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MSE, 미적분학

## [연습문제 답안 이용 안내]

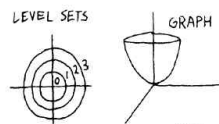
- 본 연습문제 답안의 저작권은 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

## Chapter 11 연습문제 답안

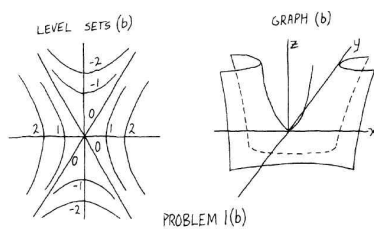
### 《Section 11.1》

1.

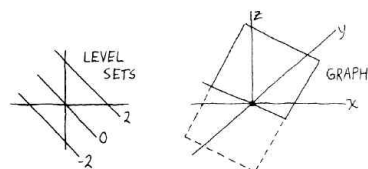
(a)



(b)

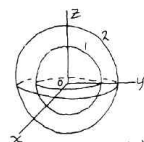


(c)

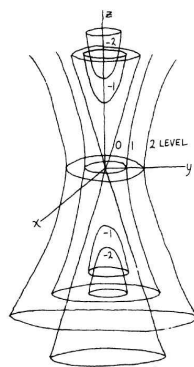


2.

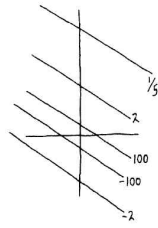
(a)



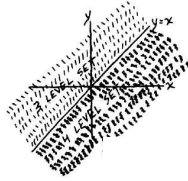
(b)



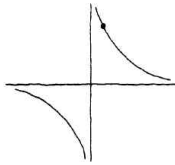
(c)



(d)



3.



4.

(a)  $f(x) = \frac{1}{3}(6 - 2x)$

6레벨  $g(x, y) = 2x + 3y$

0레벨  $h(x, y) = 2x + 3y - 6$

(b)  $f(x) = x^2 + 2y^2$

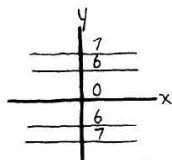
6레벨  $g(x, y, z) = z - x^2 - 2y^2$

0레벨  $h(x, y) = x^2 + 2y^2 - z$

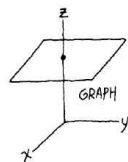
(c) 0레벨  $f(x, y, z) = z^2 + 2y^2 - x$

(d) 4레벨  $f(x, y, z) = x^2 + y^2$

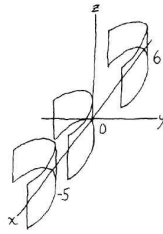
5.



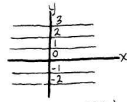
6.



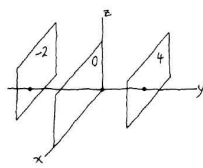
7. (a)



(b)



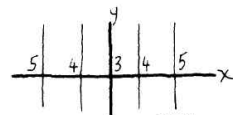
(c)



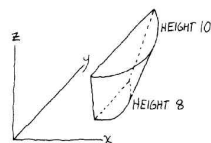
8.  $(x-2)^2 + (y-1)^2 + (z-3)^2 = \frac{1}{9}$

9. (a)  $f(3, 2)$

(b)



10.



11. 아니오

## 《Section 11.2》

1.
  - (a)  $\frac{\delta z}{\delta x} = 2x + 6x^2y^2, \frac{\delta z}{\delta y} = 4x^3y$
  - (b)  $\frac{\delta z}{\delta x} = e^{-y}, \frac{\delta z}{\delta y} = -xe^{-y}$
  - (c)  $\frac{\delta z}{\delta x} = \frac{y}{(x+y)^2}, \frac{\delta z}{\delta y} = \frac{-x}{(x+y)^2}$
  - (d)  $\frac{\delta z}{\delta x} = 8x(2x+5y)^3 + (2x+5y)^4, \frac{\delta z}{\delta y} = 20x(2x+5y)^4$
  - (e)  $\frac{\delta z}{\delta x} = \frac{-3y}{x^2}, \frac{\delta z}{\delta y} = \frac{3}{x}$
  
