7.1 연습문제

1.

풀이 (a) $\displaystyle\int\left(\frac{d}{dx}(x-x^2+\sin x)\right)dx = x-x^2+\sin x$

(b) $\displaystyle\frac{d}{dx}\left(\int(\tan x+\ln(2x+1))dx\right) = \tan x+\ln(2x+1)$

3.

풀이 (a) $f'(x)=1+x\sqrt{x}$ 이므로 $f(x)=\displaystyle\int(1+x\sqrt{x})dx = x+\frac{2}{5}x^{5/2}+C$이다. $f(1)=\dfrac{7}{5}$이므로 $f(1)=\dfrac{7}{5}+C=\dfrac{7}{5}$, $C=0$이고, 따라서 $f(x)=x+\dfrac{2}{5}x^{5/2}$이다.

$$f(x)=\frac{2}{5}x^2\sqrt{x}+\frac{1}{2}$$

(b) $f'(x)=\dfrac{2x^2-3}{x}$ 이므로 $f(x)=\displaystyle\int\frac{2x^2-3}{x}dx = x^2-3\ln|x|+C$이다. $f(1)=2$이므로 $f(1)=1+C=2$, $C=1$이고, 따라서 $f(x)=x^2+1-3\ln|x|$이다.

5.

풀이 (a) $f(x)=\displaystyle\int\left(\frac{dy}{dx}\right)dx=\int(\cos x+\sin x)dx=\sin x-\cos x+C$

$f\left(\dfrac{\pi}{4}\right)=\sin\dfrac{\pi}{4}-\cos\dfrac{\pi}{4}+C=2$; $C=2$; $f(x)=\sin x-\cos x+2$

(b) $f(x)=\displaystyle\int\left(\frac{dy}{dx}\right)dx=\int(x+e^x-1)dx=\frac{1}{2}x^2-x+e^x+C$

$f(1)=-\dfrac{1}{2}+e+C=1$; $C=\dfrac{3}{2}-e$; $f(x)=\dfrac{1}{2}x^2-x+e^x+\dfrac{3}{2}-e$

7.

풀이 (a) $f'(x)=x^2+ax-1$이므로 $f'(2)=5+2a$이다.

$\displaystyle\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=f'(2)=5+2a=2$이므로 $a=-\dfrac{3}{2}$; $f'(x)=x^2-\dfrac{3}{2}x-1$이다.

그러므로

$f(x)=\displaystyle\int\left(x^2-\frac{3}{2}x-1\right)dx=\frac{1}{3}x^3-\frac{3}{4}x^2-x+C$

$f(1)=\frac{1}{3}-\frac{3}{4}-1+C=0;\ C=\frac{17}{12};$

$f(x)=\frac{1}{3}x^3-\frac{3}{4}x^2-x-\frac{17}{12}$

(b) $f'(1)=a$이고 $\lim_{h\to 0}\frac{f(1-h)-f(1)}{h}=-f'(1)=-a=2$이므로 $a=-2$이다. 따라서

$f'(x)=x^2-2x-1$이므로

$f(x)=\int(x^2-2x-1)\,dx=\frac{1}{3}x^3-x^2-x+C;\ f(1)=\frac{1}{3}+C=1;\ C=\frac{2}{3};$

$f(x)=\frac{1}{3}x^3-x^2-x+\frac{2}{3}$

7.2 연습문제

1.

풀이 (a) $u=2x^2+1;\ \frac{1}{4}du=x\,dx$

$$\int x(2x^2+1)^2\,dx=\frac{1}{4}\int u^2\,du=\frac{1}{12}u^3+C=\frac{1}{12}(2x^2+1)^3+C$$

(b) $u=x^3+3;\ u=x^3+3\quad;\quad du=3x^2\,dx\quad;\quad \frac{1}{3}du=x^2\,dx$

$$\int 3x^2(x^3+3)^3\,dx=\int u^3\,du=\frac{1}{4}u^3+C=\frac{1}{4}(x^3+3)^4+C$$

(c) $u=x^2-2;\ \frac{1}{2}du=x\,dx$

$$\int x\sqrt{x^2-2}\,dx=\frac{1}{2}\int u^{1/2}\,du=\frac{1}{2}\frac{2}{3}u^{3/2}+C=\frac{1}{3}(x^2-2)^{3/2}+C$$

(d) $u=1+2x^2;\ du=4x\,dx$

$$\int 4x\sqrt{1+2x^2}\,dx=\int u^{1/2}\,dx=\frac{2}{3}u^{3/2}+C=\frac{2}{3}(1+2x^2)^{3/2}+C$$

