value = $\sqrt{\left(\frac{i_1^2+i_2^2+i_3^2+i_4^2+i_5^2}{5}\right)} = \sqrt{\left(\frac{0.5^2+1.5^2+2.5^2+3.5^2+4.5^2}{5}\right)}$ = **2.87 A**

(iv) Form factor = $\frac{\text{r.m.s.}}{\text{average}} = \frac{2.87}{2.50}$ = **1.15**

(v) Peak factor = $\frac{\text{maximum value}}{\text{r.m.s.}} = \frac{5}{2.87}$ = **1.74**

(b) (i) T = 4 ms, hence, **frequency, f** = $\frac{1}{T} = \frac{1}{4\times10^{-3}}$ = **250 Hz**

(ii) Average value = $\frac{\text{area under curve}}{\text{length of base}} = \frac{20\times2}{2}$ = **20 V**

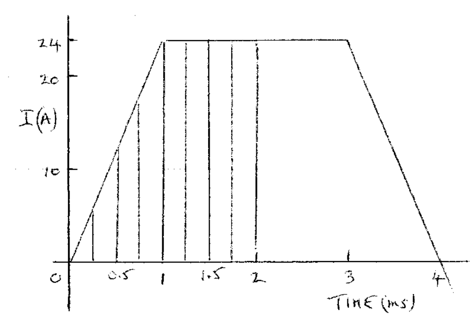
(iii) R.m.s. value = $\sqrt{\left(\frac{v_1^2+v_2^2+v_3^2+v_4^2}{4}\right)} = \sqrt{\left(\frac{20^2+20^2+20^2+20^2}{4}\right)}$ = **20 V**

(iv) Form factor = $\frac{\text{r.m.s.}}{\text{average}} = \frac{20}{20}$ = **1.0**

(v) Peak factor = $\frac{\text{maximum value}}{\text{r.m.s.}} = \frac{20}{20}$ = **1.0**

(c) (i) T = 8 ms, hence, **frequency, f** = $\frac{1}{T} = \frac{1}{8\times10^{-3}}$ = **125 Hz**

(ii) Average value = $\frac{\text{area under curve}}{\text{length of base}} = \frac{\left(\frac{1}{2}\times1\times24\right)+(2\times24)+\left(\frac{1}{2}\times1\times24\right)}{4} = \frac{72}{4}$ = **18 A**



(iii) R.m.s. value = $\sqrt{\left(\frac{i_1^2+i_2^2+i_3^2+i_4^2+....}{8}\right)}$

$$= \sqrt{\left(\frac{3^2+9^2+15^2+21^2+24^2+24^2+24^2+24^2}{8}\right)} = \mathbf{19.56\ A}$$

(iv) Form factor = $\frac{\text{r.m.s.}}{\text{average}} = \frac{19.56}{18}$ = **1.09**

(v) Peak factor = $\frac{\text{maximum value}}{\text{r.m.s.}} = \frac{24}{19.56}$ = **1.23**

(d) (i) T = 4 ms, hence, **frequency, f** = $\frac{1}{T} = \frac{1}{4\times10^{-3}}$ = **250 Hz**

(ii) Average value = $\frac{\text{area under curve}}{\text{length of base}} = \frac{0.5\times100}{2}$ = **25 V**

(iii) R.m.s. value = $\sqrt{\left(\frac{v_1^2+v_2^2+v_3^2+v_4^2}{4}\right)} = \sqrt{\left(\frac{0^2+0^2+100^2+0^2}{4}\right)}$ = **50 V**

(iv) Form factor = $\frac{\text{r.m.s.}}{\text{average}} = \frac{50}{25}$ = **2.0**

(v) Peak factor = $\frac{\text{maximum value}}{\text{r.m.s.}} = \frac{100}{50}$ = **2.0**

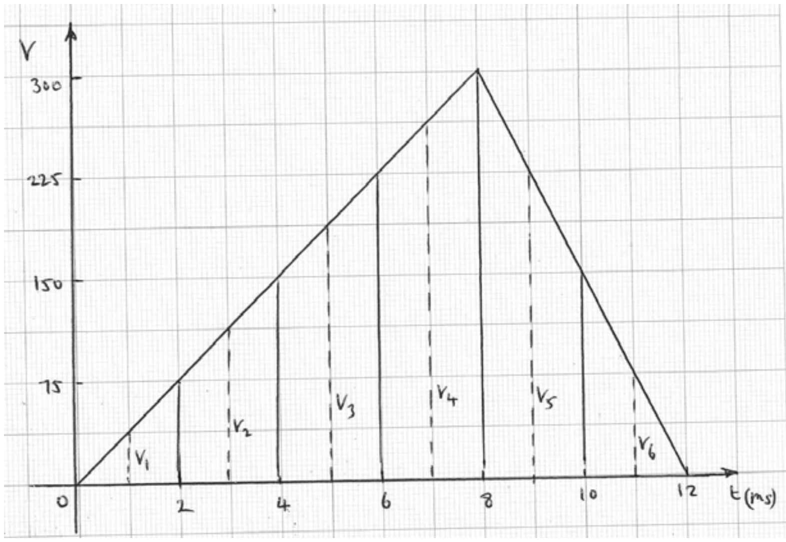
3. An alternating voltage is triangular in shape, rising at a constant rate to a maximum of 300 V in 8 ms and then falling to zero at a constant rate in 4 ms. The negative half cycle is identical in shape to the positive half cycle. Calculate (a) the mean voltage over half a cycle, and (b) the r.m.s. voltage

The waveform is shown below.

(a) Mean value = $\frac{\text{area under curve}}{\text{length of base}} = \frac{\frac{1}{2}\times12\times10^{-3}\text{s}\times300\text{ V}}{12\times10^{-3}\text{s}}$ = **150 V**

(b) R.m.s. value = $\sqrt{\left(\frac{v_1^2+v_2^2+v_3^2+v_4^2+....}{6}\right)}$

$$= \sqrt{\left(\frac{37.5^2+112.5^2+187.5^2+262.5^2+225^2+75^2}{6}\right)} = \mathbf{170\ V}$$



4. Calculate the r.m.s. value of a sinusoidal curve of maximum value 300 V.

R.m.s. value = 0.707 × peak value = 0.707 × 300 = **212.1 V**

5. Find the peak and mean values for a 200 V mains supply.

200 V is the r.m.s. value

r.m.s. value = 0.707 × peak value, from which, **peak value** = $\frac{\text{r.m.s.}}{0.707} = \frac{200}{0.707}$ = **282.9 V**

Mean value = 0.637 × peak value = 0.637 × 282.9 = **180.2 V**

6. A sinusoidal voltage has a maximum value of 120 V. Calculate its r.m.s. and average values.

R.m.s. value = 0.707 × peak value = 0.707 × 120 = **84.8 V**

Average value = 0.637 × peak value = 0.637 × 120 = **76.4 V**

7. A sinusoidal current has a mean value of 15.0 A. Determine its maximum and r.m.s. values.

Mean value = 0.637 × maximum value,

from which, **maximum value** = $\dfrac{\text{mean value}}{0.637} = \dfrac{15.0}{0.637}$ = **23.55 A**

R.m.s. value = 0.707 × maximum value = 0.707 × 23.55 = **16.65 A**

EXERCISE 189, Page 426

Answers found from within the text of the chapter, pages 420 to 426.

EXERCISE 190, Page 427

1. (b) **2.** (c) **3.** (d) **4.** (d) **5.** (a) **6.** (d) **7.** (c) **8.** (b) **9.** (c) **10.** (b)

CHAPTER 41 CAPACITORS AND INDUCTORS

EXERCISE 191, Page 432

1. Find the charge on a 10 μF capacitor when the applied voltage is 250V.

Charge, Q = C × V = $10 \times 10^{-6} \times 250 = 2.5 \times 10^{-3}$ C = **2.5 mC**

2. Determine the voltage across a 1000 pF capacitor to charge it with 2 μC.

Q = CV hence, **voltage, V** = $\frac{Q}{C} = \frac{2 \times 10^{-6}}{1000 \times 10^{-12}}$ = **2000 V** or **2 kV**

3. The charge on the plates of a capacitor is 6 mC when the potential between them is 2.4 kV. Determine the capacitance of the capacitor.

Q = CV hence, **capacitance, C** = $\frac{Q}{V} = \frac{6 \times 10^{-3}}{2.4 \times 10^{3}} = 2.5 \times 10^{-6}$ = **2.5 μF**

4. For how long must a charging current of 2 A be fed to a 5 μF capacitor to raise the p.d. between its plates by 500 V.

Charge Q = I × t and Q = C × V hence I × t = C × V

from which, **time, t** = $\frac{C \times V}{I} = \frac{5 \times 10^{-6} \times 500}{2}$ = **1.25 ms**

5. A direct current of 10 A flows into a previously uncharged 5 μF capacitor for 1 ms. Determine the p.d. between the plates.

P.d. between plates, V = $\frac{Q}{C} = \frac{I \times t}{C} = \frac{10 \times 1 \times 10^{-3}}{5 \times 10^{-6}}$ = **2000 V** or **2 kV**

6. A capacitor uses a dielectric 0.04 mm thick and operates at 30 V. What is the electric field strength across the dielectric at this voltage?

Electric field strength, E = $\frac{V}{d} = \frac{30}{0.04 \times 10^{-3}} = \mathbf{750\ kV/m}$

7. A charge of 1.5 μC is carried on two parallel rectangular plates each measuring 60 mm by 80 mm. Calculate the electric flux density. If the plates are spaced 10 mm apart and the voltage between them is 0.5 kV determine the electric field strength.

Electric flux density, D = $\frac{Q}{A} = \frac{1.5 \times 10^{-6}}{60 \times 80 \times 10^{-6}} = \mathbf{312.5\ \mu C/m^2}$

Electric field strength, E = $\frac{V}{d} = \frac{0.5 \times 10^{3}}{10 \times 10^{-3}} = \mathbf{50\ kV/m}$

EXERCISE 192, Page 434

1. A capacitor consists of two parallel plates each of area 0.01 m^2, spaced 0.1 mm in air. Calculate the capacitance in picofarads.

Capacitance, $C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.85 \times 10^{-12} \times 1 \times 0.01}{0.1 \times 10^{-3}}$ since, for air, $\varepsilon_r = 1$

$= 885 \times 10^{-12} =$ **885 pF**

2. A waxed paper capacitor has two parallel plates, each of effective area 0.2 m^2. If the capaci--tance is 4000 pF determine the effective thickness of the paper if its relative permittivity is 2.

$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ hence, **thickness of the paper, d** = $\frac{\varepsilon_0 \varepsilon_r A}{C} = \frac{8.85 \times 10^{-12} \times 2 \times 0.2}{4000 \times 10^{-12}} = 885 \times 10^{-6}$ m

= **0.885 mm**

3. How many plates has a parallel plate capacitor having a capacitance of 5 nF, if each plate is 40 mm by 40 mm and each dielectric is 0.102 mm thick with a relative permittivity of 6.

$C = \frac{\varepsilon_0 \varepsilon_r A}{d}(n-1)$ from which, $n - 1 = \frac{C\,d}{\varepsilon_0\, \varepsilon_r\, A} = \frac{5 \times 10^{-9} \times 0.102 \times 10^{-3}}{8.85 \times 10^{-12} \times 6 \times 40 \times 40 \times 10^{-6}} = 6$

Hence, **the number of plates, n** = 6 + 1 = **7**

4. A parallel plate capacitor is made from 25 plates, each 70 mm by 120 mm, interleaved with mica of relative permittivity 5. If the capacitance of the capacitor is 3000 pF determine the thickness of the mica.

$C = \frac{\varepsilon_0 \varepsilon_r A}{d}(n-1)$ from which, **dielectric thickness,**

d = $\frac{\varepsilon_0\, \varepsilon_r\, A}{C}(n-1) = \frac{8.85 \times 10^{-12} \times 5 \times 70 \times 120 \times 10^{-6}}{3000 \times 10^{-12}}(25-1)$ = 0.00297 m = **2.97 mm**

5. A capacitor is constructed with parallel plates and has a value of 50 pF. What would be the capacitance of the capacitor if the plate area is doubled and the plate spacing is halved?

If the plate area is doubled, so is the capacitance (i.e. direct proportion).

If the plate spacing is halved, then the capacitance is doubled (i.e. inverse proportion).

Hence, **capacitance of capacitor** = 4 × 50 = **200 pF**

EXERCISE 193, Page 435

1. Capacitors of 2 μF and 6 μF are connected (a) in parallel and (b) in series. Determine the equivalent capacitance in each case.

(a) In parallel, equivalent capacitance, $\mathbf{C_T}$ = 2 + 6 = **8 μF**

(b) In series, equivalent capacitance, $\mathbf{C_T} = \frac{2\times6}{2+6} = \frac{12}{8}$ = **1.5 μF**

2. Find the capacitance to be connected in series with a 10 μF capacitor for the equivalent capacitance to be 6 μF

For series connection, $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_T}$

i.e. $\frac{1}{10} + \frac{1}{C_2} = \frac{1}{6}$ from which, $\frac{1}{C_2} = \frac{1}{6} - \frac{1}{10} = 0.06666666$

and $\mathbf{C_2} = \frac{1}{0.06666666}$ = **15 μF**

3. What value of capacitance would be obtained if capacitors of 0.15 μF and 0.10 μF are connected (a) in series and (b) in parallel.

(a) In series, equivalent capacitance, $\mathbf{C_T} = \frac{0.15\times0.10}{0.15+0.10} = \frac{0.015}{0.25}$ = **0.06 μF**

(b) In parallel, equivalent capacitance, $\mathbf{C_T}$ = 0.15 + 0.10 = **0.25 μF**

4. Two 6 μF capacitors are connected in series with one having a capacitance of 12 μF. Find the total equivalent circuit capacitance. What capacitance must be added in series to obtain a capacitance of 1.2 μF?

Two 6 μF capacitors in series has a total capacitance of $\frac{6\times 6}{6+6}$ = 3 μF. (Two equal value capacitors in series will have a total capacitance of half the value of one of the capacitors).

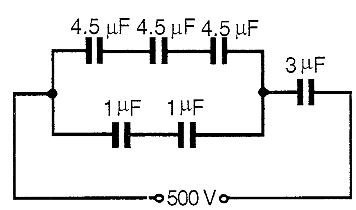
3 μF in series with 12 μF has a total capacitance of $\frac{3\times 12}{3+12}$ = **2.4 μF = total circuit capacitance.**

Let new capacitance be C_X then if new total capacitance is to be 1.2 μF then

$$\frac{1}{1.2}=\frac{1}{2.4}+\frac{1}{C_X} \quad \text{from which} \quad \frac{1}{C_X}=\frac{1}{1.2}-\frac{1}{2.4}=0.41666$$

Hence, **capacitance to be added**, $C_X = \frac{1}{0.41666}$ = **2.4 μF**

5. For the arrangement shown below find (a) the equivalent circuit capacitance and (b) the voltage across a 4.5 μF capacitor.

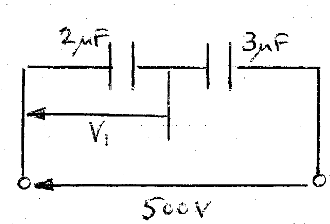


(a) Three 4.5 μF capacitors in series gives 1.5 μF and two 1 μF capacitors in series gives 0.5 μF

1.5 μF and 0.5 μF capacitors in parallel gives 1.5 + 0.5 = 2 μF

2 μF in series with 3 μF gives: $\frac{2\times 3}{2+3}=\frac{6}{5}$ = **1.2 μF = equivalent circuit capacitance**

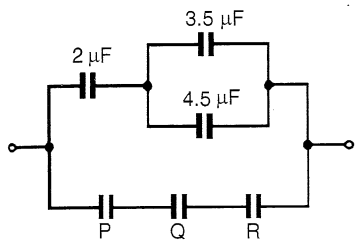
(b) The equivalent circuit is shown below where $V_1=\left(\frac{3}{2+3}\right)(500)$ = 300 V = voltage across three 4.5 μF capacitors in series. Hence, voltage across each 4.5 μF capacitor = 300/3 = **100 V**.



(Alternatively, to find V_1:

Since $C_T = 1.2\ \mu F$ then $Q_T = C_T \times V = 1.2 \times 10^{-6} \times 500 = 600\,\mu C$. This is the charge on each capacitor of the circuit shown below. Hence, $V_1 = \frac{Q_T}{C_1} = \frac{600 \times 10^{-6}}{2 \times 10^{-6}} = 300$ V)

6. In the circuit below, capacitors P, Q and R are identical and the total equivalent capacitance of the circuit is 3 μF. Determine the values of P, Q and R.



3.5 μF and 4.5 μF in parallel gives an equivalent capacitance of 3.5 + 4.5 = 8 μF

2 μF in series with 8 μF gives $\frac{2 \times 8}{2 + 8} = \frac{16}{10} = 1.6\ \mu F$

Let the equivalent capacitance of P, Q and R in series be C_X

Then $1.6 + C_X = 3$ from which, $C_X = 3 - 1.6 = 1.4\ \mu F$

Thus, $\frac{1}{1.4} = \frac{1}{C_P} + \frac{1}{C_Q} + \frac{1}{C_R} = \frac{3}{C_P}$ (since $C_P = C_Q = C_R$)

i.e. $\mathbf{C_P} = 3 \times 1.4 = \mathbf{4.2\ \mu F} = \mathbf{C_Q = C_R}$

EXERCISE 194, Page 436

1. When a capacitor is connected across a 200 V supply the charge is 4 μC. Find (a) the capacitance and (b) the energy stored.

(a) Q = CV from which, **capacitance, C** = $\frac{Q}{V} = \frac{4\times10^{-6}}{200}$ = **20 nF** or **0.02 μF**

(b) **Energy stored, W** = $\frac{1}{2}CV^2 = \frac{1}{2}\times0.02\times10^{-6}\times200^2$ = **400 μJ** or **0.4 mJ**

2. Find the energy stored in a 10 μF capacitor when charged to 2 kV

Energy stored, W = $\frac{1}{2}CV^2 = \frac{1}{2}\times10\times10^{-6}\times2000^2$ = **20 J**

3. A 3300 pF capacitor is required to store 0.5 mJ of energy. Find the p.d. to which the capacitor must be charged.

Energy, W = $\frac{1}{2}CV^2$ from which, **p.d., V** = $\sqrt{\frac{2W}{C}} = \sqrt{\left(\frac{2\times0.5\times10^{-3}}{3300\times10^{-12}}\right)}$ = **550 V**

4. A bakelite capacitor is to be constructed to have a capacitance of 0.04 μF and to have a steady working potential of 1 kV maximum. Allowing a safe value of field stress of 25 MV/m find (a) the thickness of bakelite required, (b) the area of plate required if the relative permittivity of bakelite is 5, (c) the maximum energy stored by the capacitor and (d) the average power developed if this energy is dissipated in a time of 20 μs.

(a) Field stress, E = $\frac{V}{d}$ from which,

thickness of dielectric, d = $\frac{V}{E} = \frac{1000}{25\times10^6} = 40\times10^{-6}\text{m} = 40\times10^{-3}\text{mm}$ = **0.04 mm**

(b) Capacitance, $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ from which,

cross-sectional area, A = $\frac{C\,d}{\varepsilon_0\,\varepsilon_r} = \frac{0.04\times10^{-6}\times0.04\times10^{-3}}{8.85\times10^{-12}\times5} = 0.03616\,m^2$ = $\mathbf{361.6\ cm^2}$

(c) **Maximum energy**, $W_{max} = \frac{1}{2}CV^2 = \frac{1}{2}\times0.04\times10^{-6}\times1000^2$ = **0.02 J**

(d) Energy = power × time, hence, **power, P** = $\frac{\text{energy}}{\text{time}} = \frac{0.02\,J}{20\times10^{-6}\,s}$ = **1000 W** or **1 kW**

EXERCISE 195, Page 40

1. Find the e.m.f. induced in a coil of 200 turns when there is a change of flux of 30 mWb linking with it in 40 ms.

Induced e.m.f., E $= -N\frac{d\Phi}{dt} = -200\left(\frac{30\times10^{-3}}{40\times10^{-3}}\right)$ = **- 150 V**

2. An e.m.f. of 25 V is induced in a coil of 300 turns when the flux linking with it changes by 12 mWb. Find the time, in milliseconds, in which the flux makes the change.

$|E| = N\frac{d\Phi}{dt}$ from which, **time for change, dt** $= \frac{N\,d\Phi}{E} = \frac{300\times12\times10^{-3}}{25}$ = **0.144 s** or **144 ms**

3. An ignition coil having 10000 turns has an e.m.f. of 8 kV induced in it. What rate of change of flux is required for this to happen?