2.
  - (a)  $\frac{\delta^2 f}{\delta y^2} = \frac{-9}{(2x+3y)^2}, \frac{\delta^2 f}{\delta x \delta y} = \frac{-6}{(2x+3y)^2}$
  - (b)  $\frac{\delta^2 f}{\delta y^2} = \frac{-2xy}{(x^2+y^2)^2}, \frac{\delta^2 f}{\delta x \delta y} = \frac{y^2-x^2}{(x^2+y^2)^2}$
  - (c)  $\frac{\delta^2 f}{\delta y^2} = \frac{4x}{(x-y)^3}, \frac{\delta^2 f}{\delta x \delta y} = \frac{-2x-2y}{(x-y)^3}$
  
3.
  - (a)  $g_{cba}$
  - (b)  $u_{txx}$
  
4.
  - (a)  $\frac{z}{y} \cos \frac{x}{y}$
  - (b)  $-\frac{xz}{y^2} \cos \frac{x}{y}$
  - (c)  $\sin(x/y)$
  - (d)  $\frac{1}{y} \cos \frac{x}{y}$
  
5.
  - (a)  $e^x \sin y$
  - (b)  $e^x \cos y$
  - (c)  $0$
  - (d)  $-x \cos y$
  - (e)  $-\sin y$

6. (a) 0  
(b)  $-\rho \cos \phi \sin \theta$
7.  $\delta f / \delta x < 0, \delta f / \delta y < 0$
8.  $\delta f / \delta a < 0, \delta f / \delta b > 0$
9. 달리기와 자전거 타기 모두 이익을 발생시킨다.  
자전거 타기보다 달리기가 더 많은 이익을 발생시킨다.
10. 오른쪽으로 움직일 때 : -8  
아래로 움직일 때 : 12
11.  $\delta f / \delta x > 0, \delta f / \delta y = 0$
12.  $\delta f / \delta x < 0, \delta f / \delta y = 0$
13.  $\delta f / \delta x > 0, \delta f / \delta y < 0, \delta f / \delta z = 0$
14.  $\delta g / \delta x > \delta f / \delta x$

### 《Section 11.3》

$$1. \quad \frac{\delta w}{\delta s} = \frac{\delta w}{\delta x} \frac{\delta x}{\delta s} + \frac{\delta w}{\delta y} \frac{\delta y}{\delta s} + \frac{\delta w}{\delta z} \frac{\delta z}{\delta s}$$

$$2. \quad \frac{du}{dt} = \frac{\delta u}{\delta x} \frac{\delta x}{\delta a} \frac{da}{dt} + \frac{\delta u}{\delta y} \frac{\delta y}{\delta a} \frac{da}{dt} + \frac{\delta u}{\delta z} \frac{\delta z}{\delta a} \frac{da}{dt} + \frac{\delta u}{\delta x} \frac{\delta x}{\delta b} \frac{db}{dt} + \frac{\delta u}{\delta y} \frac{\delta y}{\delta b} \frac{db}{dt} + \frac{\delta u}{\delta z} \frac{\delta z}{\delta b} \frac{db}{dt}$$

$$3. \quad \frac{\delta p}{\delta y} = \frac{dp}{dt} \frac{\delta t}{\delta y}$$

$$4. \quad (a) \quad t^2 \cos(t^3 \ln t) + 3t^2 \ln t \cos(t^3 \ln t)$$

$$(b) \quad -\frac{1}{x^2 \sin y}$$

$$5. \quad (a) \quad \frac{\delta w}{\delta t} = \frac{\delta w}{\delta x} \frac{dx}{dt} + \frac{\delta w}{\delta y} \frac{dy}{dt}$$

(b)  $t = 3$ 일 때, 온도가 2도씩 떨어진다.

$$6. \quad \frac{\delta z}{\delta u} = \cos t \frac{\delta z}{\delta x} \frac{\delta t}{\delta u} + 6t^2 \frac{\delta z}{\delta y} \frac{\delta t}{\delta u}$$

$$7. \quad \left(\frac{\delta u}{\delta x}\right)^2 + \left(\frac{\delta u}{\delta y}\right)^2 + \left(\frac{\delta u}{\delta z}\right)^2 = \frac{x^2 + y^2 + z^2}{\rho^2} \left(\frac{du}{d\rho}\right)^2 = \left(\frac{du}{d\rho}\right)^2$$

$$8. \quad u_x = -\frac{1}{x^2} \frac{\delta u}{\delta p} - \frac{1}{x^2} \frac{\delta u}{\delta q},$$