(e) $u=\ln x$라 하면, $du=\frac{1}{x}dx$이므로

$$\int\frac{\ln x}{x}dx=\int u\,du=\frac{1}{2}u^2+C=\frac{1}{2}(\ln|x|)^2+C$$

(f) $\int \operatorname{cosec}x\,dx=\int\frac{\operatorname{cosec}x(\operatorname{cosec}x+\cot x)}{\operatorname{cosec}x+\cot x}dx=\int\frac{\operatorname{cosec}^2x+\operatorname{cosec}x\cot x}{\operatorname{cosec}x+\cot x}dx$

이고, $u=\operatorname{cosec}x+\cot x$ 라 하면 $du=-(\operatorname{cosec}x\cot x+\operatorname{cosec}^2x)dx$이다. 따라서

$$\int\operatorname{cosec}x\,dx=-\int\frac{1}{u}du=-\ln|u|+C=-\ln|\operatorname{cosec}x+\cot x|+C$$

(g) $u=\ln x,\ v'=x$이라 하면, $u'=\frac{1}{x},\ v=\frac{1}{2}x^2$이므로

$$\int x\ln x\,dx=\frac{1}{2}x^2\ln|x|-\int\frac{1}{x}\left(\frac{1}{2}x^2\right)dx$$

$$=\frac{1}{2}x^2\ln|x|-\frac{1}{2}\left(\frac{1}{2}x^2\right)=\frac{1}{4}x^2(-1+2\ln|x|)+C$$

(h) $\int x^2\ln x\,dx=\frac{1}{3}x^3\ln x-3\int x^2\,dx=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+C=\frac{1}{9}x^3(-1+3\ln x)+C$

(i) $\int x\cos x\,dx=x\sin x-\cos x+C$

(j) $\int\cos^{-1}x\,dx=x\cos^{-1}x+\int\frac{x}{\sqrt{1-x^2}}dx=x\cos^{-1}x-\sqrt{1-x^2}+C$

(k) $\int (\sin x+\cos x)^2 dx = \int (1+2\sin x \cos x)dx = x+\sin^2 x + C$

(l) $\int \cos x \sin^2 x\, dx = \int (\sin x)^2 \cos x\, dx = \frac{1}{3}\sin^3 x + C$

(m) $I=\int e^{ax}\cos bx\, dx$ 그리고 $u=e^{ax},\ v'=\cos bx$ 라 하면, $u'=ae^{ax},\ v=\frac{1}{b}\sin bx$이므로

$$I=\frac{1}{b}e^{ax}\sin bx-\frac{a}{b}\int e^{ax}\sin bx\, dx$$

이다. 이제 우변의 적분에 다시 부분적분법을 적용하기 위해 $u=e^{ax},\ v'=\sin bx$ 라 하면, $u'=ae^{ax},\ v=-\frac{1}{b}\cos bx$ 이므로

$$\int e^{ax}\sin bx\, dx=-\frac{1}{b}e^{ax}\cos bx+\frac{a}{b}I+C';$$

$$I=\frac{1}{b}e^{ax}\sin bx-\frac{a}{b}\left(-\frac{1}{b}e^{ax}\cos bx+C'+\frac{a}{b}I\right)$$

이다. 따라서 구하고자 하는 적분 결과는 다음과 같다.

$$I=\frac{e^{ax}}{a^2+b^2}(a\cos bx+b\sin bx)+C,\ \ C=-\frac{ab}{a^2+b^2}C'$$

(n) $I=\int e^{ax}\sin bx\, dx$ 그리고 $u=e^{ax},\ v'=\sin bx$ 라 하면, $u'=ae^{ax},\ v=-\frac{1}{b}\cos bx$ 이므로

$$I=-\frac{1}{b}e^{ax}\cos bx+\frac{a}{b}\int e^{ax}\cos bx\, dx$$

이다. 이제 우변의 적분에 다시 부분적분법을 적용하기 위해 $u=e^{ax},\ v'=\cos bx$라면, $u'=ae^{ax},\ v=\frac{1}{b}\sin bx$ 이므로

$$\int e^{ax}\cos bx\, dx=-\frac{1}{b}e^{ax}\cos bx+\frac{a}{b}I+C';$$

$$I=-\frac{1}{b}e^{ax}\cos bx+\frac{a}{b}\left(\frac{1}{b}e^{ax}\sin bx+C'-\frac{a}{b}I\right)$$

이다. 따라서 구하고자 하는 적분 결과는 다음과 같다.

$$I=\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+C,\ \ C=-\frac{ab}{a^2+b^2}C'$$

7.3 연습문제

1.