$|E| = N\frac{d\Phi}{dt}$ from which, **rate of change of flux,** $\frac{\mathbf{d\Phi}}{\mathbf{dt}} = \frac{|E|}{N} = \frac{8\times10^{3}}{10000}$ = **0.8 Wb/s**

4. A flux of 35 mWb passing through a 125-turn coil is reversed in 25 ms. Find the magnitude of the average e.m.f. induced.

Magnitude of induced e.m.f., E $= N\frac{d\Phi}{dt} = 125\left(\frac{0.35\times10^{-3} - -0.35\times10^{-3}}{25\times10^{-3}}\right)$ = **3.5 V**

(Note that since the flux is **reversed**, it changes from 35 mWb to - 35 mWb, which is a change of 35 - - 35, i.e. 70 mWb).

5. Calculate the e.m.f. induced in a coil of inductance 6 H by a current changing at a rate of 15 A/s.

E.m.f. induced, E = $-L\frac{dI}{dt} = -6 \times \left(\frac{15}{1}\right)$ **= - 90 V**

EXERCISE 196, Page 441

1. An inductor of 20 H has a current of 2.5 A flowing in it. Find the energy stored in the magnetic field of the inductor.

Energy stored, W = $\frac{1}{2}LI^2 = \frac{1}{2} \times 20 \times (2.5)^2$ = **62.5 J**

2. Calculate the value of the energy stored when a current of 30 mA is flowing in a coil of inductance 400 mH.

Energy stored, W = $\frac{1}{2}LI^2 = \frac{1}{2} \times 400 \times 10^{-3} \times (30 \times 10^{-3})^2$ = **0.18 mJ**

3. The energy stored in the magnetic field of an inductor is 80 J when the current flowing in the inductor is 2 A. Calculate the inductance of the coil.

Energy, W = $\frac{1}{2}LI^2$ from which, **inductance, L** = $\frac{2W}{I^2} = \frac{2 \times 80}{(2)^2}$ = **40 H**

EXERCISE 197, Page 442

1. A flux of 30 mWb links with a 1200 turn coil when a current of 5 A is passing through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average e.m.f. induced if the current is reduced to zero in 0.20 s.

(a) **Inductance of coil, L** = $\frac{N\Phi}{I} = \frac{1200 \times 30 \times 10^{-3}}{5}$ = **7.2 H**

(b) **Energy stored, W** = $\frac{1}{2}LI^2 = \frac{1}{2} \times 7.2 \times (5)^2$ = **90 J**

(c) **Induced e.m.f., E** = $L\frac{dI}{dt} = 7.2\left(\frac{5-0}{0.20}\right)$ = **180 V**

2. An e.m.f. of 2 kV is induced in a coil when a current of 5 A collapses uniformly to zero in 10 ms. Determine the inductance of the coil.

Induced e.m.f., E = $L\frac{dI}{dt}$ from which, **inductance, L** = $\frac{E}{\frac{dI}{dt}} = \frac{2000}{\frac{5-0}{10 \times 10^{-3}}} = \frac{2000 \times 10 \times 10^{-3}}{5}$ = **4 H**

3. An average e.m.f. of 60 V is induced in a coil of inductance 160 mH when a current of 7.5 A is reversed. Calculate the time taken for the current to reverse.

Induced e.m.f., E = $L\frac{dI}{dt}$ hence, $60 = 160 \times 10^{-3} \times \frac{(7.5 - -7.5)}{t}$

from which, **time, t** = $160 \times 10^{-3} \times \frac{15}{60}$ = **0.04 s** or **40 ms**

4. A coil of 2500 turns has a flux of 10 mWb linking with it when carrying a current of 2 A. Calculate the coil inductance and the e.m.f. induced in the coil when the current collapses to zero in 20 ms.

Inductance, L = $\frac{N\Phi}{I} = \frac{2500 \times 10 \times 10^{-3}}{2}$ = **12.5 H**

Induced e.m.f., E = $L\frac{dI}{dt} = 12.5\left(\frac{2-0}{20 \times 10^{-3}}\right)$ = **1.25 kV**

5. When a current of 2 A flows in a coil, the flux linking with the coil is 80 μWb. If the coil inductance is 0.5 H, calculate the number of turns of the coil.

If $L = \frac{N\Phi}{I}$ then **number of turns, N** = $\frac{LI}{\Phi} = \frac{0.5 \times 2}{80 \times 10^{-6}}$ = **12,500**

EXERCISE 198, Page 442

Answers found from within the text of the chapter, pages 429 to 442.

EXERCISE 199, Page 443

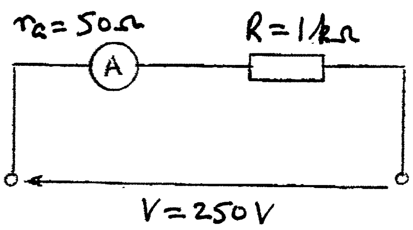
1. (a) **2.** (b) **3.** (c) **4.** (a) **5.** (b) **6.** (b) **7.** (c) **8.** (c) **9.** (c) **10.** (d) **11.** (c) **12.** (b)

13. (d) **14.** (a)

CHAPTER 42 ELECTRICAL MEASURING INSTRUMENTS AND MEASUREMENTS

EXERCISE 200, Page 448

1. A 0 – 1 A ammeter having a resistance of 50 Ω is used to measure the current flowing in a 1 kΩ resistor when the supply voltage is 250 V. Calculate: (a) the approximate value of current (neglecting the ammeter resistance), (b) the actual current in the circuit, (c) the power dissipated in the ammeter, (d) the power dissipated in the 1 kΩ resistor.



(a) Approximate value of current = $\frac{V}{R} = \frac{250}{1000}$ = **0.250 A**

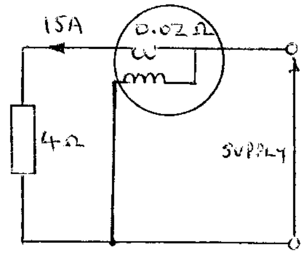
(b) Actual current = $\frac{V}{R + r_a} = \frac{250}{1000 + 50}$ = **0.238 A**

(c) Power dissipated in ammeter, P = $I^2 r_a = (0.238)^2 (50)$ = **2.832 W**

(d) Power dissipated in the 1 kΩ resistor, P = $I^2 r_a = (0.238)^2 (1000)$ = **56.64 W**

2. (a) A current of 15 A flows through a load having a resistance of 4 Ω. Determine the power dissipated in the load. (b) A wattmeter, whose current coil has a resistance of 0.02 Ω, is connected to measure the power in the load. Determine the wattmeter reading assuming the current in the load is still 15 A.

(a) Power in load, P = $I^2 R = (15)^2 (4)$ = **900 W**

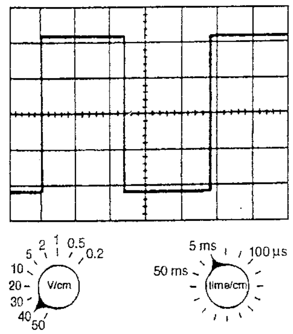


(b) Total resistance in circuit, $R_T = 4 + 0.02 = 4.02\ \Omega$

Wattmeter reading, P = $I^2R_T = (15)^2(4.02)$ = **904.5 W**

EXERCISE 201, Page 452

1. For the square voltage waveform displayed on a c.r.o. shown below, find (a) its frequency, (b) its peak-to-peak voltage

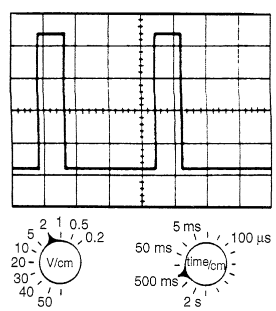


(a) Periodic time, T = 4.8 cm × 5 ms/cm = 24 ms

Hence, **frequency, f** = $\frac{1}{T} = \frac{1}{24\times10^{-3}}$ = **41.7 Hz**

(b) **Peak-to-peak voltage** = 4.4 cm × 40 V/cm = **176 V**

2. For the pulse waveform shown below, find (a) its frequency, (b) the magnitude of the pulse voltage.

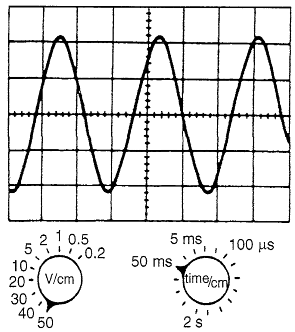


(a) Time for one cycle, T = 3.6 cm × 500 ms/cm = 1.8 s

Hence, **frequency, f** = $\frac{1}{T} = \frac{1}{1.8}$ = **0.56 Hz**

(b) **Magnitude of the pulse voltage** = 4.2 cm × 2V/cm = **8.4 V**

3. For the sinusoidal waveform shown below, determine (a) its frequency, (b) the peak-to-peak voltage, (c) the r.m.s. voltage.



(a) Periodic time, T = 2.8 cm × 50 ms/cm = 0.14 s

Hence, **frequency, f** = $\frac{1}{T} = \frac{1}{0.14}$ = **7.14 Hz**

(b) **Peak-to-peak voltage** = 4.4 cm × 50 V/cm = **220 V**

(c) Peak voltage = $\frac{220}{2}$ = 110 V and **r.m.s. voltage** = $\frac{1}{\sqrt{2}} \times 110$ = **77.78 V**

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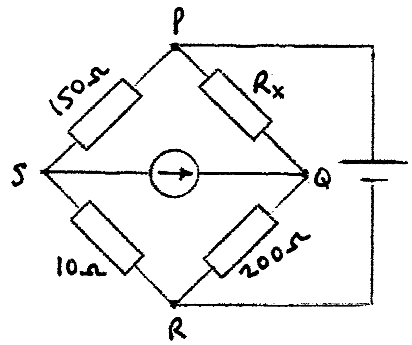
1. In a Wheatstone bridge PQRS, a galvanometer is connected between Q and S and a voltage source between P and R. An unknown resistor R_X is connected between P and Q. When the bridge is balanced, the resistance between Q and R is 200 Ω, that between R and S is 10 Ω and that between S and P is 150 Ω. Calculate the value of R_X



From the diagram, $10 \times R_X = 150 \times 200$

and **unknown resistor**, $R_X = \dfrac{150 \times 200}{10}$ = **3 kΩ**

2. Balance is obtained in a d.c. potentiometer at a length of 31.2 cm when using a standard cell of 1.0186 volts. Calculate the e.m.f. of a dry cell if balance is obtained with a length of 46.7 cm.

$\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}$ hence, $\dfrac{1.0186}{E_2} = \dfrac{31.2}{46.7}$ from which, **e.m.f. of dry cell**, $E_2 = 1.0186\left(\dfrac{46.7}{31.2}\right)$ = **1.525 V**

EXERCISE 203, Page 459

Answers found from within the text of the chapter, pages 445 to 459.

EXERCISE 204, Page 459

1. (f) **2.** (c) **3.** (a) **4.** (i) **5.** (j) **6.** (g) **7.** (c) **8.** (b) **9.** (p) **10.** (d) **11.** (o) **12.** (n)

13. (a)

CHAPTER 43 INTRODUCTION TO ENGINEERING SYSTEMS

EXERCISE 205, Page 472

Answers found from within the text of the chapter, pages 465 to 472.

EXERCISE 206, Page 472

1. (d) **2.** (a) **3.** (c) **4.** (d) **5.** (b) **6.** (a) **7.** (b) **8.** (c)

CHAPTER 5 ALGEBRA

EXERCISE 21, Page 44

1. Find the sum of 4a, - 2a, 3a, - 8a

4a + - 2a + 3a + - 8a = 4a – 2a + 3a – 8a = 4a + 3a – 2a – 8a = 7a – 10 a = **- 3a**

2. Find the sum of 2a, 5b, - 3c, - a, - 3b and 7c

2a + 5b + - 3c + - a + - 3b + 7c = 2a + 5b – 3c – a – 3b + 7c = 2a – a + 5b – 3b + 7c – 3c

= **a + 2b + 4c**

3. Simplify 5ab – 4a + ab + a

5ab – 4a + ab + a = 5ab + ab + a – 4a = **6ab – 3a**

4. Simplify 2x – 3y + 5z – x – 2y + 3z + 5x

2x – 3y + 5z – x – 2y + 3z + 5x = 2x – x + 5x – 3y – 2y + 5z + 3z

= **6x – 5y + 8z**

5. Add x – 2y + 3 to 3x + 4y – 1

(3x + 4y – 1) + (x – 2y + 3) = 3x + 4y – 1 + x – 2y + 3

= 3x + x + 4y – 2y – 1 + 3 = **4x + 2y + 2**

6. Subtract a – 2b from 4a + 3b

(4a + 3b) – (a – 2b) = 4a + 3b – a + 2b = 4a – a + 3b + 2b = **3a + 5b**

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7. From a + b – 2c take 3a + 2b – 4c

$(a + b - 2c) - (3a + 2b - 4c) = a + b - 2c - 3a - 2b + 4c$

$= a - 3a + b - 2b - 2c + 4c = \mathbf{-2a - b + 2c}$

8. Simplify $pq \times pq^2r$

$pq \times pq^2r = p \times p \times q \times q^2 \times r = p^2 \times q^3 \times r = \mathbf{p^2q^3r}$

9. Simplify $-4a \times -2a$

$-4a \times -2a = \mathbf{8a^2}$

10. Simplify $3 \times -2q \times -q$

$3 \times -2q \times -q = \mathbf{6q^2}$

11. Evaluate $3pq - 5qr - pqr$ when p = 3, q = - 2 and r = 4

When p = 3, q = - 2 and r = 4, then $3pq - 5qr - pqr = 3(3)(-2) - 5(-2)(4) - (3)(-2)(4)$

$= -18 + 40 + 24 = \mathbf{46}$

12. If x = 5 and y = 6, evaluate: $\dfrac{23(x-y)}{y+xy+2x}$

$$\frac{23(x-y)}{y+xy+2x} = \frac{23(5-6)}{6+5\times6+2\times5} = \frac{23\times-1}{6+30+10} = \frac{-23}{46} = -\frac{\mathbf{1}}{\mathbf{2}} \text{ or } \mathbf{-0.5}$$

13. If a = 4, b = 3, c = 5 and d = 6, evaluate $\dfrac{3a+2b}{3c-2d}$

If a = 4, b = 3, c = 5 and d = 6, then $\frac{3a+2b}{3c-2d} = \frac{3(4)+2(3)}{3(5)-2(6)} = \frac{12+6}{15-12} = \frac{18}{3} = \mathbf{6}$

14. Simplify $2x \div 14xy$

$$2x \div 14xy = \frac{2x}{14xy} = \frac{\mathbf{1}}{\mathbf{7y}} \text{ by cancelling}$$

15. Multiply 3a – b by a + b

$(3a - b)(a + b) = 3a^2 + 3ab - ab - b^2 = \mathbf{3a^2 + 2ab - b^2}$

16. Simplify $3a \div 9ab$

$$3a \div 9ab = \frac{3a}{9ab} = \frac{\mathbf{1}}{\mathbf{3b}} \text{ by cancelling}$$

EXERCISE 22, Page 46

1. Simplify $z^2 \times z^6$ giving the answer as a power

$$z^2 \times z^6 = z^{2+8} = \mathbf{z^8}$$

2. Simplify $a \times a^2 \times a^5$ giving the answer as a power

$$a \times a^2 \times a^5 = a^{1+2+5} = \mathbf{a^8}$$

3. Simplify $n^8 \times n^{-5}$ giving the answer as a power

$$n^8 \times n^{-5} = n^{8-5} = \mathbf{n^3}$$

4. Simplify $b^4 \times b^7$ giving the answer as a power

$$b^4 \times b^7 = b^{4+7} = \mathbf{b^{11}}$$

5. Simplify $b^2 \div b^5$ giving the answer as a power

$$b^2 \div b^5 = \frac{b^2}{b^5} = \mathbf{b^{-3}}$$

6. Simplify $c^5 \times c^3 \div c^4$ giving the answer as a power

$$c^5 \times c^3 \div c^4 = \frac{c^5 \times c^3}{c^4} = \frac{c^{5+3}}{c^4} = \frac{c^8}{c^4} = c^{8-4} = \mathbf{c^4}$$

7. Simplify $\frac{m^5 \times m^6}{m^4 \times m^3}$ giving the answer as a power

$$\frac{m^5 \times m^6}{m^4 \times m^3} = \frac{m^{5+6}}{m^{4+3}} = \frac{m^{11}}{m^7} = m^{11-7} = \mathbf{m^4}$$

8. Simplify $\frac{(x^2)(x)}{x^6}$ giving the answer as a power

$$\frac{(x^2)(x)}{x^6} = \frac{x^{2+1}}{x^6} = \frac{x^3}{x^6} = x^{3-6} = \mathbf{x^{-3}} \text{ or } \frac{\mathbf{1}}{\mathbf{x^3}}$$

9. Simplify $\left(x^3\right)^4$ giving the answer as a power

$$\left(x^3\right)^4 = x^{3\times4} = \mathbf{x^{12}}$$

10. Simplify $\left(y^2\right)^{-3}$ giving the answer as a power

$$\left(y^2\right)^{-3} = y^{2\times-3} = \mathbf{y^{-6}} \text{ or } \frac{\mathbf{1}}{\mathbf{y^6}}$$

11. Simplify $\left(t \times t^3\right)^2$ giving the answer as a power

$$\left(t \times t^3\right)^2 = \left(t^{1+3}\right)^2 = \left(t^4\right)^2 = t^{4\times2} = \mathbf{t^8}$$

12. Simplify $\left(c^{-7}\right)^{-2}$ giving the answer as a power

$$\left(c^{-7}\right)^{-2} = c^{-7\times-2} = \mathbf{c^{14}}$$

13. Simplify $\left(\frac{a^2}{a^5}\right)^3$ giving the answer as a power

$$\left(\frac{a^2}{a^5}\right)^3 = \left(a^{2-5}\right)^3 = \left(a^{-3}\right)^3 = \mathbf{a^{-9}} \text{ or } \mathbf{\frac{1}{a^9}}$$

14. Simplify $\left(\frac{1}{b^3}\right)^4$ giving the answer as a power

$$\left(\frac{1}{b^3}\right)^4 = \left(b^{-3}\right)^4 = \mathbf{b^{-12}} \text{ or } \mathbf{\frac{1}{b^{12}}}$$

15. Simplify $\left(\frac{b^2}{b^7}\right)^{-2}$ giving the answer as a power

$$\left(\frac{b^2}{b^7}\right)^{-2} = \left(b^{2-7}\right)^{-2} = \left(b^{-5}\right)^{-2} = b^{-5\times -2} = \mathbf{b^{10}}$$

16. Simplify $\frac{1}{\left(s^3\right)^3}$ giving the answer as a power

$$\frac{1}{\left(s^3\right)^3} = \frac{1}{s^{3\times 3}} = \frac{1}{s^9} = \mathbf{s^{-9}}$$

17. Simplify $\frac{a^5bc^3}{a^2b^3c^2}$ and evaluate when $a = \frac{3}{2}$, $b = \frac{1}{2}$ and $c = \frac{2}{3}$

$$\frac{a^5bc^3}{a^2b^3c^2} = a^{5-2}b^{1-3}c^{3-2} = \mathbf{a^3b^{-2}c} \text{ or } \mathbf{\frac{a^3c}{b^2}}$$

$$a^3b^{-2}c = \frac{a^3c}{b^2} = \frac{\left(\frac{3}{2}\right)^3\left(\frac{2}{3}\right)}{\left(\frac{1}{2}\right)^2} = \frac{\frac{27}{8}\times\frac{2}{3}}{\frac{1}{4}} = \frac{\frac{9}{4}}{\frac{1}{4}} = \frac{9}{4}\times\frac{4}{1} = \mathbf{9}$$

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18. Simplify $\dfrac{(abc)^2}{(a^2b^{-1}c^{-3})^3}$

$$\frac{(abc)^2}{\left(a^2b^{-1}c^{-3}\right)^3} = \frac{a^2b^2c^2}{a^6b^{-3}c^{-9}} = a^{2-6}b^{2--3}c^{2--9} = \mathbf{a^{-4}\,b^5\,c^{11}}$$