$$u_y = \frac{1}{y^2} \frac{\delta u}{\delta p}$$

$$u_z = \frac{1}{z^2} \frac{\delta u}{\delta q},$$

$$9. \quad \frac{\delta z}{\delta x} = 2x \frac{dz}{dt}, \quad \frac{\delta z}{\delta y} = 2y \frac{dz}{dt}, \quad y \frac{\delta z}{\delta x} = x \frac{dz}{dt} = 2xy \frac{dz}{dt}$$

$$10. \quad xu_x + yu_y + zu_z = 2x^2w = 2u$$

# 《Section 11.4》

$$1. \quad 9 \frac{\delta^2 p}{\delta t} + 30 \frac{\delta^2 p}{\delta a \delta b} + 25 \frac{\delta^2 p}{\delta b^2}$$

$$2. \quad 9 \frac{\delta^2 z}{\delta x^2} + 16 \frac{\delta^2 z}{\delta y^2} + 24 \frac{\delta^2 z}{\delta x \delta y}$$

$$3. \quad 6 \frac{\delta^2 u}{\delta x^2} + 2a^3 b \frac{\delta^2 u}{\delta y^2} + \frac{\delta^2 u}{\delta x \delta y} (2a^2 + 6ab) + 2a \frac{\delta u}{\delta y}$$

$$4. \quad 9t^4 \frac{\delta^2 w}{\delta x^2} + 4t^2 \frac{\delta^2 w}{\delta y^2} + 12t^3 \frac{\delta^2 w}{\delta x \delta y} + 2 \frac{\delta w}{\delta y}$$

$$5. \quad (a) \quad \frac{\delta \theta}{\delta x} = - \frac{y}{r^2}$$

$$(b) \quad \frac{\delta^2 \theta}{\delta x^2} = \frac{2xy}{r^4}$$

$$(c) \quad \frac{\delta^2 v}{\delta r^2} + \frac{1}{r} \frac{\delta v}{\delta r} + \frac{1}{r^2} \frac{\delta^2 v}{d\theta^2}$$

$$6. \quad \frac{\delta v}{\delta x} \frac{d^2 x}{dt^2} + \frac{\delta v}{\delta y} \frac{d^2 y}{dt^2} + \frac{\delta^2 v}{\delta x^2} \left( \frac{dx}{dt} \right)^2 + \frac{\delta^2 v}{\delta x^2} \left( \frac{dy}{dt} \right)^2 + 2 \frac{\delta^2 v}{\delta x \delta y} \frac{dx}{dt} \frac{dy}{dt}$$

$$7. \quad w_{xx} = \frac{\delta^2 w}{\delta p^2} + 2 \frac{\delta^2 w}{\delta p \delta q} + \frac{\delta^2 w}{\delta q^2}$$

$$w_{tt} = -c \frac{\delta}{\delta t} \left( \frac{\delta w}{\delta p} \right) + c \frac{\delta}{\delta t} \left( \frac{\delta w}{\delta q} \right)$$



## 《Section 11.5》

1.
  - (a)  $\max : 17, \min : 2$
  - (b)  $\max : 64/5, \min : 0$
  - (c)  $\max : -8, \min : -16$
  - (d)  $\max : 4.75, \min : -\sqrt{2}$
  
2.
  - (a)  $\max : (5, 3), \min : (-1/2, 0)$
  - (b)  $\max : (\frac{1}{2}, \pm \frac{1}{2} \sqrt{3}), \min : (-1/2, 0)$
  
3.
  - (a)  $\max : 10$
  - (b)  $\max : \infty, \min : -\infty$
  - (c)  $\max : \infty, \min : -8$
  - (d)  $\max : \infty, \min : -\infty$
  
4.  $(3, 2, 1)$
  
5.
  - (a) 2
  - (b) 2
  
6.  $(\frac{14}{5}, \frac{11}{5}, \frac{28}{5}), (\frac{4}{5}, \frac{9}{5}, \frac{32}{5})$
  
7.
  - (a) 높이가 낮고 밑넓이가 넓은 경우  
높이가 높고 밑넓이가 작은 경우
  - (b)  $8 \times 8 \times \frac{256}{64}$
  - (c)  $y = 20, x = 4$