풀이 (a) $\int_1^2 (2x-x^2)\,dx = \left(x^2 - \frac{1}{3}x^3\right)\Big|_1^2 = \left(4-\frac{8}{3}\right) - \left(1-\frac{1}{3}\right) = \frac{2}{3}$

(b) $\int_1^4 \frac{1}{x}\,dx = \ln x\Big|_1^4 = \ln 4 - \ln 1 = 2\ln 2$

(c) $\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$

(d) $\int_0^2 |x-1|\,dx = \int_0^1 |x-1|\,dx + \int_1^2 |x-1|\,dx = -\int_0^1 (x-1)\,dx + \int_1^2 (x-1)\,dx$

$$= -\left(\frac{1}{2}x^2 - x\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = -\left(\frac{1}{2}-1\right) + (2-2) - \left(\frac{1}{2}-1\right) = 1$$

(e) $\int_1^2 \frac{1}{x(1+\ln x)^2}\,dx = \int_1^{1+\ln 2} u^{-2}\,du = -\frac{1}{u}\Big|_1^{1+\ln 2} = \frac{\ln 2}{1+\ln 2}$

(f) $\int_0^{\ln 2} x e^x\,dx = (x-1)e^x\Big|_0^{\ln 2} = -1 + 2\ln 2$

(g) $\int_0^{\pi/2} x\sin x\,dx = -x\cos x + \sin x\Big|_0^{\pi/2} = 1$

(h) $\int_{-1/\sqrt{2}}^1 \sin^{-1}x\,dx = \sqrt{1-x^2}\,\sin^{-1}x\Big|_{-1/\sqrt{2}}^1 = -\frac{1}{8}[4\sqrt{2} + \pi(-4+\sqrt{2})]$

(i) $\int_0^1 x\tan^{-1}x\,dx = \frac{1}{2}(-x + (x^2+1)\tan^{-1}x)\Big|_0^1 = \frac{\pi-2}{4}$

(j) $\int_{-1}^1 |e^x - 1|\,dx = -\int_{-1}^0 (e^x-1)\,dx + \int_0^1 (e^x-1)\,dx$

$$= -(e^x - x)\Big|_{-1}^0 + (e^x - x)\Big|_0^1 = e + e^{-1} - 2$$

3.

풀이 (a) $\int_0^3 (2x^2+1)\,dx = 18+3 = 21 = 3(2c^2+1);\ c^2 = 3;\ c = \sqrt{3}$

(b) $\int_0^2 (x^2-1)\,dx = \frac{8}{3} - 2 = \frac{2}{3} = 2(c^2-1);\ c^4 = \frac{4}{3};\ c = \frac{2\sqrt{3}}{3}$

5.

풀이 (a) $f(x)$가 우함수이므로 $(x^5+x^3)f(x)$는 기함수이고, 따라서

$$\int_{-1}^{1}(x^5+x^3+1)f(x)\,dx=\int_{-1}^{1}(x^5+x^3)f(x)\,dx+\int_{-1}^{1}f(x)\,dx$$

$$=2\int_{0}^{1}f(x)\,dx=2\cdot 3=6$$

(b) $f(x)$가 이차함수이므로 $f(x)=ax^2+bx+c$라 하면, 조건에 의하여

$$\int_{-1}^{1}xf(x)\,dx=\int_{-1}^{1}x(ax^2+bx+c)\,dx=\int_{-1}^{1}bx^2\,dx=2b\int_{0}^{1}x^2\,dx=\frac{2}{3}b=2$$

이고 따라서 $b=3$이다.

$$\int_{-1}^{1}x^3f(x)\,dx=\int_{-1}^{1}x^3(ax^2+3x+c)\,dx=3\int_{-1}^{1}x^4\,dx$$

$$=6\int_{0}^{1}x^4\,dx=\frac{6}{5}x^5\Big|_0^1=\frac{6}{5}$$

7.

풀이 (a) $F'(x)=\dfrac{d}{dx}\displaystyle\int_{1}^{x}u^3\,du=x^3$

(b) x^3의 원시함수는 $F(x)=\displaystyle\int x^3\,dx=\frac{1}{4}x^4$ 이므로

$$\int_{1}^{4}x^3\,dx=F(x)\Big|_1^4=\left(\frac{1}{4}x^4\right)\Big|_1^4=\frac{1}{4}(4^4)-\frac{1}{4}(1^4)=\frac{256-1}{4}=\frac{255}{4}$$

9.

풀이 (a) $\dfrac{d}{dx}\displaystyle\int_{1}^{x}(t^2+1)\,dt=x^2+1$

(b) $\dfrac{d}{dx}\displaystyle\int_{1}^{x^2}\frac{t}{1+t^2}\,dt=\frac{x^2}{1+(x^2)^2}\cdot 2x=\frac{2x^3}{1+x^4}$

(c) $\dfrac{d}{dx}\displaystyle\int_{-x}^{x^2}(x-t)^2(t+2)\,dt=(x-x^2)^2(x^2+2)\cdot(2x)-(x-(-x))^2(-x+2)\cdot(-1)$

$$=2x^3(x-1)^2(x^2+2)-4x^2(x-2)$$

$$=2x^7-\frac{14}{3}x^6+7x^5-10x^4+\frac{7}{3}x^3+14x^2$$

(d) $\dfrac{d}{dx}\displaystyle\int_{x}^{\sqrt{x}}\sqrt{t+1}\,dt=\frac{\sqrt{1+\sqrt{x}}}{2\sqrt{x}}-\sqrt{x+1}$

7.4 연습문제

1.