19. Simplify $\dfrac{(a^3b^{1/2}c^{-1/2})(ab)^{1/3}}{(\sqrt{a^3}\,\sqrt{b}\,c)}$

$$\frac{\left(a^3b^{\frac{1}{2}}c^{-\frac{1}{2}}\right)(ab)^{\frac{1}{3}}}{\left(\sqrt{a^3}\,\sqrt{b}\,c\right)} = \frac{a^3b^{\frac{1}{2}}c^{-\frac{1}{2}}a^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{3}{2}}b^{\frac{1}{2}}c} = a^{\left(3+\frac{1}{3}-\frac{3}{2}\right)}b^{\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{2}\right)}c^{-\left(-\frac{1}{2}-1\right)} = a^{\left(\frac{18+2-9}{6}\right)}b^{\frac{1}{3}}c^{-\frac{3}{2}}$$

$$= \mathbf{a^{\frac{11}{6}}b^{\frac{1}{3}}c^{-\frac{3}{2}}} \text{ or } \frac{\mathbf{\sqrt[6]{a^{11}}\,\sqrt[3]{b}}}{\mathbf{\sqrt{c^3}}}$$

EXERCISE 23, Page 47

1. Expand the brackets: (x + 2)(x + 3)

$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = \mathbf{x^2 + 5x + 6}$

2. Expand the brackets: (x + 4)(2x + 1)

$(x + 4)(2x + 1) = 2x^2 + x + 8x + 4 = \mathbf{2x^2 + 9x + 4}$

3. Expand the brackets: $(2x + 3)^2$

$(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9 = \mathbf{4x^2 + 12x + 9}$

4. Expand the brackets: (2j – 4)(j + 3)

$(2j - 4)(j + 3) = 2j^2 + 6j - 4j - 12 = \mathbf{2j^2 + 2j - 12}$

5. Expand the brackets: (2x + 6)(2x + 5)

$(2x + 6)(2x + 5) = 4x^2 + 10x + 12x + 30 = \mathbf{4x^2 + 22x + 30}$

6. Expand the brackets: (pq + r)(r + pq)

$(pq + r)(r + pq) = pqr + p^2q^2 + r^2 + pqr = \mathbf{2pqr + p^2q^2 + r^2}$

7. Expand the brackets: $(x + 6)^2$

$(x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 = \mathbf{x^2 + 12x + 36}$

8. Expand the brackets: $(5x + 3)^2$

$(5x + 3)^2 = (5x + 3)(5x + 3) = 25x^2 + 15x + 15x + 9 = \mathbf{25x^2 + 30x + 9}$

9. Expand the brackets: $(2x - 6)^2$

$(2x - 6)^2 = (2x - 6)(2x - 6) = 4x^2 - 12x - 12x + 36 = \mathbf{4x^2 - 24x + 36}$

10. Expand the brackets: $(2x - 3)(2x + 3)$

$(2x - 3)(2x + 3) = (2x - 3)(2x + 3) = 4x^2 + 6x - 6x - 9 = \mathbf{4x^2 - 9}$

11. Expand the brackets: $3a(b - 2a)$

$3a(b - 2a) = \mathbf{3ab - 6a^2}$

12. Expand the brackets: $2x(x - y)$

$2x(x - y) = \mathbf{2x^2 - 2xy}$

13. Expand the brackets: $(2a - 5b)(a + b)$

$(2a - 5b)(a + b) = 2a^2 + 2ab - 5ab - 5b^2 = \mathbf{2a^2 - 3ab - 5b^2}$

14. Expand the brackets: $3(3p - 2q) - (q - 4p)$

$3(3p - 2q) - (q - 4p) = 9p - 6q - q + 4p = \mathbf{13p - 7q}$

15. Expand the brackets: $(3x - 4y) + 3(y - z) - (z - 4x)$

$(3x - 4y) + 3(y - z) - (z - 4x) = 3x - 4y + 3y - 3z - z + 4x$

$= \mathbf{7x - y - 4z}$

16. Expand the brackets: (2a + 5b)(2a – 5b)

$(2a + 5b)(2a - 5b) = 4a^2 - 10ab + 10ab - 25b^2 = \mathbf{4a^2 - 25b^2}$

17. Expand the brackets: $(x-2y)^2$

$(x - 2y)^2 = (x - 2y)(x - 2y) = x^2 - 2xy - 2xy + 4y^2 = \mathbf{x^2 - 4xy + 4y^2}$

18. Expand the brackets: 2x + [y - (2x + y)]

$2x + [y - (2x + y)] = 2x + y - 2x - y = \mathbf{0}$

19. Expand the brackets: 3a + 2[a - (3a - 2)]

$3a + 2[a - (3a - 2)] = 3a + 2a - 2(3a - 2) = 3a + 2a - 6a + 4 = \mathbf{-a + 4}$ or $\mathbf{4 - a}$

20. Expand the brackets: $4\left[a^2 - 3a(2b+a) + 7ab\right]$

$$4\left[a^2 - 3a(2b+a) + 7ab\right] = 4\left[a^2 - 6ab - 3a^2 + 7ab\right] = 4\left[-2a^2 + ab\right]$$

$$= \mathbf{-8a^2 + 4ab} \text{ or } \mathbf{4ab - 8a^2}$$

EXERCISE 24, Page 48

1. Factorise and simplify: $2x + 4$

$2x + 4 = \mathbf{2(x + 2)}$

2. Factorise and simplify: $2xy – 8xz$

$2xy – 8xz = \mathbf{2x(y – 4z)}$

3. Factorise and simplify: $pb + 2pc$

$pb + 2pc = \mathbf{p(b + 2c)}$

4. Factorise and simplify: $2x + 4xy$

$2x + 4xy = \mathbf{2x(1 + 2y)}$

5. Factorise and simplify: $4d^2 - 12df^5$

$4d^2 - 12df^5 = \mathbf{4d\left(d - 3f^5\right)}$

6. Factorise and simplify: $4x + 8x^2$

$4x + 8x^2 = \mathbf{4x(1 + 2x)}$

7. Factorise and simplify: $2q^2 + 8qn$

$2q^2 + 8qn = \mathbf{2q(q + 4n)}$

8. Factorise and simplify: $rs + rp + rt$

rs + rp +rt = **r(s + p + t)**

9. Factorise and simplify: $x + 3x^2 + 5x^3$

$x + 3x^2 + 5x^3 = \mathbf{x\left(1 + 3x + 5x^2\right)}$

10. Factorise and simplify: $abc + b^3c$

$abc + b^3c = \mathbf{bc\left(a + b^2\right)}$

11. Factorise and simplify: $3x^2y^4 - 15xy^2 + 18xy$

$3x^2y^4 - 15xy^2 + 18xy = \mathbf{3xy\left(xy^3 - 5y + 6\right)}$

12. Factorise and simplify: $4p^3q^2 - 10pq^3$

$4p^3q^2 - 10pq^3 = \mathbf{2pq^2\left(2p^2 - 5q\right)}$

13. Factorise and simplify: $21a^2b^2 - 28ab$

$21a^2b^2 - 28ab = \mathbf{7ab(3ab - 4)}$

14. Factorise and simplify: $2xy^2 + 6x^2y + 8x^3y$

$2xy^2 + 6x^2y + 8x^3y = \mathbf{2xy(y + 3x + 4x^2)}$

15. Factorise and simplify: $2x^2y - 4xy^3 + 8x^3y^4$

$2x^2y - 4xy^3 + 8x^3y^4 = \mathbf{2xy\left(x - 2y^2 + 4x^2y^3\right)}$

EXERCISE 25, Page 49

1. Simplify: $3x + 2x \times 4x - x$

$3x + 2x \times 4x - x = 3x + 8x^2 - x$ (M)

$= \mathbf{2x + 8x^2}$ (S)

2. Simplify: $(2y + y) \times 4y - 3y$

$(2y + y) \times 4y - 3y = 3y \times 4y - 3y$ (B)

$= \mathbf{12y^2 - 3y}$ (M)

3. Simplify: $4b + 3b \times (b - 6b)$

$4b + 3b \times (b - 6b) = 4b + 3b \times - 5b$ (B)

$= \mathbf{4b - 15b^2}$ (M)

4. Simplify: $8a \div 2a + 6a - 3a$

$8a \div 2a + 6a - 3a = \dfrac{8a}{2a} + 6a - 3a$ (D)

$= 4 + 6a - 3a$ by cancelling

$\mathbf{= 4 + 3a}$ (S)

5. Simplify: $6x \div (3x + x) - 4x$

$6x \div (3x + x) - 4x = 6x \div 4x - 4x$ (B)

$= \dfrac{6x}{4x} - 4x$ (D)

$= \mathbf{\dfrac{3}{2} - 4x}$ by cancelling

6. Simplify: $4t \div (5t - 3t + 2t)$

$4t \div (5t - 3t + 2t) = 4t \div 4t$ (B)

$= \frac{4t}{4t}$ (D)

$= \mathbf{1}$ by cancelling

7. Simplify: $3y + 2y \times 5y + 2y \div 8y - 6y$

$3y + 2y \times 5y + 2y \div 8y - 6y = 3y + 2y \times 5y + \frac{2y}{8y} - 6y$ (D)

$= 3y + 2y \times 5y + \frac{1}{4} - 6y$ by cancelling

$= 3y + 10y^2 + \frac{1}{4} - 6y$ (M)

$= \mathbf{10y^2 - 3y + \frac{1}{4}}$ (S)

8. Simplify: $(x + 2x)3x + 2x \div 6x - 4x$

$(x + 2x)3x + 2x \div 6x - 4x = 3x \times 3x + 2x \div 6x - 4x$ (B)

$= 3x \times 3x + \frac{2x}{6x} - 4x$ (D)

$= 3x \times 3x + \frac{1}{3} - 4x$ by cancelling

$= \mathbf{9x^2 + \frac{1}{3} - 4x}$ (M)

9. Simplify: $5a + 2a \times 3a + a \div (2a - 9a)$

$5a + 2a \times 3a + a \div (2a - 9a) = 5a + 2a \times 3a + a \div -7a$ (B)

$= 5a + 2a \times 3a + \dfrac{a}{-7a}$ (D)

$= 5a + 2a \times 3a - \dfrac{1}{7}$ by cancelling

$= 5a + 6a^2 - \dfrac{1}{7}$ (M)

$= \mathbf{6a^2 + 5a - \dfrac{1}{7}}$

---

10. Simplify: (3t + 2t)(5t + t) ÷ (t – 3t)

(3t + 2t)(5t + t) ÷ (t – 3t) = 5t × 6t ÷ - 2t (B)

$= \dfrac{5t \times 6t}{-2t}$ (D)

$= \dfrac{5t \times 3}{-1}$ by cancelling (D)

= **- 15t** (M)

CHAPTER 6 SIMPLE EQUATIONS

EXERCISE 26, Page 52

1. Solve the equation: $2x + 5 = 7$

Since $2x + 5 = 7$ then $2x = 7 - 5$

i.e. $2x = 2$ from which, $\mathbf{x} = \frac{2}{2} = \mathbf{1}$

2. Solve the equation: $8 - 3t = 2$

Since $8 - 3t = 2$ then $8 - 2 = 3t$

i.e. $6 = 3t$ from which, $\mathbf{t} = \frac{6}{3} = \mathbf{2}$

3. Solve the equation: $\frac{2}{3}c - 1 = 3$

Since $\frac{2}{3}c - 1 = 3$ then $\frac{2}{3}c = 3 + 1$

i.e. $\frac{2}{3}c = 4$ from which, $\mathbf{c} = \frac{4 \times 3}{2 =} = \mathbf{6}$

4. Solve the equation: $2x - 1 = 5x + 11$

Since $2x - 1 = 5x + 11$ then $-1 - 11 = 5x - 2x$

i.e. $-12 = 3x$ from which, $\mathbf{x} = \frac{-12}{3} = \mathbf{-4}$

5. Solve the equation: $7 - 4p = 2p - 5$

Since $7 - 4p = 2p - 5$ then $7 + 5 = 2p + 4p$

i.e. $12 = 6p$ from which, $\mathbf{p} = \frac{12}{6} = \mathbf{2}$

6. Solve the equation: $2a + 6 - 5a = 0$

Since $2a + 6 – 5a = 0$ then $6 = 5a – 2a$

i.e. $6 = 3a$ from which, $\mathbf{a} = \frac{6}{3} = \mathbf{2}$

7. Solve the equation: $3x - 2 - 5x = 2x - 4$

Since $3x - 2 - 5x = 2x – 4$ then $4 – 2 = 2x – 3x + 5x$

i.e. $2 = 4x$ from which, $\mathbf{x} = \frac{2}{4} = \mathbf{\frac{1}{2}}$

8. Solve the equation: $20d - 3 + 3d = 11d + 5 - 8$

Since $20d – 3 + 3d = 11d + 5 – 8$ then $20d + 3d – 11d = 5 – 8 + 3$

i.e. $12d = 0$ from which, $\mathbf{d = 0}$

9. Solve the equation: $2(x - 1) = 4$

Since $2(x - 1) = 4$ then $2x – 2 = 4$

i.e. $2x = 4 + 2 = 6$ from which, $\mathbf{x} = \frac{6}{2} = \mathbf{3}$

10. Solve the equation: $16 = 4(t + 2)$

Since $16 = 4(t + 2)$ then $16 = 4t + 8$

i.e. $16 – 8 = 4t$

i.e. $8 = 4t$ from which, $\mathbf{t} = \frac{8}{4} = \mathbf{2}$

11. Solve the equation: $5(f - 2) - 3(2f + 5) + 15 = 0$

Since $5(f-2)-3(2f+5)+15=0$ then $5f-10-6f-15+15=0$

i.e. $5f-6f=10+15-15$

and $-f=10$ from which, $\mathbf{f=-10}$

12. Solve the equation: $2x = 4(x - 3)$

Since $2x = 4(x - 3)$ then $2x = 4x - 12$

i.e. $12 = 4x - 2x$

i.e. $12 = 2x$ from which, $\mathbf{x} = \dfrac{12}{2} = \mathbf{6}$

13. Solve the equation: $6(2 - 3y) - 42 = - 2(y - 1)$

Since $6(2-3y)-42=-2(y-1)$ then $12-18y-42=-2y+2$

i.e. $-18y+2y=2-12+42$

and $-16y=32$

from which, $\mathbf{y} = \dfrac{32}{-16} = -\dfrac{32}{16} = \mathbf{-2}$

14. Solve the equation: $2(3g - 5) - 5 = 0$

Since $2(3g - 5) - 5 = 0$ then $6g-10-5=0$

i.e. $6g = 10 + 5$

i.e. $6g = 15$ from which, $\mathbf{g} = \dfrac{15}{6} = \mathbf{2.5}$

15. Solve the equation: $4(3x + 1) = 7(x + 4) - 2(x + 5)$

Since $4(3x+1)=7(x+4)-2(x+5)$ then $12x+4=7x+28-2x-10$

i.e. $12x-7x+2x=28-10-4$

and $7x = 14$ from which, $\mathbf{x} = \frac{14}{7} = \mathbf{2}$

EXERCISE 27, Page 54

1. Solve the equations: $\frac{1}{5}d + 3 = 4$

Since $\frac{1}{5}d + 3 = 4$ then $\frac{1}{5}d = 4 - 3$

i.e. $\frac{1}{5}d = 1$ from which, $\mathbf{d} = \frac{5 \times 1}{1} = \mathbf{5}$

2. Solve the equations: $2 + \frac{3}{4}y = 1 + \frac{2}{3}y + \frac{5}{6}$

Multiplying each term by 12 (the lowest common denominator of 3, 4 and 6) gives:

$$(12)(2) + (12)\frac{3}{4}y = (12)(1) + (12)\frac{2}{3}y + (12)\frac{5}{6}$$

i.e. $24 + 9y = 12 + 8y + 10$

and $9y - 8y = 12 + 10 - 24$

i.e. $\mathbf{y = -2}$

3. Solve the equations: $\frac{1}{4}(2x - 1) + 3 = \frac{1}{2}$

Multiplying each term by 4 gives:

$$(4)\frac{1}{4}(2x - 1) + (4)(3) = (4)\frac{1}{2}$$

i.e. $2x - 1 + 12 = 2$

and $2x = 2 + 1 - 12$

i.e. $2x = -9$ from which, $\mathbf{x} = -\frac{9}{2} = \mathbf{-4\frac{1}{2}}$

4. Solve the equations: $\frac{1}{5}(2f - 3) + \frac{1}{6}(f - 4) + \frac{2}{15} = 0$

Multiplying each term by 30 gives:

$$(30)\frac{1}{5}(2f-3)+(30)\frac{1}{6}(f-4)+(30)\frac{2}{15}=0$$

i.e. $6(2f-3) + 5(f-4) + 4 = 0$

and $12f - 18 + 5f - 20 + 4 = 0$

i.e. $12f + 5f = 18 + 20 - 4$

and $17f = 34$ from which, $\mathbf{f} = \frac{34}{17} = \mathbf{2}$

5. Solve the equations: $\frac{x}{3} - \frac{x}{5} = 2$

Multiplying each term of $\frac{x}{3} - \frac{x}{5} = 2$ by 15 gives:

$$(15)\frac{x}{3} - (15)\frac{x}{5} = (15)2$$

i.e. $5x - 3x = 30$

i.e. $2x = 30$ from which, $\mathbf{x} = \frac{30}{2} = \mathbf{15}$

6. Solve the equations: $1 - \frac{y}{3} = 3 + \frac{y}{3} - \frac{y}{6}$

Multiplying each term by 6 gives:

$$(6)(1)-(6)\frac{y}{3}=(6)(3)+(6)\frac{y}{3}-(6)\frac{y}{6}$$

i.e. $6 - 2y = 18 + 2y - y$

and $-2y - 2y + y = 18 - 6$

i.e. $-3y = 12$

from which, $\mathbf{y} = \frac{12}{-3} = -\frac{12}{3} = \mathbf{-4}$

7. Solve the equations: $\frac{2}{a} = \frac{3}{8}$

Multiplying each term by 8a gives:

$$(8a)\frac{2}{a} = (8a)\frac{3}{8}$$

i.e. $8(2) = a(3)$

i.e. $16 = 3a$ from which, $\mathbf{a} = \frac{16}{3} = \mathbf{5\frac{1}{3}}$

8. Solve the equations: $\frac{1}{3n} + \frac{1}{4n} = \frac{7}{24}$

Multiplying each term by 24n gives:

$$(24n)\frac{1}{3n} + (24n)\frac{1}{4n} = (24n)\frac{7}{24}$$

i.e. $8 + 6 = 7n$ i.e. $14 = 7n$ from which, $\mathbf{n} = \frac{14}{7} = \mathbf{2}$

9. Solve the equations: $\frac{x+3}{4} = \frac{x-3}{5} + 2$

Multiplying each term by 20 gives:

$$(20)\frac{x+3}{4} = (20)\frac{x-3}{5} + (20)(2)$$

i.e. $5(x + 3) = 4(x - 3) + 40$

i.e. $5x + 15 = 4x - 12 + 40$

and $5x - 4x = -12 + 40 - 15$

from which, $\mathbf{x = 13}$

10. Solve the equations: $\frac{2}{a-3} = \frac{3}{2a+1}$

Multiplying each term by (a – 3)(2a + 1) gives:

$$(a-3)(2a+1)\frac{2}{a-3} = (a-3)(2a+1)\frac{3}{2a+1}$$

i.e. $2(2a + 1) = 3(a – 3)$

i.e. $4a + 2 = 3a – 9$

and $4a – 3a = - 9 - 2$

from which, $\mathbf{a = - 11}$

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11. Solve the equations: $\frac{x}{4} - \frac{x+6}{5} = \frac{x+3}{2}$

Multiplying each term by 20 gives:

$$(20)\frac{x}{4} - (20)\frac{x+6}{5} = (20)\frac{x+3}{2}$$

i.e. $5x – 4(x + 6) = 10(x + 3)$

i.e. $5x – 4x – 24 = 10x + 30$

and $5x – 4x – 10x = 30 + 24$

i.e. $– 9x = 54$ from which, $\mathbf{x} = \frac{54}{-9} = -\frac{54}{9} = \mathbf{- 6}$

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12. Solve the equations: $3\sqrt{t} = 9$

Since $3\sqrt{t} = 9$ then $\sqrt{t} = \frac{9}{3} = 3$

Squaring both sides gives: $\mathbf{t} = 3^2 = \mathbf{9}$

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13. Solve the equations: $2\sqrt{y} = 5$

Since $2\sqrt{y} = 5$ then $\sqrt{y} = \frac{5}{2}$

Squaring both sides gives: $\mathbf{y} = \left(\frac{5}{2}\right)^2 = \frac{25}{4} = \mathbf{6\frac{1}{4}}$

14. Solve the equations: $4 = \sqrt{\left(\frac{3}{a}\right)} + 3$

Since $4 = \sqrt{\left(\frac{3}{a}\right)} + 3$ then $4 - 3 = \sqrt{\left(\frac{3}{a}\right)}$

i.e. $\sqrt{\left(\frac{3}{a}\right)} = 1$

Squaring both sides gives: $\frac{3}{a} = 1^2 = 1$ from which, $3 = a$ or $\mathbf{a = 3}$

15. Solve the equations: $10 = 5\sqrt{\left(\frac{x}{2} - 1\right)}$

Dividing both sides by 5 gives: $2 = \sqrt{\frac{x}{2} - 1}$

Squaring both sides gives: $4 = \frac{x}{2} - 1$

i.e. $4 + 1 = \frac{x}{2}$ i.e. $5 = \frac{x}{2}$ from which, $\mathbf{x} = (5)(2) = \mathbf{10}$

16. Solve the equations: $16 = \frac{t^2}{9}$

Since $16 = \frac{t^2}{9}$ then $(16)(9) = t^2$

i.e. $144 = t^2$

and $\mathbf{t} = \sqrt{144} = \mathbf{\pm 12}$

EXERCISE 28, Page 55

1. A formula used for calculating resistance of a cable is R = $\frac{\rho L}{a}$. Given R = 1.25, L = 2500 and a = 2×10^{-4} find the value of ρ.