《Section 11.6》

1. (a)  $14/\sqrt{2}$   
 (c)  $-4$   
 (e)  $10\cos 157.5 + 4\sin 157.5$

- (b)  $-4/\sqrt{40}$   
 (d)  $\sqrt{116}$

2.  $16/\sqrt{2}$

3. (a)  $12/\sqrt{29}$   
 (c)  $3/\sqrt{6}$

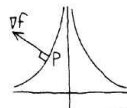
- (b)  $(-1, 2, 0), (5, -8, -2)$   
 (d)  $-2\vec{i} - \vec{j} - \vec{k}, \sqrt{6}/m$

4. 떨어지는 것

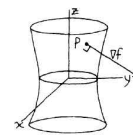
5.  $4/\sqrt{13}$

6.  $\nabla f = (-4, -2) |\nabla f| = \sqrt{20}$

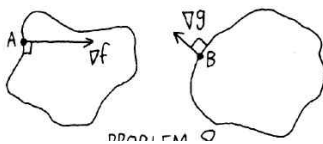
7. (a)  $\nabla f = (2xy, x^2), (-4, 1)$



(b)  $\nabla f = (2x, 4y, -2z), (2, 8, -2)$

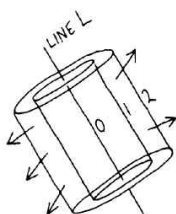


8.  $\nabla f > \nabla g$



9.  $\nabla f = \vec{0}$

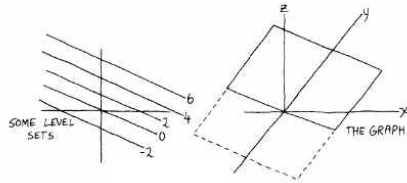
10.  $\nabla f = \vec{0}$



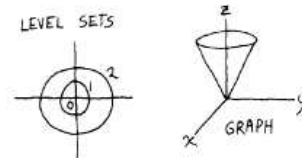
11. (a) 12 (b)  $\sqrt{45}$   
 (c)  $-3/\sqrt{2}$  (d)  $x = 1 + 12t, y = 2 + 6t, z = 6 + 2t$
12. (a)  $-1$  (b)  $\sqrt{52}$   
 (c)  $2/\sqrt{2}$  (d)  $x = 1 - 6t, y = -1 - 4t, z = 1 + t$
13. (a) 11 (b)  $(1, 3, 11)$
14. (a)  $(2, 4, 12)$  (b)  $40/\sqrt{164}$   
 (c)  $(1, -1, -1), (3, 2, 4), (0, -2, 2)$  (d)  $(-3, 2, 4)$   
 (e)  $-\frac{1}{2\sqrt{2}}$

《복습문제》

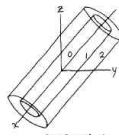
1. (a)  $z = 2x + 3y$



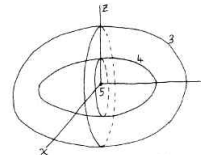
(b)  $z^2 = x^2 + y^2$



2. (a)  $y = z = 0$



(b)  $x^2 + 2y^2 + 3z^2 = 8$



3. (a)  $y$

(b) 0

4.  $(\frac{\delta z}{\delta r})^2 + \frac{1}{r^2}(\frac{\delta z}{\delta \theta})^2 = (\frac{\delta z}{\delta x})^2(\cos^2\theta + \sin^2\theta) + (\frac{\delta z}{\delta y})^2(\cos^2\theta + \sin^2\theta)$

5.  $8\frac{\delta^2 u}{\delta x \delta y} + 4c\frac{\delta^2 u}{\delta z \delta y} + 2a\frac{\delta^2 u}{\delta x \delta z} + ac\frac{\delta^2 u}{\delta z^2} + \frac{\delta u}{\delta z}$

6. max : 0, min : -12

7.  $\|\nabla f\| = \sqrt{64 - 28x^2} \quad (-1 \leq x \leq 1)$

8. 출발할 때 :  $\frac{25}{\sqrt{14}}$ , 도착할 때 :  $\frac{1}{\sqrt{30}}$

9. (a)  $\nabla T = (2x, -1)$

(b)  $-4\vec{i} - \vec{j}$

10. (a)  $z = x^2 - y$

(b)  $(-4, -1, -1)$

(c)  $3/\sqrt{2}$

(d)  $\nabla z = -4\vec{i} - \vec{j}$

11.  $(2\pi h + 4\pi r)dr = 2\pi r dh$

12.  $y = x + \frac{1}{x}$