풀이 (a) $\int \frac{dx}{x^2+4} = \frac{1}{4}\int \frac{dx}{(x/2)^2+1}$ 이므로 $u = \frac{x}{2}$ 라 하면, $2du = dx$ 이므로

$$\int \frac{dx}{x^2+4} = \frac{1}{4}\int \frac{2du}{u^2+1} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

(b) $\int \frac{1}{(x+1)(x+2)}dx = \int\left(\frac{1}{x+1} - \frac{1}{x+2}\right)dx$

$$= \ln|x+1| - \ln|x+2| + C = \ln\left|\frac{x+1}{x+2}\right| + C$$

(c) $\frac{x^2}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$ 이라 놓고 미정계수 A, B, C를 구하면 다음과 같다.

$$x^2 = Ax^2 + (-2A+B)x + A - B + C;\ A = 1,\ B = 2,\ C = 1$$

그러므로

$$\int \frac{x^2}{(x-1)^3}dx = \int\left(\frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{1}{(x-1)^3}\right)dx$$

$$= \ln|x-1| - \frac{4x-3}{2(x-1)^2} + C$$

(d) $\int \frac{x^3-1}{x-1}dx = \int (x^2+x+1)dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$

(e) $\frac{x^3+x-1}{x^2-1} = x + \frac{2x-1}{x^2-1} = x + \frac{2x}{x^2-1} - \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$

이므로

$$\int \frac{x^3+x-1}{x^2-1}dx = \int\left(x + \frac{2x}{x^2-1} - \frac{1}{2}\frac{1}{x-1} + \frac{1}{2}\frac{1}{x+1}\right)dx$$

$$= \frac{x^2}{2} + \ln|x^2-1| - \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + C$$

$$= \frac{1}{2}\left(x^2 + \ln|(x-1)(x+1)^3|\right) + C$$

(f) $\frac{x^3+x-1}{x^2+1} = x - \frac{1}{x^2+1}$ 이므로

$$\int \frac{x^3+x-1}{x^2+1}dx = \int\left(x - \frac{1}{x^2+1}\right)dx = \frac{x^2}{2} + \tan^{-1}x + C$$

$$\text{(g)} \int \frac{1}{x^3+x}dx = \int\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + C = \frac{1}{2}\ln\frac{x^2}{x^2+1} + C$$

$$\text{(h)} \int \frac{2x^2+x-1}{x(x+1)^2}dx = \int\left(\frac{1}{x}-\frac{1}{1+x}-\frac{1}{(x+1)^2}\right)dx$$

$$= \frac{1}{x+1} + \ln|x| - \ln|x+1| + C$$

$$= \frac{1}{x+1} + \ln\left|\frac{x}{x+1}\right| + C$$

$$\text{(i)} \int \frac{x^2-x+1}{(x-1)^3}dx = \int\left(\frac{1}{x-1}-\frac{1}{(x-1)^2}+\frac{1}{(x-1)^3}\right)dx$$

$$= \frac{1-2x}{2(x-1)^2} + \ln|x-1| + C$$

$$\text{(j)} \int \frac{x+4}{x^3-2x^2}dx = \int\left(\frac{3}{2}\frac{1}{x-2}-\frac{3}{2}\frac{1}{x}-\frac{2}{x^2}\right)dx$$

$$= \frac{2}{x} + \frac{3}{2}\ln|x-2| - \frac{3}{2}\ln|x| + C$$

$$= \frac{2}{x} + \frac{3}{2}\ln\left|\frac{x-2}{x}\right| + C$$

$$\text{(k)} \int \frac{x+1}{(x-2)^3}dx = \int\left(\frac{1}{(x-2)^2}+\frac{3}{(x-2)^3}\right)dx = \frac{1-2x}{2(x-2)^2} + C$$

$$\text{(l)} \int \frac{x^2+x-1}{x(x+1)(x-1)}dx = \int\left(\frac{1}{x}+\frac{1}{2}\frac{1}{x-1}-\frac{1}{2}\frac{1}{x+1}\right)dx$$

$$= \ln|x| + \frac{1}{2}(\ln|x-1| - \ln|x+1|) + C$$

$$= \frac{1}{2}\ln\left|\frac{x^2(x-1)}{x+1}\right| + C$$

3.

풀이 $f'(x) = \dfrac{1}{x^2-3x+2}$; $f(x) = \displaystyle\int \frac{1}{x^2-3x+2}dx = \ln\left|\frac{x-2}{x-1}\right| + C$

$f(0) = \ln 2 + C = 1$; $f(0) = 1 - \ln 2$; $f(x) = 1 + \ln\left|\dfrac{x-2}{2(x-1)}\right|$

7.5 연습문제

1.