Since $R = \frac{\rho l}{A}$ then $1.25 = \frac{\rho \times 2500}{2 \times 10^{-4}}$

Multiplying both sides by 2×10^{-4} gives: $(2 \times 10^{-4})(1.25) = \rho(2500)$

Dividing both sides by 2500 gives: $\frac{(2 \times 10^{-4})(1.25)}{2500} = \rho$

from which, by calculator, $\boldsymbol{\rho = 0.0000001}$ or $\mathbf{10^{-7}}$

2. Force F newtons is given by F = ma, where m is the mass in kilograms and a is the acceleration in metres per second squared. Find the acceleration when a force of 4 kN is applied to a mass of 500 kg

Since F = ma then 4000 = (500)(a)

Dividing both sides by 500 gives: $\frac{4000}{500} = a$

from which, **acceleration, a = 8 $\mathbf{m/s^2}$**

3. PV = mRT is the characteristic gas equation. Find the value of m when P = 100×10^3, V = 3.00, R = 288 and T = 300.

Dividing both sides of PV = mRT by RT gives: $\frac{PV}{RT} = m$

from which, $\mathbf{m} = \frac{(100 \times 10^3)(3.00)}{(288)(300)}$ = **3.472**, correct to 4 significant figures.

4. When three resistors R_1, R_2 and R_3 are connected in parallel the total resistance R_T is determined from $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

(a) Find the total resistance when $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$ amd $R_3 = 18\ \Omega$

(b) Find the value of R_3 given that $R_T = 3\ \Omega$, $R_1 = 5\ \Omega$ and $R_2 = 10\ \Omega$

(a) $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{18} = \frac{6+3+1}{18} = \frac{10}{18}$

Turning both sides upside down gives: $\mathbf{R_T} = \frac{18}{10} = \mathbf{1.8\ \Omega}$

(b) $\frac{1}{3} = \frac{1}{5} + \frac{1}{10} + \frac{1}{R_3}$ from which, $\frac{1}{R_3} = \frac{1}{3} - \frac{1}{5} - \frac{1}{10} = \frac{10-6-3}{30} = \frac{1}{30}$

Turning both sides upside down gives: $\mathbf{R_3} = \frac{30}{1} = \mathbf{30\ \Omega}$

5. Ohm's law may be represented by I = V/R, where I is the current in amperes, V is the voltage in volts and R is the resistance in ohms. A soldering iron takes a current of 0.30 A from a 240 V supply. Find the resistance of the element.

Multiplying both sides of $I = \frac{V}{R}$ by R gives: IR = V

and dividing both sides by I gives: $R = \frac{V}{I}$

Thus, **resistance, R** $= \frac{240}{0.3} = \mathbf{800\ \Omega}$

6. The stress, σ Pascal's, acting on the reinforcing rod in a concrete column is given in the following equation: $500\times10^{-6}\sigma + 2.67\times10^{5} = 3.55\times10^{5}$

Find the value of the stress in MPa.

Since $500\times10^{-6}\sigma+2.67\times10^{5}=3.55\times10^{5}$

then $500\times10^{-6}\sigma=3.55\times10^{5}-2.67\times10^{5}$

and **stress, σ** $=\dfrac{3.55\times10^{5}-2.67\times10^{5}}{500\times10^{-6}}=\dfrac{0.88\times10^{5}}{500\times10^{-6}}=\mathbf{176\times10^{6}\ Pa=176\ MPa}$

EXERCISE 29, Page 56

1. Given $R_2 = R_1(1 + \alpha t)$, find α given $R_1 = 5.0$, $R_2 = 6.03$ and $t = 51.5$

Substituting into $R_2 = R_1(1+\alpha t)$ gives: $6.03 = 5.0[1 + \alpha(51.5)]$

and $\frac{6.03}{5.0} = 1 + \alpha(51.5)$

i.e. $1.206 = 1 + \alpha(51.5)$

and $1.206 - 1 = \alpha(51.5)$

i.e. $0.206 = \alpha(51.5)$

from which, $\alpha = \frac{0.206}{51.5} = \mathbf{0.004}$

2. If $v^2 = u^2 + 2as$, find u given $v = 24$, $a = - 40$ and $s = 4.05$

Substituting into $v^2 = u^2 + 2as$ gives: $24^2 = u^2 + 2(-40)(4.05)$

i.e. $576 = u^2 - 324$

and $576 + 324 = u^2$

i.e. $u^2 = 900$

and $\mathbf{u} = \sqrt{900} = \mathbf{30}$

3. The relationship between the temperature on a Fahrenheit scale and that on a Celsius scale is given by $F = \frac{9}{5}C + 32$. Express 113°F in degrees Celsius.

Since $F = \frac{9}{5}C + 32$ then $113 = \frac{9}{5}C + 32$

i.e. $113 - 32 = \frac{9}{5}C$

i.e. $81 = \frac{9}{5}C$

Multiplying both sides by 5 gives: $(81)(5) = 9C$

Dividing both sides by 9 gives: $\frac{(81)(5)}{9} = C$ from which, $C = 45$

Hence, **113° Fahrenheit is equivalent to 45° Celsius**

4. If $t = 2\pi\sqrt{\frac{w}{Sg}}$, find the value of S given w = 1.219, g = 9.81 and t = 0.3132

Since $t = 2\pi\sqrt{w/Sg}$ then $0.3132 = 2\pi\sqrt{\left(\frac{1.219}{S \times 9.81}\right)}$

and $$\frac{0.3132}{2\pi} = \sqrt{\left(\frac{1.219}{S \times 9.81}\right)}$$

from which, $$\left(\frac{0.3132}{2\pi}\right)^2 = \frac{1.219}{S \times 9.81}$$

Thus, $$S\left(\frac{0.3132}{2\pi}\right)^2 = \frac{1.219}{9.81}$$

from which, $$\mathbf{S} = \frac{1.219}{9.81\left(\frac{0.3132}{2\pi}\right)^2} = \mathbf{50} \text{ using a calculator}$$

5. A rectangular laboratory has a length equal to one and a half times its width and a perimeter of 40 m. Find its length and width.

Let length of laboratory = l and width = w

Length, l = 1.5w and perimeter = 40 = 2l + 2w

Hence, 40 = 2(1.5w) + 2w

i.e. 40 = 3w + 2w = 5w

from which, **width, w** = $\frac{40}{5}$ = **8 m**

and **length, l** = 1.5w = 1.5(8) = **12 m**

6. Applying the principle of moments to a beam results in the following equation:

$$F \times 3 = (5 - F) \times 7$$

where F is the force in Newtons. Determine the value of F.

Since $F \times 3 = (5 - F) \times 7$ then $3F = 35 - 7F$

i.e. $3F + 7F = 35$

i.e. $10F = 35$ from which, $\mathbf{F} = \frac{35}{10} = \mathbf{3.5\ N}$

CHAPTER 7 TRANSPOSITION OF FORMULAE

EXERCISE 30, Page 60

1. Make d the subject of the formula: $a + b = c - d - e$

Since $a + b = c - d - e$ then $\mathbf{d = c - e - a - b}$

2. Make x the subject of the formula: $y = 7x$

Dividing both sides of $y = 7x$ by 7 gives: $\mathbf{x} = \mathbf{\dfrac{y}{7}}$

3. Make v the subject of the formula: $pv = c$

Dividing both sides of $pv = c$ by p gives: $\mathbf{v} = \mathbf{\dfrac{c}{p}}$

4. Make 'a' the subject of the formula: $v = u + at$

Since $v = u + at$ then $v - u = at$

and dividing both sides by t gives: $\mathbf{\dfrac{v-u}{t}} = \mathbf{a}$ or $\mathbf{a} = \mathbf{\dfrac{v-u}{t}}$

5. Make y the subject of the formula: $x + 3y = t$

Since $x + 3y = t$ then $3y = t - x$

and dividing both sides by 3 gives: $\mathbf{y = \dfrac{t-x}{3}}$ or $\mathbf{y = \dfrac{1}{3}(t-x)}$

6. Make r the subject of the formula: $c = 2\pi r$

Dividing both sides of $c = 2\pi r$ by 2π gives: $\frac{c}{2\pi} = r$ or $\mathbf{r = \frac{c}{2\pi}}$

7. Make x the subject of the formula: $y = mx + c$

Since $y = mx + c$ then $y - c = mx$

and dividing both sides by m gives: $\frac{\mathbf{y - c}}{\mathbf{m}} = \mathbf{x}$ or $\mathbf{x} = \frac{\mathbf{y - c}}{\mathbf{m}}$

8. Make T the subject of the formula: $I = PRT$

Dividing both sides of $I = PRT$ by PR gives: $\frac{I}{PR} = T$ or $\mathbf{T = \frac{I}{PR}}$

9. Make L the subject of the formula: $X_L = 2\pi f L$

Dividing both sides of $X_L = 2\pi f L$ by $2\pi f$ gives: $\frac{X_L}{2\pi f} = L$ or $\mathbf{L = \frac{X_L}{2\pi f}}$

10. Make R the subject of the formula: $I = \frac{E}{R}$

Multiplying both sides of $I = \frac{E}{R}$ by R gives: $I R = E$

and dividing both sides by I gives: $\mathbf{R = \frac{E}{I}}$

11. Make x the subject of the formula: $y = \frac{x}{a} + 3$

Since $y = \frac{x}{a} + 3$ then $y - 3 = \frac{x}{a}$

Multiplying both sides by 'a' gives: $a(y - 3) = x$ or $\mathbf{x = a(y - 3)}$

12. Make C the subject of the formula: $F = \frac{9}{5}C + 32$

Rearranging $F = \frac{9}{5}C + 32$ gives: $F - 32 = \frac{9}{5}C$

Multiplying both sides by $\frac{5}{9}$ gives: $\frac{5}{9}(F - 32) = \left(\frac{5}{9}\right)\left(\frac{9}{5}C\right)$

i.e. $\frac{5}{9}(F - 32) = C$ or $\mathbf{C = \frac{5}{9}(F - 32)}$

EXERCISE 31, Page 61

1. Make r the subject of the formula: $S = \frac{a}{1-r}$

Multiplying both sides of $S = \frac{a}{1-r}$ by $(1 - r)$ gives: $S(1 - r) = a$

i.e. $S - Sr = a$

from which, $S - a = Sr$

and dividing both sides by S gives: $\frac{S-a}{S} = r$ i.e. $\mathbf{r = \frac{S-a}{S}}$ or $\mathbf{r = 1 - \frac{a}{S}}$

2. Make x the subject of the formula: $y = \frac{\lambda(x-d)}{d}$

Multiplying both sides of $y = \frac{\lambda(x-d)}{d}$ by d gives: $yd = \lambda(x - d)$

Dividing both sides by λ gives: $\frac{yd}{\lambda} = x - d$

and $d + \frac{yd}{\lambda} = x$ or $\mathbf{x = d + \frac{yd}{\lambda}}$

Alternatively, from the first step, $yd = \lambda(x - d)$

i.e. $yd = \lambda x - \lambda d$

and $yd + \lambda d = \lambda x$

from which, $x = \frac{yd + \lambda d}{\lambda} = \frac{d(y+\lambda)}{\lambda}$ i.e. $\mathbf{x = \frac{d}{\lambda}(y+\lambda)}$

3. Make f the subject of the formula: $A = \frac{3(F-f)}{L}$

Multiplying both sides of $A = \frac{3(F-f)}{L}$ by L gives: $AL = 3(F - f)$

Dividing both sides by 3 gives: $\frac{AL}{3} = F - f$

and $\mathbf{f = F - \frac{AL}{3}}$ or $\mathbf{f = \frac{3F - AL}{3}}$

4. Make D the subject of the formula: $y = \frac{AB^2}{5CD}$

Multiplying both sides of $y = \frac{AB^2}{5CD}$ by D gives: $yD = \frac{AB^2}{5C}$

Dividing both sides by y gives: $\mathbf{D = \frac{AB^2}{5Cy}}$

5. Make t the subject of the formula: $R = R_0(1 + \alpha t)$

Removing the bracket in $R = R_0(1+\alpha t)$ gives: $R = R_0 + R_0 \alpha t$

from which, $R - R_0 = R_0 \alpha t$

and $\frac{R - R_0}{R_0\alpha} = t$ or $\mathbf{t = \frac{R - R_0}{R_0\alpha}}$

6. Make R_2 the subject of the formula: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Rearranging $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ gives: $\frac{1}{R} - \frac{1}{R_1} = \frac{1}{R_2}$

i.e. $\frac{1}{R_2} = \frac{1}{R} - \frac{1}{R_1} = \frac{R_1 - R}{R R_1}$

Turning both sides upside down gives: $\mathbf{R_2 = \frac{R R_1}{R_1 - R}}$

7. Make R the subject of the formula: $I = \dfrac{E - e}{R + r}$

Multiplying both sides by $(R + r)$ gives: $I(R + r) = E - e$

i.e. $IR + Ir = E - e$

and $IR = E - e - Ir$

and dividing both sides by I gives: $\mathbf{R = \dfrac{E - e - Ir}{I}}$ or $\mathbf{R = \dfrac{E - e}{I} - r}$

8. Make b the subject of the formula: $y = 4ab^2c^2$

Dividing both sides by $4ac^2$ gives: $\dfrac{y}{4ac^2} = b^2$ or $b^2 = \dfrac{y}{4ac^2}$

Taking the square root of both sides gives: $\mathbf{b = \sqrt{\dfrac{y}{4ac^2}}}$

9. Make x the subject of the formula: $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$

Rearranging $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$ gives: $\dfrac{a^2}{x^2} = 1 - \dfrac{b^2}{y^2} = \dfrac{y^2 - b^2}{y^2}$

Turning both sides upside down gives: $\dfrac{x^2}{a^2} = \dfrac{y^2}{y^2 - b^2}$

Multiplying both sides by a^2 gives: $x^2 = a^2\left(\dfrac{y^2}{y^2 - b^2}\right) = \dfrac{a^2y^2}{y^2 - b^2}$

Taking the square root of both sides gives: $x = \sqrt{\dfrac{a^2y^2}{y^2 - b^2}} = \dfrac{\sqrt{a^2y^2}}{\sqrt{y^2 - b^2}} = \dfrac{ay}{\sqrt{y^2 - b^2}}$

i.e. $\mathbf{x = \dfrac{ay}{\sqrt{y^2 - b^2}}}$

10. Make L the subject of the formula: $t = 2\pi\sqrt{\frac{L}{g}}$

Dividing both sides of $t = 2\pi\sqrt{\frac{L}{g}}$ by 2π gives: $\frac{t}{2\pi} = \sqrt{\frac{L}{g}}$

Squaring both sides gives: $\left(\frac{t}{2\pi}\right)^2 = \frac{L}{g}$ or $\frac{L}{g} = \left(\frac{t}{2\pi}\right)^2$

Multiplying both sides by g gives: $\mathbf{L = g\left(\frac{t}{2\pi}\right)^2}$ or $\mathbf{L = \frac{g\,t^2}{4\pi^2}}$

11. Make u the subject of the formula: $v^2 = u^2 + 2as$

Since $v^2 = u^2 + 2as$ then $v^2 - 2as = u^2$ or $u^2 = v^2 - 2as$

Taking the square root of each side gives: $\mathbf{u = \sqrt{\left(v^2 - 2as\right)}}$

12. Make 'a' the subject of the formula: $N = \sqrt{\left(\frac{a+x}{y}\right)}$

Squaring both sides of $N = \sqrt{\frac{a+x}{y}}$ gives: $N^2 = \frac{a+x}{y}$

Multiplying both sides by y gives: $N^2y = a + x$ or $a + x = N^2y$

from which, $\mathbf{a = N^2y - x}$

13. The lift force, L, on an aircraft is given by: $L = \frac{1}{2}\rho v^2 a c$ where ρ is the density, v is the velocity, a is the area and c is the lift coefficient. Transpose the equation to make the velocity the subject.

Since $\quad L = \dfrac{1}{2}\rho v^2 a c \quad$ then $\quad \dfrac{2L}{\rho a c} = v^2$

from which, **velocity,** $\mathbf{v} = \sqrt{\dfrac{\mathbf{2L}}{\boldsymbol{\rho}\mathbf{ac}}}$

EXERCISE 32, Page 62

1. Make 'a' the subject of the formula: $y = \dfrac{a^2m - a^2n}{x}$

Multiplying both sides of $y = \dfrac{a^2m - a^2n}{x}$ by x gives: $xy = a^2m - a^2n$

and factorising gives: $xy = a^2(m - n)$

Dividing both sides by (m – n) gives: $\dfrac{xy}{m-n} = a^2$ or $a^2 = \dfrac{xy}{m-n}$

Taking the square root of both sides gives: $\mathbf{a = \sqrt{\dfrac{xy}{m-n}}}$

2. Make R the subject of the formula: $M = \pi(R^4 - r^4)$

Dividing both sides of $M = \pi(R^4 - r^4)$ by π gives: $\dfrac{M}{\pi} = R^4 - r^4$

and rearranging gives: $\dfrac{M}{\pi} + r^4 = R^4$ or $R^4 = \dfrac{M}{\pi} + r^4$

Taking the fourth root of both sides gives: $\mathbf{R = \sqrt[4]{\dfrac{M}{\pi} + r^4}}$

3. Make r the subject of the formula: $x + y = \dfrac{r}{3+r}$

Multiplying both sides of $x + y = \dfrac{r}{3+r}$ by (3 + r) gives: $(x + y)(3 + r) = r$

Multiplying the brackets gives: $3x + xr + 3y + yr = r$

and rearranging gives: $xr + yr - r = -3x - 3y$

Factorising gives: $r(x + y - 1) = -3(x + y)$

Dividing both sides by (x + y – 1) gives: $r = \dfrac{-3(x+y)}{x+y-1}$

Multiplying numerator and denominator by -1 gives: $\mathbf{r = \dfrac{3(x+y)}{1-x-y}}$

4. Make L the subject of the formula: $m = \dfrac{\mu L}{L + rCR}$

Multiplying both sides of $m = \dfrac{\mu L}{L + rCR}$ by (L + rCR) gives: $m(L + rCR) = \mu L$

Removing brackets gives: $mL + mrCR = \mu L$

and rearranging gives: $mrCR = \mu L - mL$

Factorising gives: $mrCR = L(\mu - m)$

Dividing both sides by $(\mu - m)$ gives: $\mathbf{L = \dfrac{mrCR}{\mu - m}}$

5. Make b the subject of the formula: $a^2 = \dfrac{b^2 - c^2}{b^2}$

Multiplying both sides by b^2 gives: $a^2b^2 = b^2 - c^2$

and rearranging gives: $c^2 = b^2 - a^2b^2$ or $b^2 - a^2b^2 = c^2$

Factorising gives: $b^2\left(1 - a^2\right) = c^2$

Dividing both sides by $\left(1 - a^2\right)$ gives: $b^2 = \dfrac{c^2}{1 - a^2}$

Taking the square root of both sides gives: $b = \sqrt{\dfrac{c^2}{1 - a^2}} = \dfrac{\sqrt{c^2}}{\sqrt{1 - a^2}}$

Hence, $\mathbf{b = \dfrac{c}{\sqrt{1 - a^2}}}$

6. Make r the subject of the formula: $\dfrac{x}{y} = \dfrac{1 + r^2}{1 - r^2}$

Rearranging by 'cross-multiplying' gives: $x(1-r^2)=y(1+r^2)$

Removing brackets gives: $x-xr^2=y+yr^2$

and rearranging gives: $x-y=yr^2+xr^2$ or $yr^2+xr^2=x-y$

Factorising gives: $r^2(x+y)=x-y$

Dividing both sides by $(x+y)$ gives: $r^2=\dfrac{x-y}{x+y}$

Taking the square root of both sides gives: $\mathbf{r=\sqrt{\dfrac{x-y}{x+y}}}$

7. A formula for the focal length, f, of a convex lens is: $\dfrac{1}{f}=\dfrac{1}{u}+\dfrac{1}{v}$. Transpose the formula to make v the subject and evaluate v when f = 5 and u = 6

Rearranging $\dfrac{1}{f}=\dfrac{1}{u}+\dfrac{1}{v}$ gives: $\dfrac{1}{v}=\dfrac{1}{f}-\dfrac{1}{u}=\dfrac{u-f}{uf}$

Turning each side upside down gives: $\mathbf{v=\dfrac{uf}{u-f}}$

When f = 5 and u = 6, then $\mathbf{v}=\dfrac{uf}{u-f}=\dfrac{(6)(5)}{6-5}=\dfrac{30}{1}=\mathbf{30}$

8. The quantity of heat, Q, is given by the formula $Q = mc(t_2 - t_1)$. Make t_2 the subject of the formula and evaluate t_2 when $m = 10$, $t_1 = 15$, $c = 4$ and $Q = 1600$

Removing the brackets in $Q=mc(t_2-t_1)$ gives: $Q=mct_2-mct_1$

and rearranging gives: $Q+mct_1=mct_2$

or $mct_2=Q+mvt_1$

Dividing both sides by mc gives: $\mathbf{t_2=\dfrac{Q+mvt_1}{mc}}$ or $\mathbf{t_2=\dfrac{Q}{mc}+t_1}$ or $\mathbf{t_2=t_1+\dfrac{Q}{mc}}$

When $m = 10$, $t_1 = 15$, $c = 4$ and $Q = 1600$,

$$\mathbf{t_2} = t_1 + \frac{Q}{mc} = 15 + \frac{1600}{(10)(4)} = 15 + \frac{1600}{40} = 15 + 40 = \mathbf{55}$$

9. The velocity, v, of water in a pipe appears in the formula $h = \frac{0.03Lv^2}{2dg}$. Express v as the subject of the formula and evaluate v when h = 0.712, L = 150, d = 0.30 and g = 9.81

Multiplying both sides of $h = \frac{0.03Lv^2}{2dg}$ by $2dg$ gives: $2dgh = 0.03Lv^2$

Dividing both sides by 0.03L gives: $\frac{2dgh}{0.03L} = v^2$ or $v^2 = \frac{2dgh}{0.03L}$

Taking the square root of each side gives: $\mathbf{v = \sqrt{\frac{2dgh}{0.03L}}}$

When h = 0.712, L = 150, d = 0.30 and g = 9.81,

$$\mathbf{v} = \sqrt{\frac{2dgh}{0.03L}} = \sqrt{\frac{2(0.30)(9.81)(0.712)}{0.03(150)}} = \sqrt{0.931296} = \mathbf{0.965}$$

10. The sag S at the centre of a wire is given by the formula: $S = \sqrt{\left(\frac{3d(l-d)}{8}\right)}$

Make l the subject of the formula and evaluate l when d = 1.75 and S = 0.80

Squaring both sides of $S = \sqrt{\frac{3d(l-d)}{8}}$ gives: $S^2 = \frac{3d(l-d)}{8}$

Multiplying both sides by 8 gives: $8S^2 = 3d(l-d)$

Removing the bracket gives: $8S^2 = 3dl - 3d^2$

Rearranging gives: $8S^2 + 3d^2 = 3dl$

or $3dl = 8S^2 + 3d^2$

Dividing both sides by 3d gives: $l = \frac{8S^2 + 3d^2}{3d} = \frac{8S^2}{3d} + \frac{3d^2}{3d}$

i.e. $$\mathbf{l = \frac{8S^2}{3d} + d}$$

When d = 1.75 and S = 0.80, $\mathbf{l} = \frac{8S^2}{3d} + d = \frac{8(0.80)^2}{3(1.75)} + 1.75 = 0.975 + 1.75 = \mathbf{2.725}$

11. An approximate relationship between the number of teeth, T, on a milling cutter, the diameter of cutter, D, and the depth of cut, d, is given by: $T = \frac{12.5D}{D + 4d}$

Determine the value of D when T = 10 and d = 4 mm.

Multiplying both sides of $T = \frac{12.5D}{D + 4d}$ by $D + 4d$ gives: $T(D + 4d) = 12.5D$

Removing brackets gives: $TD + 4dT = 12.5D$

Rearranging gives: $4dT = 12.5D - TD$

or $12.5D - TD = 4dT$

Factorising gives: $D(12.5 - T) = 4dT$

Dividing both sides by (12.5 – T) gives: $D = \frac{4dT}{12.5 - T}$

When T = 10 and d = 4 mm, then $\mathbf{D} = \frac{4dT}{12.5 - T} = \frac{4(4)(10)}{12.5 - 10} = \frac{160}{2.5} = \mathbf{64\ mm}$

12. A simply supported beam of length L has a centrally applied load F and a uniformly distributed load of w per metre length of beam. The reaction at the beam support is given by:

$$R = \frac{1}{2}(F + wL)$$

Rearrange the equation to make w the subject. Hence determine the value of w when L = 4 m, F = 8 kN and R = 10 kN

Since $R = \frac{1}{2}(F + wL)$ then $2R = F + wL$

and $\quad 2R - F = wL$

from which, $\quad \mathbf{w = \dfrac{2R - F}{L}}$

When L = 4 m, F = 8 kN and R = 10 kN, $\quad w = \dfrac{2(10) - 8}{4} = \dfrac{12}{4} = \mathbf{3\ kN/m}$

13. The rate of heat conduction through a slab of material, Q, is given by the formula $Q = \dfrac{kA(t_1 - t_2)}{d}$ where t_1 and t_2 are the temperatures of each side of the material, A is the area of the slab, d is the thickness of the slab, and k is the thermal conductivity of the material.

Rearrange the formula to obtain an expression for t_2

Since $\quad Q = \dfrac{kA(t_1 - t_2)}{d}$ then $\quad Qd = kA\left(t_1 - t_2\right)$

i.e. $\quad \dfrac{Qd}{kA} = t_1 - t_2$

from which, $\quad \mathbf{t_2 = t_1 - \dfrac{Qd}{kA}}$

14. The slip, s, of a vehicle is given by: $s = \left(1 - \dfrac{r\omega}{v}\right) \times 100\%$ where r is the tyre radius, ω is the angular velocity and v the velocity. Transpose to make r the subject of the formula.

Since $\quad s = \left(1 - \dfrac{r\omega}{v}\right) \times 100\%$ then $\quad \dfrac{s}{100} = 1 - \dfrac{r\omega}{v}$

and $\quad \dfrac{r\omega}{v} = 1 - \dfrac{s}{100}$

from which, $\quad \mathbf{r = \dfrac{v}{\omega}\left(1 - \dfrac{s}{100}\right)}$

15. The critical load, F newtons, of a steel column may be determined from the formula

$L\sqrt{\dfrac{F}{EI}} = n\pi$ where L is the length, EI is the flexural rigidity, and n is a positive integer.

Transpose for F and hence determine the value of F when n = 1, $E = 0.25 \times 10^{12}\ N/m^2$, $I = 6.92 \times 10^{-6}\ m^4$ and L = 1.12 m

Since $L\sqrt{\dfrac{F}{EI}} = n\pi$ then $\sqrt{\dfrac{F}{EI}} = \dfrac{n\pi}{L}$

and $\dfrac{F}{EI} = \left(\dfrac{n\pi}{L}\right)^2$

i.e. $\mathbf{F = EI\left(\dfrac{n\pi}{L}\right)^2}$

When n = 1, $E = 0.25 \times 10^{12}\ N/m^2$, $I = 6.92 \times 10^{-6}\ m^4$ and L = 1.12 m,

$$\textbf{load, F} = EI\left(\frac{n\pi}{L}\right)^2 = \left(0.25\times 10^{12}\right)\left(6.92\times 10^{-6}\right)\left(\frac{1\times\pi}{1.12}\right)^2 = 13.61\ \times 10^6\ N = \mathbf{13.61\ MN}$$

16. The flow of slurry along a pipe on a coal processing plant is given by: $V = \dfrac{\pi p r^4}{8\eta\ell}$

Transpose the equation for r

Since $V = \dfrac{\pi p r^4}{8\eta\ell}$ then $8\eta\ell V = \pi p r^4$

and $\dfrac{8\eta\ell V}{\pi p} = r^4$

from which, $\mathbf{r = \sqrt[4]{\left(\dfrac{8\eta\ell V}{\pi p}\right)}}$

CHAPTER 8 SIMULTANEOUS EQUATIONS

EXERCISE 33, Page 66

1. Solve the simultaneous equations: $2x - y = 6$
$x + y = 6$

Numbering the equations gives: $2x - y = 6$ (1)

$x + y = 6$ (2)

Equation (1) + equation (2) gives: $3x = 12$

from which, $\mathbf{x} = \frac{12}{3} = \mathbf{4}$

Substituting x = 4 in equation (2) gives: $4 + y = 6$ from which, $\mathbf{y} = 6 - 4 = \mathbf{2}$

(Checking in equation (1): L.H.S. = 2(4) – 2 = 8 – 2 = 6 = R.H.S.)

2. Solve the simultaneous equations: $2x - y = 2$
$x - 3y = -9$

Numbering the equations gives: $2x - y = 2$ (1)

$x - 3y = -9$ (2)

3 × equation (1) gives: $6x - 3y = 6$ (3)

Equation (3) – equation (2) gives: $5x = 6 - -9 = 15$

from which, $\mathbf{x} = \frac{15}{5} = \mathbf{3}$

Substituting x = 3 in equation (1) gives: $6 - y = 2$ from which, $\mathbf{y} = 6 - 2 = \mathbf{4}$

(Checking in equation (2): L.H.S. = 3 – 3(4) = 3 – 12 = - 9 = R.H.S.)

3. Solve the simultaneous equations: $x - 4y = -4$
$5x - 2y = 7$

Numbering the equations gives: $x - 4y = -4$ (1)

$5x - 2y = 7$ (2)

$2 \times$ equation (2) gives: $10x - 4y = 14$ (3)

Equation (3) – equation (1) gives: $9x \quad = 14 - -4 = 18$

from which, $\mathbf{x} = \frac{18}{9} = \mathbf{2}$

Substituting $x = 2$ in equation (1) gives: $2 - 4y = -4$ from which, $2 + 4 = 4y$

from which, $4y = 6$ and $\mathbf{y} = \frac{6}{4} = \mathbf{1.5}$

(Checking in equation (2): L.H.S. = 5(2) – 2(1.5) = 10 – 3 = 7 = R.H.S.)

4. Solve the simultaneous equations: $3x - 2y = 10$

$5x + y = 21$

Numbering the equations gives: $3x - 2y = 10$ (1)

$5x + y = 21$ (2)

$2 \times$ equation (2) gives: $10x + 2y = 42$ (3)

Equation (1) + equation (3) gives: $13x \quad = 52$

from which, $\mathbf{x} = \frac{52}{13} = \mathbf{4}$

Substituting $x = 4$ in equation (1) gives: $12 - 2y = 10$ from which, $12 - 10 = 2y$

from which, $2y = 2$ and $\mathbf{y = 1}$

(Checking in equation (2): L.H.S. = 5(4) + 1 = 20 + 1 = 21 = R.H.S.)

5. Solve the simultaneous equations: $2x - 7y = -8$

$3x + 4y = 17$

Numbering the equations gives: $2x - 7y = -8$ (1)

$3x + 4y = 17$ (2)

3 × equation (1) gives: $6x - 21y = -24$ (3)

2 × equation (2) gives: $6x + 8y = 34$ (4)

Equation (4) - equation (3) gives: $8y - -21y = 34 - -24$ i.e. $29y = 58$

from which, $\mathbf{y} = \frac{58}{29} = \mathbf{2}$

Substituting $y = 2$ in equation (1) gives: $2x - 14 = -8$ from which, $2x = 14 - 8 = 6$

from which, $\mathbf{x} = \frac{6}{2} = \mathbf{3}$

(Checking in equation (2): L.H.S. = $3(3) + 4(2) = 9 + 8 = 17$ = R.H.S.)

6. Solve the simultaneous equations: $a + 2b = 8$

$b - 3a = -3$

Numbering the equations gives: $a + 2b = 8$ (1)

$-3a + b = -3$ (2)

2 × equation (2) gives: $-6a + 2b = -6$ (3)

Equation (1) - equation (3) gives: $a - -6a = 8 - -6$ i.e. $7a = 14$

from which, $\mathbf{a} = \frac{14}{7} = \mathbf{2}$

Substituting $a = 2$ in equation (1) gives: $2 + 2b = 8$ from which, $2b = 8 - 2 = 6$

from which, $\mathbf{b} = \frac{6}{2} = \mathbf{3}$

(Checking in equation (2): L.H.S. = $-3(2) + 3 = -6 + 3 = -3$ = R.H.S.)

7. Solve the simultaneous equations: $a + b = 7$

$a - b = 3$

Numbering the equations gives: $a + b = 7$ (1)

$a - b = 3$ (2)

Equation (1) + equation (2) gives: $2a = 10$

from which, $\mathbf{a} = \frac{10}{2} = \mathbf{5}$

Substituting $a = 5$ in equation (1) gives: $5 + b = 7$ from which, $\mathbf{b} = 7 – 5 = \mathbf{2}$

(Checking in equation (2): L.H.S. = 5 - 2 = 3 = R.H.S.)

8. Solve the simultaneous equations: $2x + 5y = 7$
$x + 3y = 4$

$2x + 5y = 7$ (1)

$x + 3y = 4$ (2)

$2 \times$ equation (2) gives: $2x + 6y = 8$ (3)

(3) – (1) gives: $\mathbf{y} = \mathbf{1}$

Substituting in (1) gives: $2x + 5 = 7$ i.e. $2x = 7 – 5 = 2$ and $\mathbf{x} = \frac{2}{2} = \mathbf{1}$

Thus, **x = 1** and **y = 1** and may be checked by substituting into both of the original equations

9. Solve the simultaneous equations: $3s + 2t = 12$
$4s - t = 5$

$3s + 2t = 12$ (1)

$4s - t = 5$ (2)

$2 \times$ equation (2) gives: $8s – 2t = 10$ (3)

(1) + (3) gives: $11s = 22$ from which, $\mathbf{s} = \frac{22}{11} = \mathbf{2}$

Substituting in (1) gives: $6 + 2t = 12$ i.e. $2t = 12 – 6 = 6$ and $\mathbf{t} = \frac{6}{2} = \mathbf{3}$

Thus, **s = 2** and **t = 3** and may be checked by substituting into both of the original equations

10. Solve the simultaneous equations: $3x - 2y = 13$
$2x + 5y = - 4$

$3x - 2y = 13$ (1)

$2x + 5y = -4$ (2)

$2 \times$ equation (1) gives: $6x - 4y = 26$ (3)

$3 \times$ equation (2) gives: $6x + 15y = -12$ (4)

(3) - (4) gives: $-19y = 38$ from which, $\mathbf{y} = \frac{38}{-19} = \mathbf{-2}$

Substituting in (1) gives: $3x + 4 = 13$ i.e. $3x = 13 - 4 = 9$ and $\mathbf{x} = \frac{9}{3} = \mathbf{3}$

Thus, **x = 3** and **y = - 2** and may be checked by substituting into both of the original equations

11. Solve the simultaneous equations: $5m - 3n = 11$
$3m + n = 8$

$5m - 3n = 11$ (1)

$3m + n = 8$ (2)

$3 \times$ equation (2) gives: $9m + 3n = 24$ (3)

(1) + (3) gives: $14m = 35$ from which, $\mathbf{m} = \frac{35}{14} = \mathbf{2.5}$

Substituting in (1) gives: $12.5 - 3n = 11$ i.e. $12.5 - 11 = 3n$ i.e. $3n = 1.5$

from which, $\mathbf{n} = \frac{1.5}{3} = \mathbf{0.5}$

Thus, **m = 2.5** and **n = 0.5** and may be checked by substituting into both of the original equations

12. Solve the simultaneous equations: $8a - 3b = 51$
$3a + 4b = 14$

$$8a - 3b = 51 \quad (1)$$

$$3a + 4b = 14 \quad (2)$$

$4 \times$ equation (1) gives: $32a - 12b = 204 \quad (3)$

$3 \times$ equation (2) gives: $9a + 12b = 42 \quad (4)$

(3) + (4) gives: $41a = 246$ from which, $\mathbf{a} = \frac{246}{41} = \mathbf{6}$

Substituting in (1) gives: $48 - 3b = 51$ i.e. $48 - 51 = 3b$ and $\mathbf{b} = \frac{-3}{3} = \mathbf{-1}$

Thus, **a = 6** and **b = - 1** and may be checked by substituting into both of the original equations

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EXERCISE 34, Page 68

1. Solve the simultaneous equations: $7p + 11 + 2q = 0$

$$-1 = 3q - 5p$$

Rearranging gives: $7p + 2q = -11$ (1)

$5p - 3q = 1$ (2)

$3 \times$ equation (1) gives: $21p + 6q = -33$ (3)

$2 \times$ equation (2) gives: $10p - 6q = 2$ (4)

(3) + (4) gives: $31p = -31$ from which, $p = -1$

Substituting in (1) gives: $-7 + 2q = -11$ i.e. $2q = -11 + 7 = -4$ and $q = \frac{-4}{2} = -2$

Thus, $\mathbf{p = -1}$ and $\mathbf{q = -2}$ and may be checked by substituting into both of the original equations.

2. Solve the simultaneous equations: $\frac{x}{2} + \frac{y}{3} = 4$

$$\frac{x}{6} - \frac{y}{9} = 0$$

Rearranging gives: $(6)\frac{x}{2} + (6)\frac{y}{3} = (6)(4)$ i.e. $3x + 2y = 24$ (1)

and $(18)\frac{x}{6} - (18)\frac{y}{9} = (18)(0)$ i.e. $3x - 2y = 0$ (2)

(1) - (2) gives: $4y = 24$ from which, $y = 6$

Substituting in (1) gives: $3x + 12 = 24$ i.e. $3x = 24 - 12 = 12$ and $x = \frac{12}{3} = 4$

Thus, $\mathbf{x = 4}$ and $\mathbf{y = 6}$ and may be checked by substituting into both of the original equations.

3. Solve the simultaneous equations: $\frac{a}{2} - 7 = -2b$

$$12 = 5a + \frac{2}{3}b$$

Rearranging gives: $(2)\frac{a}{2}-(2)7=-(2)(2b)$ i.e. a + 4b = 14 (1)

and $(3)(12) = (3)(5a) + (3)\frac{2}{3}b$ i.e. 15a + 2b = 36 (2)

2 × equation (2) gives: 30a + 4b = 72 (3)

(3) - (1) gives: 29a = 58 from which, a = 2

Substituting in (1) gives: 2 + 4b = 14 i.e. 4b = 14 - 2 = 12 and $b = \frac{12}{4} = 3$

Thus, **a = 2** and **b = 3** and may be checked by substituting into both of the original equations.

4. Solve the simultaneous equations: $\frac{3}{2}s - 2t = 8$

$\frac{s}{4} + 3t = -2$

Rearranging gives: $(2)\frac{3}{2}s-(2)(2t)=(2)(8)$ i.e. 3s – 4t = 16 (1)

and $(4)\ \frac{s}{4} + (4)(3t) = (4)(-2)$ i.e. s + 12t = - 8 (2)

3 × equation (2) gives: 3s + 36t = - 24 (3)

(1) - (3) gives: - 40t = 40 from which, **t = - 1**

Substituting in (1) gives: 1.5s + 2 = 8 i.e. 1.5s = 8 - 2 and $\mathbf{s} = \frac{6}{1.5} = \mathbf{4}$

Thus, **s = 4** and **t = - 1** and may be checked by substituting into both of the original equations.