풀이 (a) $\int \sin^5 x\,dx = \int (1-\cos^2 x)^2 \sin x\,dx = -\int (t^4 - 2t^2 + 1)\,dt \quad (\cos x = t)$

$$= -\left(\frac{1}{5}t^5 - \frac{2}{3}t^3 + t\right) + C = -\frac{1}{5}\cos^5 x + \frac{2}{3}\cos x + \cos x + C$$

(b) $\int \cos^3 x\,dx = \int (1-\sin^2 x)\cos x\,dx = \int (1-t^2)\,dt \qquad (\sin x = t)$

$$= t - \frac{1}{3}t^3 + C = \sin x - \frac{1}{3}\sin^3 x + C$$

(c) $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C = \frac{1}{4}(2x - \sin 2x) + C$$

(d) $\int \cos^4 x\,dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\,dx$

$$= \frac{1}{4}\int \left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right)dx$$

$$= \frac{1}{32}(12x + 8\sin 2x + \sin 4x) + C$$

3.

풀이 (a) $\int \tan^5 x\,dx = \int \tan^3 x\,\tan^2 x\,dx = \int \tan^3 x\,(\sec^2 x - 1)\,dx$

$$= \int \tan^3 x\sec^2 x\,dx - \int \tan^3 x\,dx$$

$$= \int \tan^3 x\sec^2 x\,dx - \int \tan x\,(\sec^2 x - 1)\,dx$$

$$= \int (u^3 - u)\,du + \int \tan x\,dx$$

$$= \frac{1}{4}u^4 - \frac{1}{2}u^2 - \ln|\cos x| + C$$

$$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \ln|\cos x| + C$$

(b) $\int \tan^6 x\,dx = \int \tan^4 x\,(\sec^2 x - 1)dx$

$$= \int \tan^4 x \sec^2 x\,dx - \int \tan^4 x\,dx$$

$$= \int \tan^4 x \sec^2 x\,dx - \int \tan^2 x\,(\sec^2 x - 1)dx$$

$$= \int (\tan^4 x - \tan^2 x)\sec^2 x\,dx + \int \tan^2 x\,dx$$

$$= \int (\tan^4 x - \tan^2 x + 1)\sec^2 x\,dx - \int 1\,dx$$

$$= -x + \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x + C$$

(c) $\int \cot^5 x\,dx = \int \cot^3 x \cot^2 x\,dx = \int \cot^3 x\,(\mathrm{cosec}^2 x - 1)dx$

$$= \int \cot^3 x\,\mathrm{cosec}^2 x\,dx - \int \cot^3 x\,dx$$

$$= \int \cot^3 x\,\mathrm{cosec}^2 x\,dx - \int \cot x\,(\mathrm{cosec}^2 x - 1)dx$$

$$= \int (u - u^3)\,du + \int \cot x\,dx$$

$$= -\frac{1}{4}u^4 + \frac{1}{2}u^2 + \ln|\sin x| + C$$

$$= -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \ln|\sin x| + C$$

(d) $\int \cot^6 x\,dx = \int \cot^4 x \cot^2 x\,dx = \int \cot^4 x\,\mathrm{cosec}^2 x\,dx - \int \cot^4 x\,dx$

$$= \int \cot^4 x\,\mathrm{cosec}^2 x\,dx - \int \cot^2 x\,(\mathrm{cosec}^2 x - 1)dx$$

$$= \int (\cot^4 x - \cot^2 x)\,\mathrm{cosec}^2 x\,dx + \int \cot^2 x\,dx$$

$$= \int (\cot^4 x - \cot^2 x + 1)\,\mathrm{cosec}^2 x\,dx + \int (-1)\,dx$$

$$= -x + \frac{1}{5}\cot^5 x - \frac{1}{3}\cot^3 x + \cot x + C$$

5.

풀이 (a) $\int \sin 2x \cos x\,dx = \frac{1}{2}\int (\sin 3x + \sin x)\,dx = -\frac{1}{6}(\cos 3x + 3\cos x) + C$

(b) $\int \sin 3x \cos 4x\,dx = \frac{1}{2}\int (\sin 7x - \sin x)\,dx = \frac{1}{14}(-\cos 7x + 7\cos x) + C$

(c) $\int \cos 3x \cos 5x \, dx = \frac{1}{2} \int (\cos 8x + \cos 2x) \, dx = \frac{1}{16} (\sin 8x + 4 \sin 2x) + C$

(d) $\int \cos x \cos 2x \, dx = \frac{1}{2} \int (\cos 3x + \cos x) \, dx = \frac{1}{6} (\sin 3x + 3 \sin x) + C$

7.6 연습문제

1.