5. Solve the simultaneous equations: $\frac{x}{5} + \frac{2y}{3} = \frac{49}{15}$

$\frac{3x}{7} - \frac{y}{2} + \frac{5}{7} = 0$

Rearranging gives: $(15)\frac{x}{5}+(15)\frac{2y}{3}=(15)\frac{49}{15}$ i.e. $3x + 10y = 49$ (1)

and $(14)\frac{3x}{7}-(14)\frac{y}{2}+(14)\frac{5}{7}=0$ i.e. $6x - 7y = -10$ (2)

$2 \times$ equation (1) gives: $6x + 20y = 98$ (3)

(3) - (2) gives: $27y = 108$ from which, $y = \frac{108}{27} = 4$

Substituting in (1) gives: $3x + 40 = 49$ i.e. $3x = 49 - 40 = 9$ and $x = \frac{9}{3} = 3$

Thus, **x = 3** and **y = 4** and may be checked by substituting into both of the original equations.

6. Solve the simultaneous equations: $v - 1 = \frac{u}{12}$

$$u + \frac{v}{4} - \frac{25}{2} = 0$$

Rearranging gives: $(12)v - (12)(1) = (12)\frac{u}{12}$ i.e. $-u + 12v = 12$ (1)

and $(4)u + (4)\frac{v}{4} - (4)\frac{25}{2}$ i.e. $4u + v = 50$ (2)

$4 \times$ equation (1) gives: $-4u + 48v = 48$ (3)

(2) + (3) gives: $49v = 98$ from which, $\mathbf{v} = \frac{98}{49} = \mathbf{2}$

Substituting in (1) gives: $2 - 1 = \frac{u}{12}$ i.e. $1 = \frac{u}{12}$ and **u = 12**

Thus, **u = 12** and **v = 2** and may be checked by substituting into both of the original equations.

EXERCISE 35, Page 69

1. In a system of pulleys, the effort P required to raise a load W is given by P = aW + b, where a and b are constants. If W = 40 when P = 12 and W = 90 when P = 22, find the values of a and b.

P = aW + b, hence if W = 40 when P = 12, then: $12 = 40a + b$ (1)

and if W = 90 when P = 22, then: $22 = 90a + b$ (2)

Equation (2) – equation (1) gives: $10 = 50a$

from which, $a = \frac{10}{50} = \frac{1}{5}$

Substituting in (1) gives: $12 = 40\left(\frac{1}{5}\right) + b$ i.e. $12 = 8 + b$

from which, $b = 4$

Thus, $\mathbf{a = \frac{1}{5}}$ or **0.2** and **b = 4** and may be checked by substituting into both of the original equations.

2. Applying Kirchhoff's laws to an electrical circuit produces the following equations:

$$5 = 0.2I_1 + 2(I_1 - I_2)$$

$$12 = 3I_2 + 0.4I_2 - 2(I_1 - I_2)$$

Determine the values of currents I_1 and I_2

Rearranging $5 = 0.2I_1 + 2(I_1 - I_2)$ gives: $5 = 0.2I_1 + 2I_1 - 2I_2$ i.e. $2.2I_1 - 2I_2 = 5$

Rearranging $12 = 3I_2 + 0.4I_2 - 2(I_1 - I_2)$ gives: $12 = 3I_2 + 0.4I_2 - 2I_1 + 2I_2$ i.e. $-2I_1 + 5.4I_2 = 12$

Thus, $2.2I_1 - 2I_2 = 5$ (1)

and $-2I_1 + 5.4I_2 = 12$ (2)

2 × equation (1) gives: $4.4I_1 - 4I_2 = 10$ (3)

2.2 × equation (2) gives: $-4.4I_1 + 11.88I_2 = 26.4$ (4)

(3) + (4) gives: $7.88 I_2 = 36.4$ from which, $I_2 = \frac{36.4}{7.88} = 4.62$

Substituting in (1) gives: $2.2 I_1 - 9.24 = 5$ i.e. $2.2 I_1 = 14.24$ and $I_1 = \frac{14.24}{2.2} = 6.47$

Thus, $\mathbf{I_1 = 6.47}$ and $\mathbf{I_2 = 4.62}$ and may be checked by substituting into both of the original equations.

3. Velocity v is given by the formula v = u + at. If v = 20 when t = 2 and v = 40 when t = 7 find the values of u and a. Hence find the velocity when t = 3.5

v = u + at, hence if v = 20 when t = 2, then: 20 = u + 2a (1)

and if v = 40 when t = 7, then: 40 = u + 7a (2)

Equation (2) – equation (1) gives: 20 = 5a

from which, $\mathbf{a} = \frac{20}{5} = \mathbf{4}$

Substituting in (1) gives: 20 = u + 8 from which, **u** = **12**

Thus, **a = 4** and **u = 12** and may be checked by substituting into both of the original equations.

When **t = 3.5**, **velocity, v** = u + at = 12 + (4)(3.5) = **26**

4. y = mx + c is the equation of a straight line of slope m and y-axis intercept c. If the line passes through the point where x = 2 and y = 2, and also through the point where x = 5 and y = 0.5, find the slope and y-axis intercept of the straight line.

y = mx + c, hence if x = 2 when y = 2, then: 2 = 2m + c (1)

and if x = 5 when y = 0.5, then: 0.5 = 5m + c (2)

Equation (2) – equation (1) gives: - 1.5 = 3m

from which, $m = \frac{-1.5}{3} = -\frac{1}{2} = -0.5$

Substituting in (1) gives: $2 = 2(-0.5) + c$ i.e. 2 = - 1 + c from which, $c = 3$

Thus, $\mathbf{m = -0.5}$ and $\mathbf{c = 3}$ which may be checked by substituting into both of the original equations.

5. The molar heat capacity of a solid compound is given by the equation $c = a + bT$. When $c = 52$, $T = 100$ and when $c = 172$, $T = 400$. Find the values of a and b.

$c = a + bT$, hence if $c = 52$ when $T = 100$, then: $52 = a + 100b$ (1)

and if $c = 172$ when $T = 400$, then: $172 = a + 400b$ (2)

Equation (2) – equation (1) gives: $120 = 300b$

from which, $\mathbf{b} = \dfrac{120}{300} = \mathbf{0.40}$

Substituting in (1) gives: $52 = a + 40$ from which, $\mathbf{a = 12}$

Thus, $\mathbf{a = 12}$ and $\mathbf{b = 0.40}$ and may be checked by substituting into both of the original equations.

6. In a system of forces, the relationship between two forces F_1 and F_2 is given by:

$$5F_1 + 3F_2 + 6 = 0$$
$$3F_1 + 5F_2 + 18 = 0$$

Solve for F_1 and F_2

Rearranging gives: $5F_1 + 3F_2 = -6$ (1)

$3F_1 + 5F_2 = -18$ (2)

$3 \times$ equation (1) gives: $15F_1 + 9F_2 = -18$ (3)

$5 \times$ equation (2) gives: $15F_1 + 25F_2 = -90$ (4)

(3) - (4) gives: $-16F_2 = -18 - -90 = 72$ from which, $\mathbf{F_2} = \dfrac{72}{-16} = \mathbf{-4.5}$

Substituting in (1) gives: $5F_1 - 13.5 = -6$ i.e. $5F_1 = 13.5 - 6 = 7.5$ and $\mathbf{F_1} = \dfrac{7.5}{5} = \mathbf{1.5}$

Thus, $\mathbf{F_1 = 1.5}$ and $\mathbf{F_2 = -4.5}$ and may be checked by substituting into both of the original equations.

7. For a balanced beam, the equilibrium of forces is given by: $R_1 + R_2 = 12.0\,\text{kN}$

As a result of taking moments: $0.2R_1 + 7\times0.3 + 3\times0.6 = 0.8R_2$

Determine the values of the reaction forces R_1 and R_2

Rearranging gives: $R_1 + R_2 = 12.0$ (1)

$0.2R_1 - 0.8R_2 = -3.9$ (2)

5 × (2) gives: $R_1 - 4.0R_2 = -19.5$ (3)

(1) – (3) gives: $5.0\,R_2 = 31.5$

from which, $\mathbf{R_2} = \dfrac{31.5}{5} = \mathbf{6.3\ kN}$

Substituting in (1) gives: $R_1 + 6.3 = 12.0$

Hence, $\mathbf{R_1} = 12.0 - 6.3 = \mathbf{5.7\ kN}$

CHAPTER 9 STRAIGHT LINE GRAPHS

EXERCISE 36, Page 75

1. Assuming graph paper measuring 20 cm by 20 cm is available, suggest suitable scales for the following ranges of values:

(a) Horizontal axis: 3 V to 55 V   Vertical axis: 10 Ω to 180 Ω

(b) Horizontal axis: 7 m to 86 m   Vertical axis: 0.3 V to 1.69 V

(c) Horizontal axis: 5 N to 150 N   Vertical axis: 0.6 mm to 3.4 mm

(a) Horizontal scale: 55 – 3 = 52V; 52 ÷ 20 ≈ 2.5 V

Hence, **1 cm = 4 V** (or even **1 cm = 5 V**) would be the best scale to use

Vertical scale: 180 - 10 = 170 Ω; 170 ÷ 20 = 8.5 Ω

Hence, **1 cm = 10 Ω** would be the best scale to use

(b) Horizontal scale: 86 – 7 = 79 m; 79 ÷ 20 ≈ 4 m

Hence, **1 cm = 5 m** would be the best scale to use

Vertical scale: 1.69 – 0.3 = 1.66 V; 1.66 ÷ 20 ≈ 0.08 V

Hence, **1 cm = 0.1 V** would be the best scale to use

(c) Horizontal scale: 150 – 5 = 145 N; 145 ÷ 20 ≈ 7 N

Hence, **1 cm = 10 N** would be the best scale to use

Vertical scale: 3.4 – 0.6 = 2.8 mm; 2.8 ÷ 20 ≈ 0.14 mm

Hence, **1 cm = 0.2 mm** would be the best scale to use

2. Corresponding values obtained experimentally for two quantities are:

x	- 5	- 3	- 1	0	2	4
y	- 13	- 9	- 5	- 3	1	5

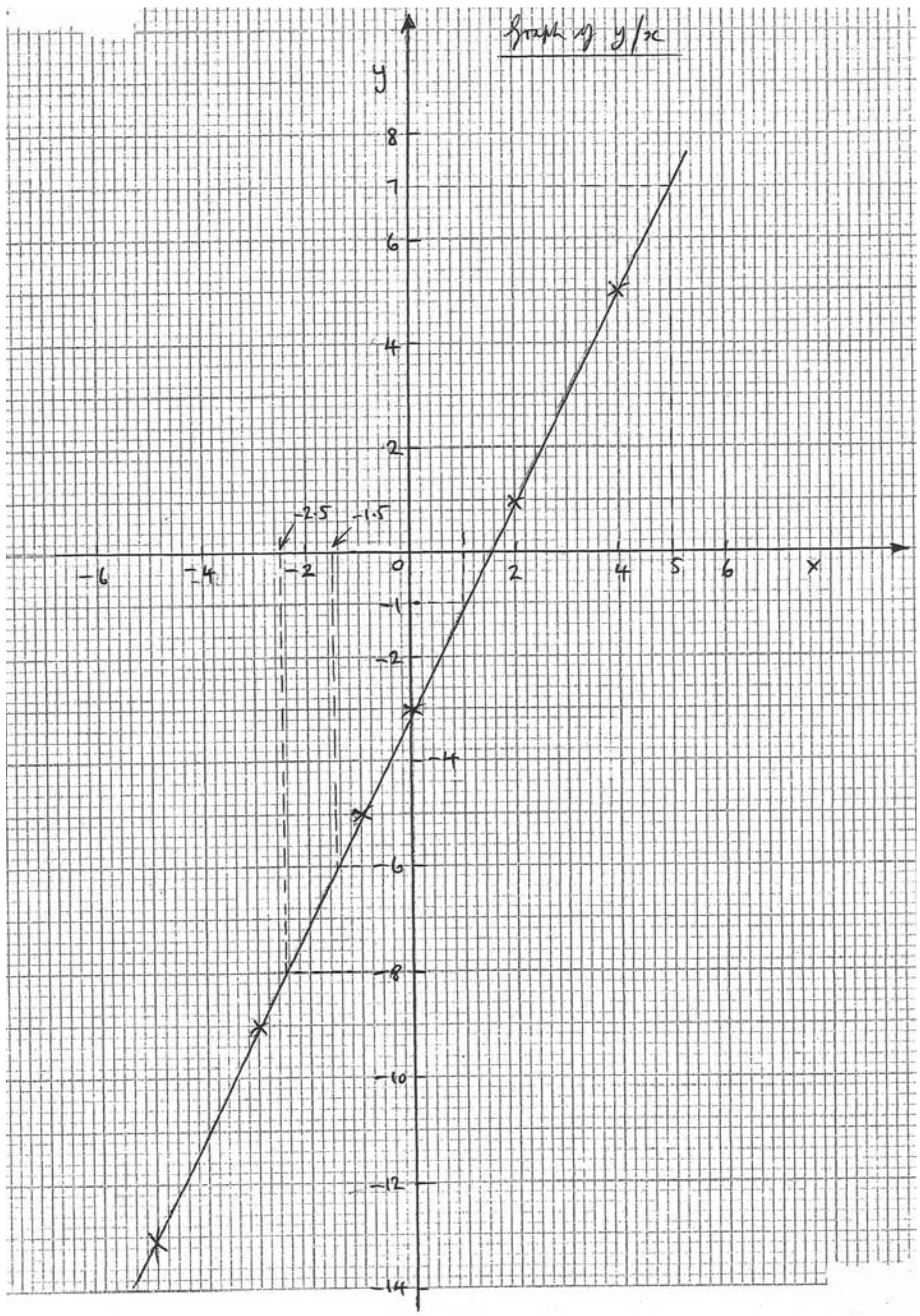
Plot a graph of y (vertically) against x (horizontally) to scales of 2 cm = 1 for the horizontal x-axis and 1 cm = 1 for the vertical y-axis. (This graph will need the whole of the graph paper with the origin somewhere in the centre of the paper).

From the graph find: (a) the value of y when x = 1

(b) the value of y when x = - 2.5

(c) the value of x when y = - 6

(d) the value of x when y = 7



From the above graph:

(a) When x = 1, **y = - 1**

(b) When x = - 2.5, **y = - 8**

(c) When y = - 6, **x = - 1.5**

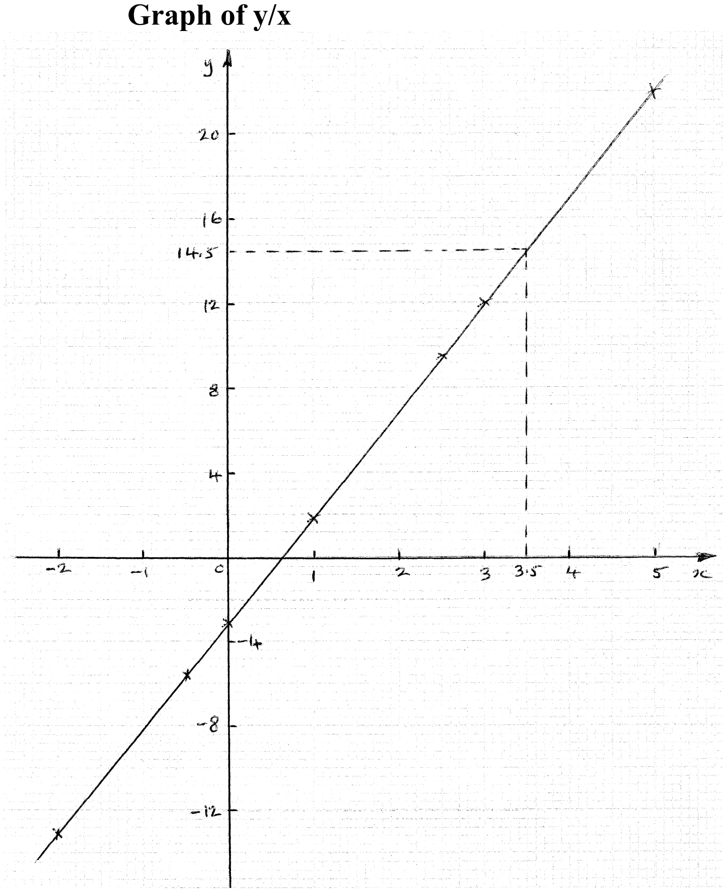
(d) When y = 7, **x = 5**

3. Corresponding values obtained experimentally for two quantities are:

x	- 2.0	- 0.5	0	1.0	2.5	3.0	5.0
y	- 13.0	- 5.5	- 3.0	2.0	9.5	12.0	22.0

Use a horizontal scale for x of 1 cm = $\frac{1}{2}$ unit and a vertical scale for y of 1 cm = 2 units and draw a graph of x against y. Label the graph and each of its axes. By interpolation, find from the graph the value of y when x is 3.5

Graph of y/x



The graph of y against x is shown plotted above.

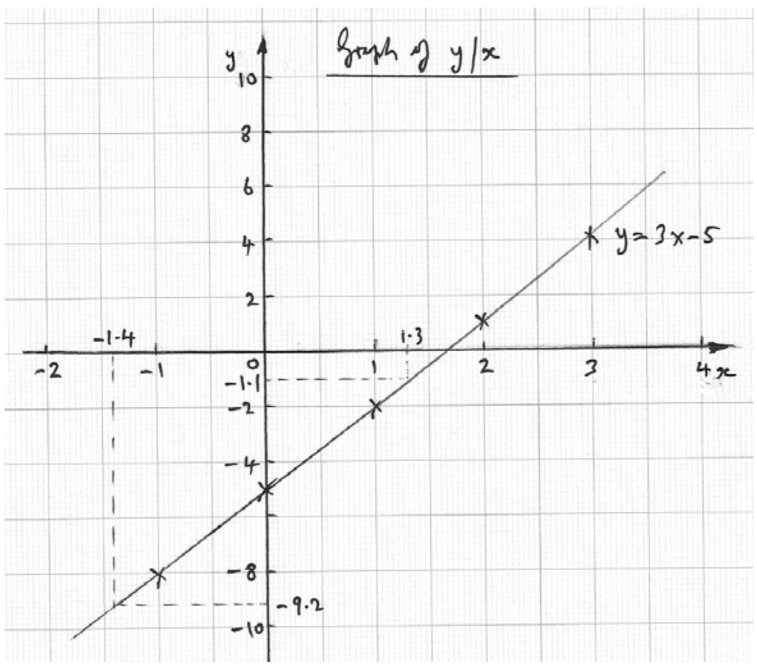
From the graph, when x = 3.5, **y = 14.5**

4. Draw a graph of y - 3x + 5 = 0 over a range of x = - 2 to x = 4. Hence determine (a) the value of y when x = 1.3 and (b) the value of x when y = - 9.2

y – 3x + 5 = 0 i.e. y = 3x – 5

x	0	1	2
y	- 5	- 2	1

A graph of y = 3x – 5 is shown below.



(a) When x = 1.3, **y = - 1.1**

(b) When y = - 9.2, **x = - 1.4**

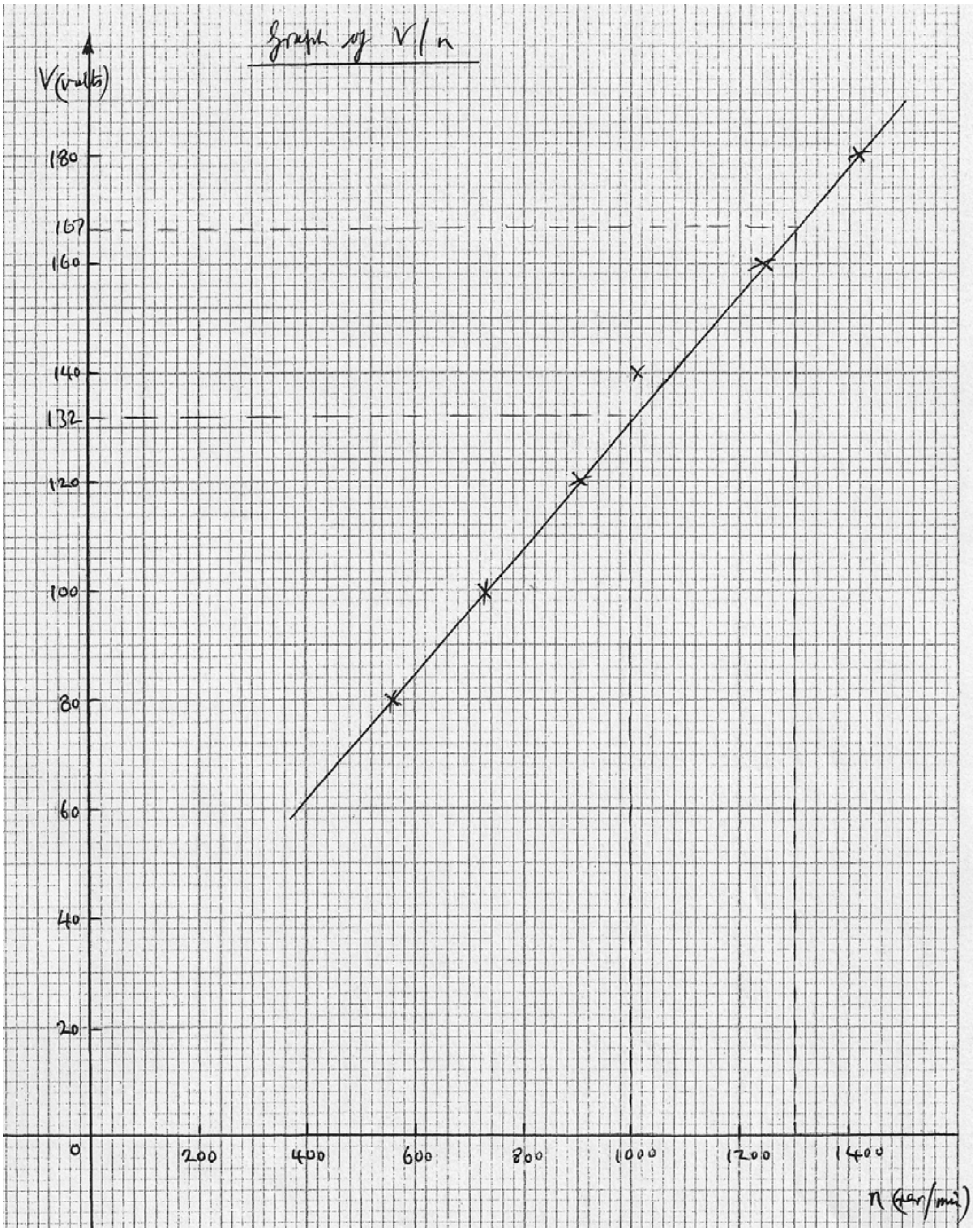
5. The speed n rev/min of a motor changes when the voltage V across the armature is varied. The results are shown in the following table:

n (rev/min)	560	720	900	1010	1240	1410
V (volts)	80	100	120	140	160	180

It is suspected that one of the readings taken of the speed is inaccurate. Plot a graph of speed (horizontally) against voltage (vertically) and find this value. Find also (a) the speed at a voltage of 132 V, and (b) the voltage at a speed of 1300 rev/min.

A graph of V/n is shown below.

The 1010 rev/min reading should be closer to **1070 rev/min**.



(a) When the voltage is 132 V, the speed is **1000 rev/min**

(b) When the speed is 1300 rev/min, the voltage is **167 V**

EXERCISE 37, Page 79

1. The equation of a line is 4y = 2x + 5. A table of corresponding values is produced and is shown below. Complete the table and plot a graph of y against x. Find the gradient of the graph.

x	- 4	- 3	- 2	- 1	0	1	2	3	4
y		- 0.25			1.25				3.25

$4y = 2x + 5$ from which, $y = \frac{2}{4}x + \frac{5}{4}$ i.e. $y = \frac{1}{2}x + \frac{5}{4}$

Hence, when **x = - 4**, $y = \frac{1}{2}(-4) + \frac{5}{4} = -2 + 1.25 = \mathbf{-0.75}$

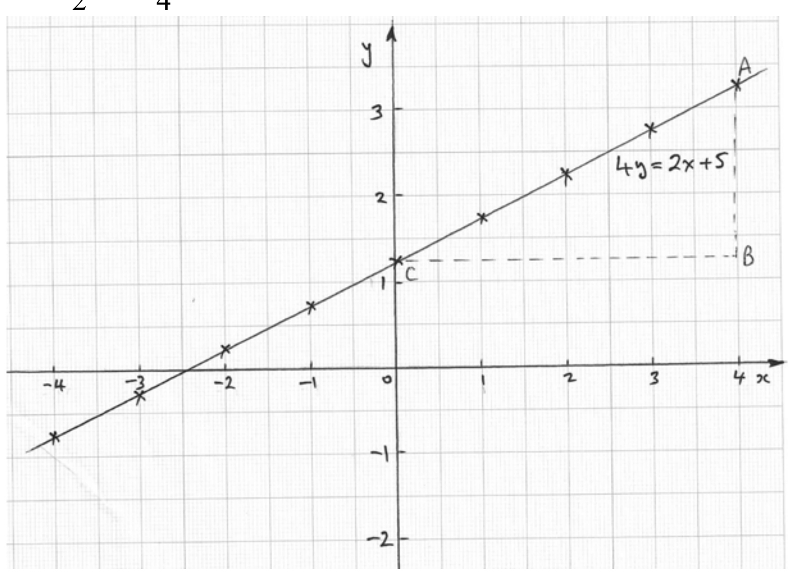
when **x = - 2**, $y = \frac{1}{2}(-2) + \frac{5}{4} = -1 + 1.25 = \mathbf{0.25}$

when **x = - 1**, $y = \frac{1}{2}(-1) + \frac{5}{4} = -0.5 + 1.25 = \mathbf{0.75}$

when **x = 1**, $y = \frac{1}{2}(1) + \frac{5}{4} = 0.5 + 1.25 = \mathbf{1.75}$

when **x = 2**, $y = \frac{1}{2}(2) + \frac{5}{4} = 1 + 1.25 = \mathbf{2.25}$

when **x = 3**, $y = \frac{1}{2}(3) + \frac{5}{4} = 1.5 + 1.25 = \mathbf{2.75}$



A graph of $y = \frac{1}{2}x + \frac{5}{4}$ is shown above.

Gradient of graph $= \frac{AB}{BC} = \frac{3.25 - 1.25}{4 - 0} = \frac{2}{4} = \mathbf{\frac{1}{2}}$

2. Determine the gradient and intercept on the y-axis for each of the following equations:

(a) $y = 4x - 2$ (b) $y = -x$ (c) $y = -3x - 4$ (d) $y = 4$

(a) Since $y = 4x - 2$, then **gradient = 4** and **y-axis intercept = - 2**

(b) Since $y = -x$, then **gradient = - 1** and **y-axis intercept = 0**

(c) Since $y = -3x - 4$, then **gradient = - 3** and **y-axis intercept = - 4**

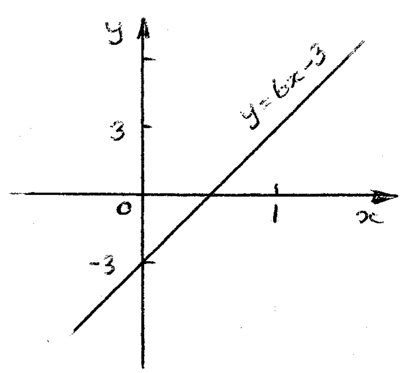
(d) Since $y = 4$ i.e. $y = 0x + 4$, then **gradient = 0** and **y-axis intercept = 4**

3. Determine the gradient and y-axis intercept for each of the following equations. Sketch the graphs.

(a) $y = 6x - 3$ (b) $y = -2x + 4$ (c) $y = 3x$ (d) $y = 7$

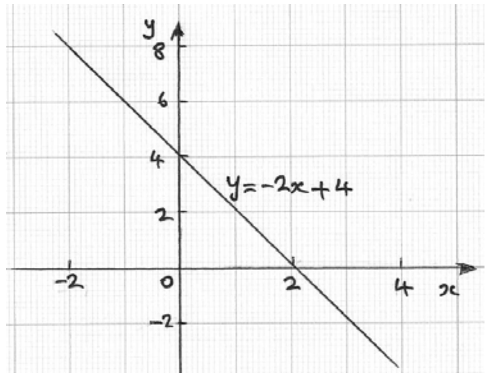
(a) Since $y = 6x - 3$, then **gradient = 6** and **y-axis intercept = - 3**

A sketch of $y = 6x - 3$ is shown below.



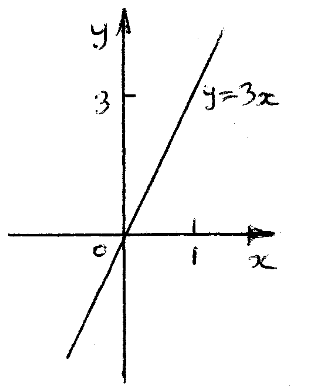
(b) Since $y = -2x + 4$, then **gradient = - 2** and **y-axis intercept = 4**

A sketch of $y = -2x + 4$ is shown below.



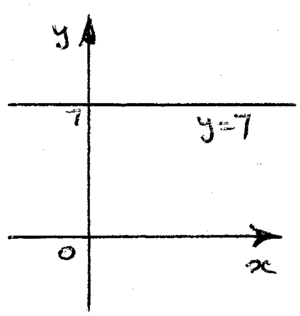
(c) Since y = 3x, then **gradient = 3** and **y-axis intercept = 0**

A sketch of y = 3x is shown below.



(d) Since y = 7, then **gradient = 0** and **y-axis intercept = 7**

A sketch of y = 7 is shown below.



4. Determine the gradient of the straight line graphs passing through the co-ordinates:

(a) (2, 7) and (- 3, 4) (b) (- 4, - 1) and (- 5, 3) (c) $\left(\frac{1}{4},-\frac{3}{4}\right)$ and $\left(-\frac{1}{2},\frac{5}{8}\right)$

(a) From page 72 of textbook, gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-7}{-3-2} = \frac{-3}{-5} = \mathbf{\frac{3}{5}}$

(b) Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3--1}{-5--4} = \frac{4}{-1} = \mathbf{-4}$

(c) Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{5}{8} - -\frac{3}{4}}{-\frac{1}{2} - \frac{1}{4}} = \frac{\frac{11}{8}}{-\frac{3}{4}} = -\frac{11}{8} \times \frac{4}{3} = -\frac{44}{24} = -\frac{11}{6} = \mathbf{-1\frac{5}{6}}$

5. State which of the following equations will produce graphs which are parallel to one another:

(a) $y - 4 = 2x$ (b) $4x = -(y + 1)$ (c) $x = \frac{1}{2}(y + 5)$

(d) $1 + \frac{1}{2}y = \frac{3}{2}x$ (e) $2x = \frac{1}{2}(7 - y)$

(a) Since $y - 4 = 2x$ then $\mathbf{y = 2x + 4}$

(b) Since $4x = -(y + 1)$ then $\mathbf{y = -4x - 1}$

(c) Since $x = \frac{1}{2}(y+5)$ then $2x = y + 5$ and $\mathbf{y = 2x - 5}$

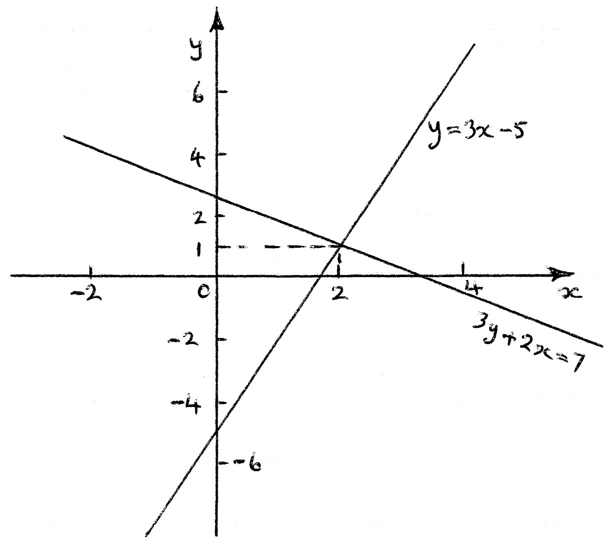
(d) Since $1 + \frac{1}{2}y = \frac{3}{2}x$ then $2 + y = 3x$ and $\mathbf{y = 3x - 2}$

(e) Since $2x = \frac{1}{2}(7 - y)$ then $4x = 7 - y$ and $\mathbf{y = -4x + 7}$

Thus, **(a) and (c) are parallel** (since their gradients are the same)**, and (b) and (e) are parallel.**

6. Draw on the same axes the graphs of $y = 3x - 5$ and $3y + 2x = 7$. Find the co-ordinates of the point of intersection. Check the result obtained by solving the two simultaneous equations algebraically.

The graphs of $y = 3x - 5$ and $3y + 2x = 7$, i.e. $y = -\frac{2}{3}x + \frac{7}{3}$ are shown below.



The two graphs intersect at x = 2 and y = 1, i.e. the co-ordinate (2, 1)

Solving simultaneously gives:

$y = 3x - 5$ i.e. $y - 3x = -5$ (1)

$y = -\frac{2}{3}x + \frac{7}{3}$ i.e. $3y + 2x = 7$ (2)

$3 \times (1)$ gives: $3y - 9x = -15$ (3)

(2) – (3) gives: $11x = 22$ from which, $\mathbf{x = 2}$

Substituting in (1) gives: $y - 6 = -5$ from which, $\mathbf{y = 1}$ as obtained graphically above.

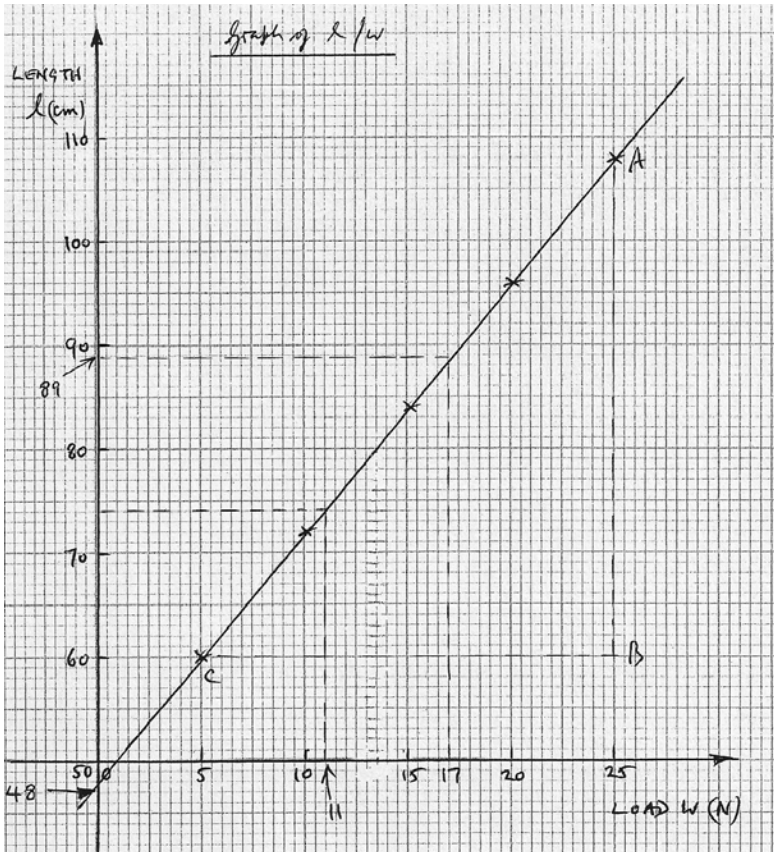
7. A piece of elastic is tied to a support so that it hangs vertically, and a pan, on which weights can be placed, is attached to the free end. The length of the elastic is measured as various weights are added to the pan and the results obtained are as follows:

Load, W (N)	5	10	15	20	25
Length, l (cm)	60	72	84	96	108

Plot a graph of load (horizontally) against length (vertically) and determine:

(a) the value length when the load is 17 N, (b) the value of load when the length is 74 cm,

(c) its gradient, and (d) the equation of the graph.



From the graph:

(a) When the load is 17 N, the **length = 89 cm**

(b) When the length is 74 cm, the **load = 11 N**

(c) **Gradient of graph** = $\frac{AB}{BC} = \frac{108-60}{25-5} = \frac{48}{20} = \mathbf{2.4}$

(d) The vertical axis intercept = 48, the equation of the graph is: **l = 2.4W + 48**

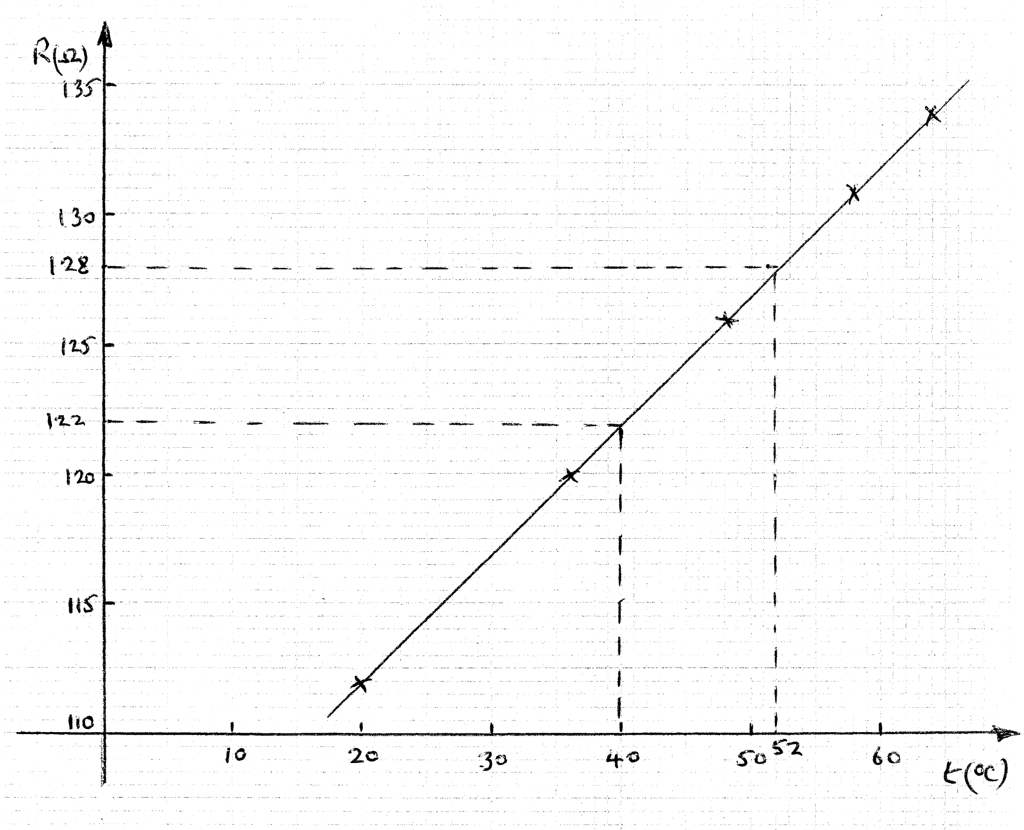
EXERCISE 38, Page 83

1. The resistance R ohms of a copper winding is measured at various temperatures t°C and the results are as follows:

R ohms	112	120	126	131	134
t°C	20	36	48	58	64

Plot a graph of R (vertically) against t (horizontally) and find from it (a) the temperature when the resistance is 122 Ω and (b) the resistance when the temperature is 52°C

A graph of resistance R against temperature t is shown below.