풀이 (a) $$\int \frac{1}{\sqrt{4-x^2}}\,dx = \int \frac{1}{2\sqrt{1-(x/2)^2}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,du \qquad (u=x/2)$$

$$= \sin^{-1}u + C = \sin^{-1}\frac{x}{2} + C$$

(b) $$\int \frac{dx}{\sqrt{x^2-4}} = \int \frac{dx}{2\sqrt{(x/2)^2-1}} = \int \frac{dx}{\sqrt{u^2-1}} \qquad (u=x/2)$$

$$= \cosh^{-1}u + C = \cosh^{-1}\frac{x}{2} + C$$

(c) $\sqrt{2x+3} = t$ 라 하면, $x = \frac{1}{2}(t^2-3)$, $dx = t\,dt$

$$\int \sqrt{2x+3}\,dx = \int t^2\,dt = \frac{1}{3}t^3 + C = \frac{1}{3}(2x+3)\sqrt{2x+3} + C$$

(d) $\sqrt{1-3x} = t$라 하면, $x = \frac{1}{3}(1-t^2)$, $dx = -\frac{2}{3}t\,dt$

$$\int \sqrt{1-3x}\,dx = -\frac{2}{3}\int t^2\,dt = -\frac{2}{9}t^3 + C = \frac{2}{9}(3x-1)\sqrt{1-3x} + C$$

(e) $\sqrt{3-x} = t$ 라 하면, $x = 3-t^2$, $dx = -2t\,dt$

$$\int \frac{x}{\sqrt{3-x}}\,dx = \int \frac{3-t^2}{t}(-2t)\,dt = -2\int (3-t^2)\,dt$$

$$= -2\left(3t - \frac{1}{3}t^3\right) + C = -\frac{2}{3}(6+x))\sqrt{3-x} + C$$

(f) $\sqrt{3-x} = t$ 라 하면, $x = 3-t^2$, $dx = -2t\,dt$

$$\int \frac{x^2}{\sqrt{3-x}}\,dx = \int \frac{(3-t^2)^2}{t}(-2t)\,dt = -2\int (9-6t^2+t^4)\,dt$$

$$= -2\left(9t - 2t^3 + \frac{1}{5}t^5\right) + C$$

$$= -2\left(9 - 2(3-x) + \frac{1}{5}(3-x)^2\right)\sqrt{3-x} + C$$

$$= -\frac{2}{5}(24+4x+x^2)\sqrt{3-x} + C$$

(g) $\sqrt{\frac{x+2}{2-x}} = t$ 라 하면, $x = \frac{2(t^2-1)}{1+t^2}$, $dx = \frac{8t}{(1+t^2)^2}\,dt$

$$\int \frac{1}{x}\sqrt{\frac{x+2}{2-x}}\,dx = 4\int \frac{t^2}{(t^2-1)(t^2+1)}\,dt$$

$$= 4\int\left(\frac{1}{4(t-1)} - \frac{1}{4(t+1)} + \frac{1}{2(t^2+1)}\right)dt$$

$$= \ln\left|\frac{t-1}{t+1}\right| + 2\tan^{-1}t + C$$

$$= \ln\left|\frac{\sqrt{2+x}-\sqrt{2-x}}{\sqrt{2+x}+\sqrt{2-x}}\right| + 2\tan^{-1}\sqrt{\frac{x+2}{2-x}} + C$$

(h) $\sqrt{1-x} = t$ 라 하면, $x = 1-t^2$, $dx = -2t\,dt$, $1+x = 2-t^2$

$$\int \frac{1+x}{\sqrt{1-x}}\,dx = \int \frac{2-t^2}{t}(-2t)\,dt = 2\int (t^2-2)\,dt$$

$$= \frac{2}{3}(t^2-6)t + C = -\frac{2}{3}(x+5)\sqrt{1-x} + C$$

(i) $\sqrt[6]{x} = t$ 라 하면, $x = t^6$, $x = 6t^5\,dt$, $\sqrt[3]{x} = t^2$

$$\int \frac{x}{\sqrt{x}\,(1+\sqrt[3]{x})}\,dx = 6\int \frac{t^8}{1+t^2}\,dt = 6\int\left(t^6 - t^4 + t^2 - 1 + \frac{1}{1+t^2}\right)dt$$

$$= 6\left(\frac{1}{7}t^7 - \frac{1}{5}t^5 + \frac{1}{3}t^3 - t + \tan^{-1}t\right)$$

$$= 6\left(\frac{1}{7}x^{7/6} - \frac{1}{5}x^{5/6} + \frac{1}{3}x^{1/2} - x^{1/6} + \tan^{-1}x^{1/6}\right)$$

(j) $\sqrt{x^2-4x+5} = t - x$라 하면,

$$x = \frac{t^2-5}{2(t-2)},\ dx = \frac{t^2-4t+5}{2(t-2)^2}dt,\ \sqrt{2x^2-4x+5} = \frac{t^2-4t+5}{2(t-2)}$$

$$\int \frac{1}{\sqrt{x^2-4x+5}}\,dx = \int \frac{2(t-2)}{t^2-4t+5}\cdot\frac{t^2-4t+5}{2(t-2)^2}\,dt$$

$$= \int \frac{1}{t-2}\,dt = \ln|t-2| + C$$

$$= \ln\left|x-2+\sqrt{x^2-4x+5}\right| + C$$

(k) $\sqrt{1-x^2} = \sqrt{(1-x)(1+x)} = (1-x)\sqrt{\frac{1+x}{1-x}}$; $\sqrt{\frac{1+x}{1-x}} = t$라 하면,

$$x = \frac{t^2-1}{t^2+1},\ dx = \frac{4t}{(t^2+1)^2}dt,\ 1-x = \frac{2}{t^2+1},\ 2x+1 = \frac{3t^2-1}{t^2+1}$$

$$\int \frac{dx}{(2x+1)\sqrt{1-x^2}} = \int \frac{t^2+1}{3t^2-1} \cdot \frac{t^2+1}{2t} \cdot \frac{4t}{(t^2+1)^2}\,dt = \int \frac{2}{3t^2-1}\,dt$$

$$= \int \left(\frac{1}{\sqrt{3}\ t-1} - \frac{1}{\sqrt{3}\ t+1} \right) dt = \frac{\sqrt{3}}{3} \ln \left| \frac{\sqrt{3}\ t-1}{\sqrt{3}\ t+1} \right| + C$$

$$= \frac{\sqrt{3}}{3} ln \left| \frac{\sqrt{3(1+x)} - \sqrt{1-x}}{\sqrt{3(1+x)} + \sqrt{1-x}} \right| + C$$

(l) $x = 2\sin\theta$ 라 하면, $\sqrt{4-x^2} = 2\cos\theta$, $dx = 2\cos\theta\, d\theta$

$$\int \frac{dx}{x\sqrt{4-x^2}} = \int \frac{2\cos\theta}{2\sin\theta\, 2\cos\theta} d\theta = \frac{1}{2} \int \operatorname{cosec}\theta\, d\theta$$

$$= -\frac{1}{2} \ln|\operatorname{cosec}\theta + \cot\theta| + C = -\frac{1}{2} \ln \left| \frac{2+\sqrt{4-x^2}}{x} \right| + C$$

(m) $x = \sec\theta$ 라 하면, $\sqrt{x^2-1} = \tan\theta$, $dx = \tan\theta \sec\theta\, d\theta$

$$\int \frac{\sqrt{x^2-1}}{x} dx = \int \frac{\tan\theta}{\sec\theta} \tan\theta \sec\theta\, d\theta = \int \tan^2\theta\, d\theta$$

$$= \int (1+\sec^2\theta)\, d\theta = \theta + \tan\theta = \sqrt{x^2-1} + \tan^{-1}\sqrt{x^2-1}$$

(n) $x = 3\tan\theta$ 라 하면, $\sqrt{x^2+9} = 3\sec\theta$, $dx = 3\sec^2\theta\, d\theta$

$$\int x\sqrt{x^2+9}\, dx = \int 3\tan\theta\, 3\sec\theta\, 3\sec^2\theta\, d\theta = 27 \int \tan\theta \sec^3\theta\, d\theta$$

$$= 27 \int \sec^2\theta \tan\theta \sec\theta d\theta = 27 \int u^2\, du$$

$$= 9u^2 + C = 9\sec^3\theta + C = \frac{\sqrt{x^2+9}}{3}(x^2+9) + C$$

7.7 연습문제

1.

풀이 (a) $S=\int_0^{2\pi}|\sin x|\,dx=\int_0^{\pi}\sin x\,dx-\int_{\pi}^{2\pi}\sin x\,dx$

$$=\left(-\cos x\right)\Big|_0^{\pi}+\cos x\Big|_{\pi}^{2\pi}=2+2=4$$

(b) $S=\int_{-\pi/3}^{\pi/3}|\tan x|\,dx=-\int_{-\pi/3}^{0}\tan x\,dx+\int_0^{\pi/3}\tan x\,dx$

$$=\left(\ln|\cos x|\right)\Big|_{-\pi/3}^{0}+\left(-\ln|\cos x|\right)\Big|_0^{\pi/3}=\ln 2+\ln 2=2\ln 2$$

(c) $S=-\int_{-2}^{1}(x^2+x-2)\,dx=-\left(\frac{1}{3}x^3+\frac{1}{2}x^2-2x\right)\Big|_{-2}^{1}=\frac{9}{2}$

(d) $S=\int_1^3|(x-1)(x-2)(x-3)|\,dx$

$$=\int_1^2(x-1)(x-2)(x-3)|\,dx-\int_2^3(x-1)(x-2)(x-3)\,dx$$

$$=\left(\frac{1}{4}x^4-2x^3+\frac{11}{2}x^2-6x\right)\Big|_1^2-\left(\frac{1}{4}x^4-2x^3+\frac{11}{2}x^2-6x\right)\Big|_2^3$$

$$=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

3.