From the graph:

(a) the temperature when the resistance is 122 Ω is **40°C**

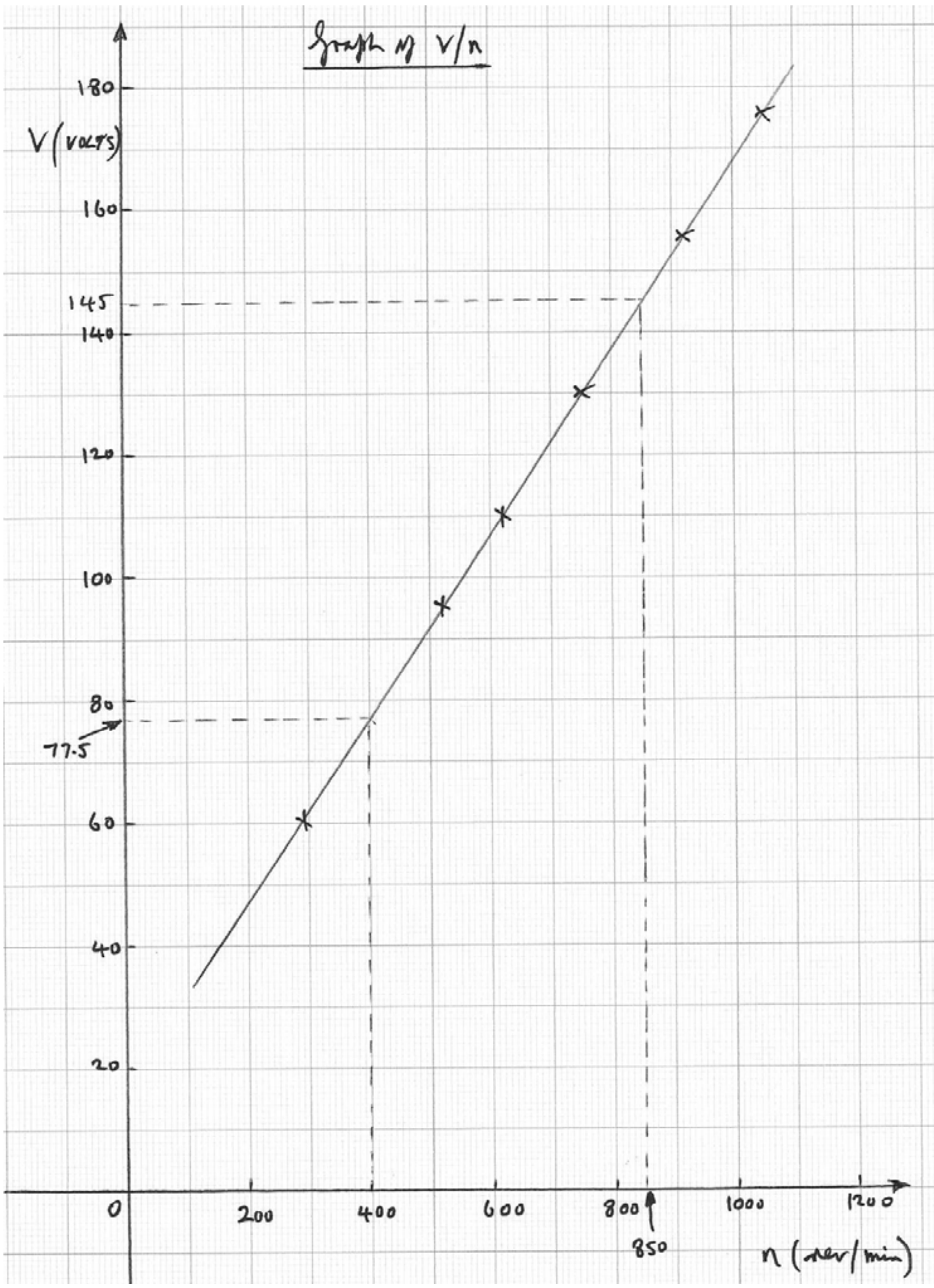
(b) the resistance when the temperature is 52°C is **128 Ω**

2. The speed of a motor varies with armature voltage as shown by the following experimental results:

n (rev/min)	285	517	615	750	917	1050
V volts	60	95	110	130	155	175

Plot a graph of speed (horizontally) against voltage (vertically) and draw the best straight line through the points. Find from the graph (a) the speed at a voltage of 145 V, and (b) the voltage at a speed of 400 rev/min.

A graph of V/n is shown below.



(a) At a voltage of 145 V, the **speed is 850 rev/min**

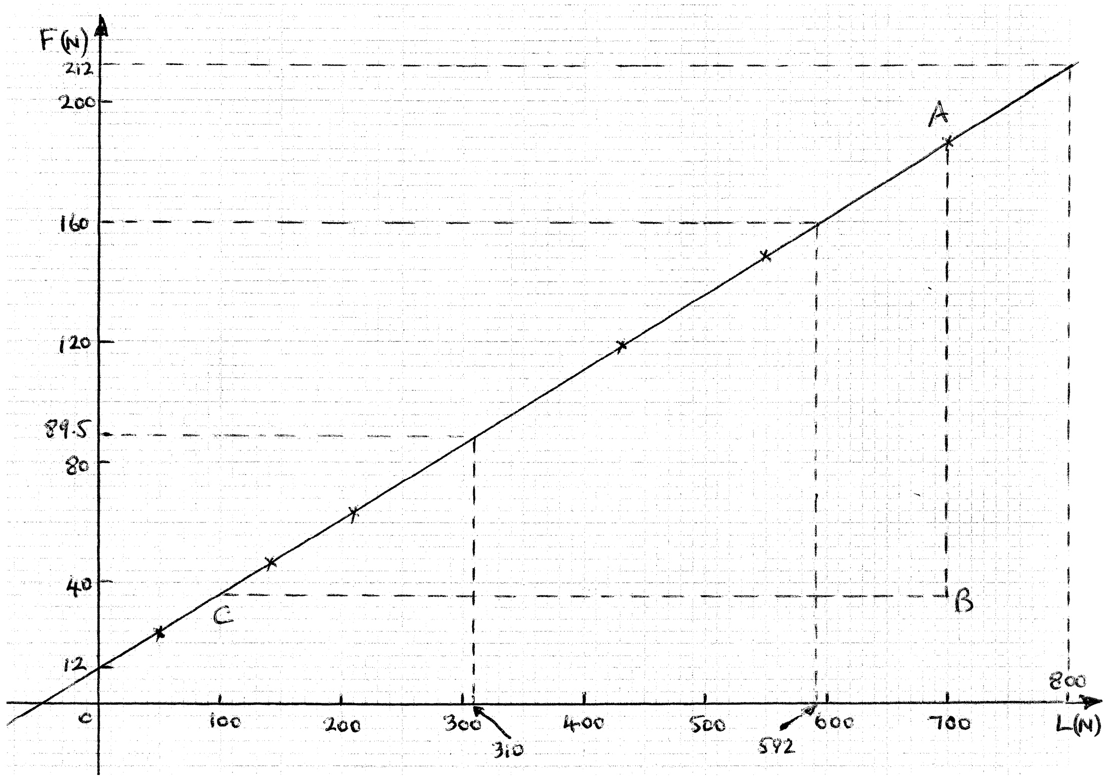
(b) At a speed of 400 rev/min, the **voltage is 77.5 V**

3. The following table gives the force F Newtons which, when applied to a lifting machine, overcomes a corresponding load of L Newtons.

Force F Newtons	25	47	64	120	149	187
Load L Newtons	50	140	210	430	550	700

Choose suitable scales and plot a graph of F (vertically) against L (horizontally). Draw the best straight line through the points. Determine from the graph (a) the gradient, (b) the F-axis intercept, (c) the equation of the graph, (d) the force applied when the load is 310 N, and (e) the load that a force of 160 N will overcome. (f) If the graph were to continue in the same manner, what value of force will be needed to overcome a 800 N load?

A graph of F against L is shown below.



From the graph:

(a) the **gradient** = $\frac{AB}{BC} = \frac{187-37}{700-100} = \frac{150}{600}$ = **0.25**

(b) the F-axis intercept = **12 N**

(c) the equation of the graph is: **F = 0.25L + 12**

(d) the force applied when the load is 310 N is **89.5 N**

(e) the load that a force of 160 N will overcome is **592 N**

(f) If the graph were to continue in the same manner the force needed to overcome a 800 N load is

212 N. From the equation of the graph, F = 0.25L + 12 = 0.25(800) + 12 = 200 + 12 = **212 N**

4. The velocity v of a body after varying time intervals t was measured as follows:

t (seconds)	2	5	8	11	15	18
v (m/s)	16.9	19.0	21.1	23.2	26.0	28.1

Plot v vertically and t horizontally and draw a graph of velocity against time. Determine from the graph (a) the velocity after 10 s, (b) the time at 20 m/s and (c) the equation of the graph.

A graph of velocity v against time t is shown below.

From the graph:

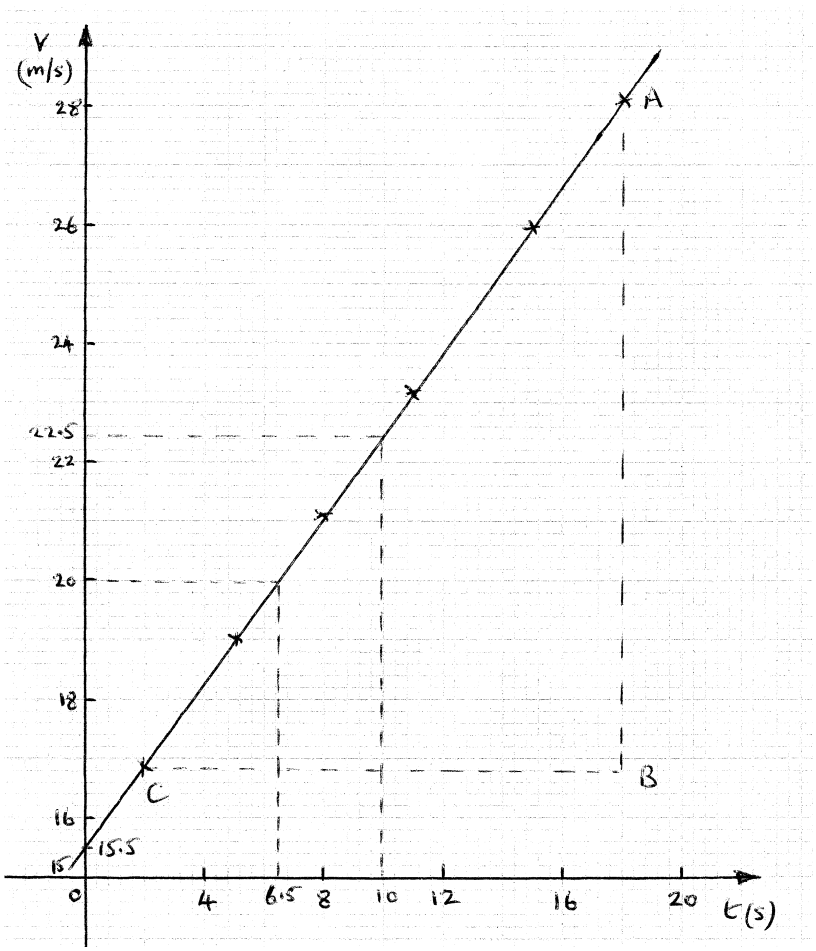
(a) After 10 s, the velocity = **22.5 m/s**

(b) At 20 m/s, the time = **6.5 s**

(c) Gradient of graph = $\frac{AB}{BC} = \frac{28.1-16.9}{18-2} = \frac{11.2}{16}$ = 0.7

Vertical axis intercept at t = 0, is v = 15.5 m/s

Hence, the equation of the graph is: **v = 0.7t + 15.5**



5. The mass m of a steel joist varies with length L as follows:

mass, m (kg)	80	100	120	140	160
length, L (m)	3.00	3.74	4.48	5.23	5.97

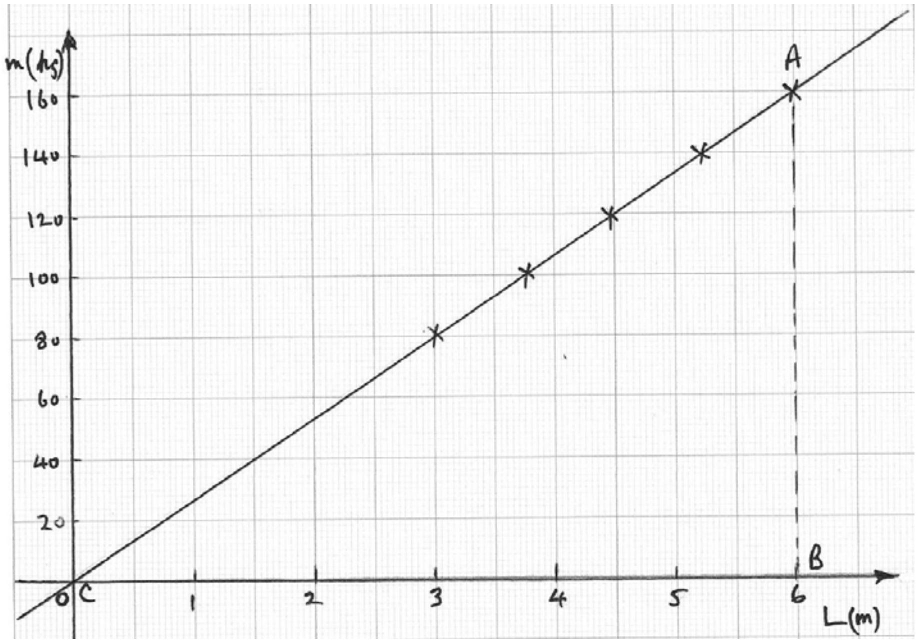
Plot a graph of mass (vertically) against length (horizontally). Determine the equation of the graph

A graph of m/L is shown below.

$$\text{Gradient of graph} = \frac{AB}{BC} = \frac{160-0}{5.97-0} = \frac{160}{5.97} = 26.8$$

Vertical axis intercept = 0

Hence, the equation of the graph is: **m = 26.8L**



6. An experiment with a set of pulley blocks gave the following results:

Effort, E (newtons)	9.0	11.0	13.6	17.4	20.8	23.6
Load, L (newtons)	15	25	38	57	74	88

Plot a graph of effort (vertically) against load (horizontally) and determine (a) the gradient, (b) the vertical axis intercept, (c) the law of the graph, (d) the effort when the load is 30 N and (e) the load when the effort is 19 N.

A graph of effort E against load L is shown below.

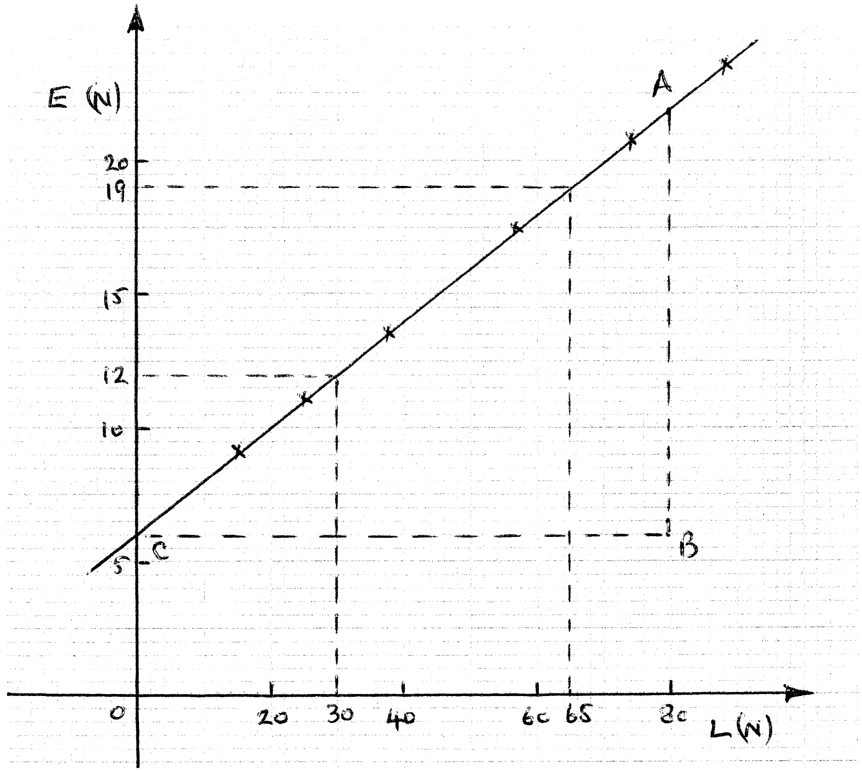
(a) **Gradient of straight line** = $\frac{AB}{BC} = \frac{22-6}{80-0} = \frac{16}{80} = \mathbf{\frac{1}{5}}$ or **0.2**

(b) **Vertical axis intercept = 6**

(c) The law of the graph is: $\mathbf{E = \frac{1}{5}L + 6}$

(d) From the graph, when the load is 30 N, **effort, E = 12 N**

(e) From the graph, when the effort is 19 N, **load, L = 65 N**



7. The variation of pressure p in a vessel with temperature T is believed to follow a law of the form $p = aT + b$, where 'a' and 'b' are constants. Verify this law for the results given below and determine the approximate values of 'a' and 'b'. Hence determine the pressures at temperatures of 285 K and 310 K and the temperature at a pressure of 250 kPa.

pressure, p kPa	244	247	252	258	262	267
temperature, T K	273	277	282	289	294	300

A graph of pressure p against temperature T is shown below.

Plotting the values of p against T produces a straight line, hence will be of the form $p = aT + b$

Taking points A and B on the straight line gives:

For point A, (300, 267) $\quad 267 = 300a + b \quad$ (1)

For point B, (273, 244), $\quad 244 = 273a + b \quad$ (2)

(1) – (2) gives: $\quad 23 = 27a \quad$ from which, $\quad \mathbf{a} = \dfrac{23}{27} = \mathbf{0.85}$

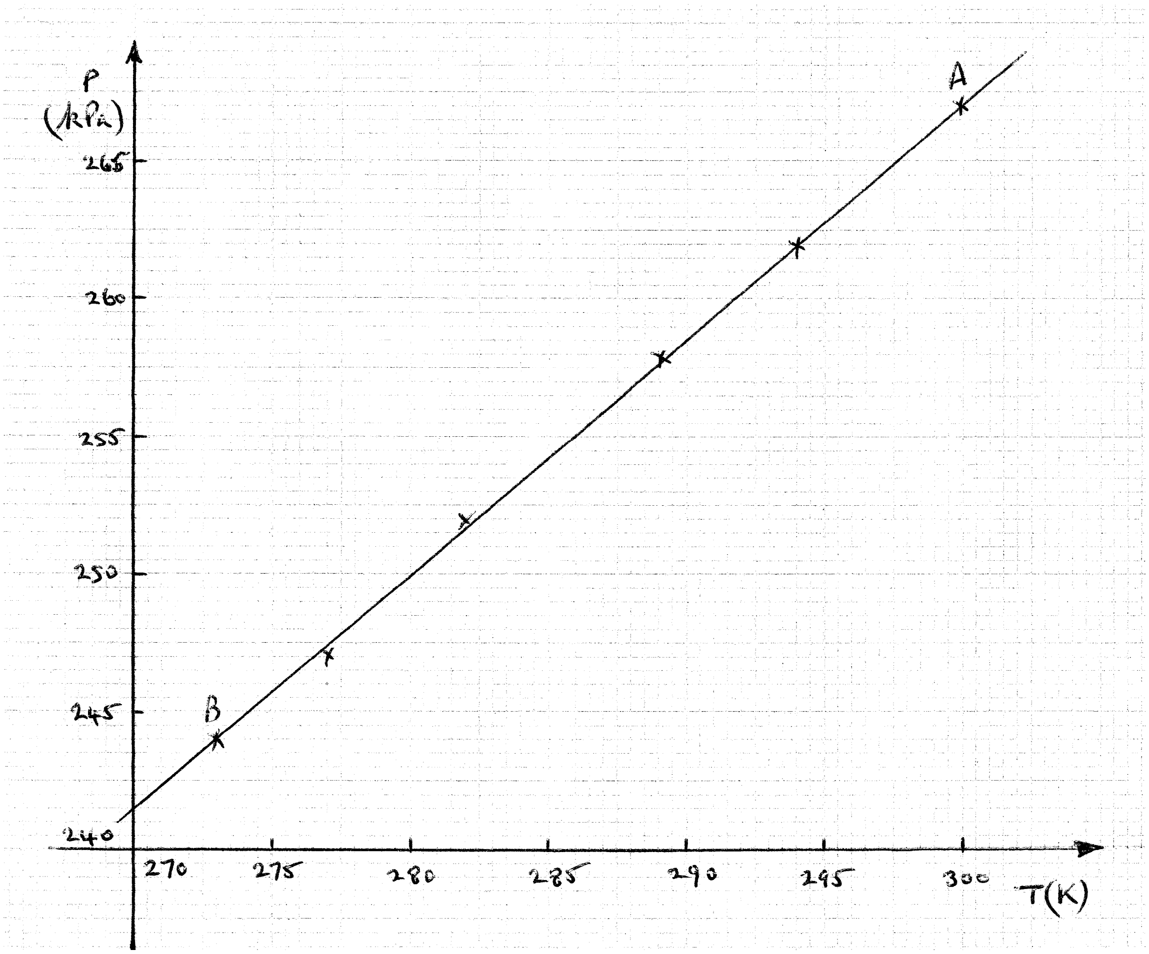
Substituting in (1) gives: $267 = 300(0.85) + b$

from which, $\mathbf{b} = 267 - 300(0.85) = \mathbf{12}$

Hence, the law of the graph is: $\mathbf{p = 0.85T + 12}$

When T = 285 K, pressure, $p = 0.85(285) + 12 = \mathbf{254.3\ kPa}$

When T = 310 K, pressure, $p = 0.85(310) + 12 = \mathbf{275.5\ kPa}$



When p = 250 kPa, then $250 = 0.85(T) + 12$

from which, $250 - 12 = 0.85T$ and **temperature,** $\mathbf{T} = \frac{238}{0.85} = \mathbf{280\ K}$