풀이 (a) $x^2=\sqrt{x}$, $x(x^3-1)=0$으로부터 교점의 x좌표는 $x=0$, $x=1$이다.

$$S=\int_0^4|\sqrt{x}-x^2|\,dx=\int_0^1\left(\sqrt{x}-x^2\right)dx+\int_1^4\left(x^2-\sqrt{x}\right)dx$$

$$=\left(\frac{2}{3}x\sqrt{x}-\frac{1}{3}x^3\right)\Big|_0^1+\left(\frac{1}{3}x^3-\frac{2}{3}x\sqrt{x}\right)\Big|_1^4=\frac{1}{3}+\frac{49}{3}=\frac{50}{3}$$

(b) $x^2-x+1=-x^2+2x+3$으로부터 교점의 x좌표는 $x=-\frac{1}{2}$, $x=2$이다.

$$S=\int_{-1}^{2}|(x^2-x+1)-(-x^2+2x+3)|\,dx$$

$$=\int_{-1}^{-1/2}(2x^2-3x-2)\,dx-\int_{-1/2}^{2}(2x^2-3x-2)\,dx$$

$$=\left(\frac{2}{3}x^3-\frac{3}{2}x^2-2x\right)\Big|_{1}^{-1/2}-\left(\frac{2}{3}x^3-\frac{3}{2}x^2-2x\right)\Big|_{-1/2}^{2}$$

$$=\frac{17}{24}+\frac{125}{24}=\frac{71}{12}$$

(c) $\sin x=\cos x$ 로부터 교점의 x좌표는 $x=\frac{\pi}{4}$ 이다.

$$S=\int_{0}^{\pi/4}(\cos x-\sin x)\,dx+\int_{\pi/4}^{\pi}(\sin x-\cos x)\,dx$$

$$=(\sin x+\cos x)\big|_{0}^{\pi/4}-(\sin x+\cos x)\big|_{\pi/4}^{\pi}=2\sqrt{2}$$

(d) $\sin x=\sin 2x$ 로부터 교점의 x좌표는 $x=0,\ x=\frac{\pi}{3}$ 이다.

$$S=\int_{0}^{\pi}|\sin x-\sin 2x|\,dx=\int_{0}^{\pi/3}(\sin 2x-\sin x)\,dx+\int_{\pi/3}^{\pi}(\sin x-\sin 2x)\,dx$$

$$=\left(-\frac{1}{2}\cos 2x+\cos x\right)\Big|_{0}^{\pi/3}+\left(-\cos x+\frac{1}{2}\cos 2x\right)\Big|_{0}^{\pi/3}=\frac{1}{4}+\frac{9}{4}=\frac{5}{2}$$

5.

풀이 (a) $V=\pi\int_{0}^{\pi}\sin^2 x\,dx=\frac{\pi}{2}\int_{0}^{\pi}(1-\cos 2x)\,dx=\frac{\pi^2}{2}$

(b) $V=\pi\int_{0}^{\pi/4}\tan^2 x\,dx=\pi\int_{0}^{\pi/4}(\sec^2 x-1)\,dx$, x-축, $x=\pi/4$

$$=\pi(\tan x-x)\big|_{0}^{\pi/4}=\frac{\pi}{4}(4-\pi)$$

(c) $V=\pi\int_{0}^{\pi/2}\cos^2 x\,dx=\frac{\pi}{2}\int_{0}^{\pi}(1+\cos 2x)\,dx=\frac{\pi^2}{2}$

(d) $V=\pi\int_{0}^{\pi/3}(\sin^2 2x-\sin^2 x)\,dx=\frac{\pi}{2}\int_{0}^{\pi/3}(\cos 2x-\cos 4x)\,dx$

$$=\frac{\pi}{2}\left(\frac{1}{2}\sin 2x-\frac{1}{4}\sin 4x\right)\Big|_{0}^{\pi/3}=\frac{3\sqrt{3}}{16}\pi$$

$$=\frac{\pi}{2}\left(\frac{1}{2}sin 2x-\frac{1}{4}sin 4x\right)\Big|_{0}^{\pi/3}=\frac{3\sqrt{3}}{16}\pi$$

7.

풀이 곡선 $y=4-x^2$과 직선 $y=2$로 둘러싸인 부분을 $y=2$를 중심으로 회전한 회전체는 곡선 $y=2-x^2$과 x축으로 둘러싸인 부분을 x축을 중심으로 회전한 회전체와 동일하다. 한편 $4-x^2=2$를 만족하는 x축 좌표는 $x=-\sqrt{2}$, $x=\sqrt{2}$이므로 구하고자 하는 회전체의 부피는 다음과 같다.

$$V=\pi\int_{-\sqrt{2}}^{\sqrt{2}}(2-x^2)^2\,dx=\pi\left(\frac{1}{5}x^5-\frac{4}{3}x^3+4x\right)\Bigg|_{-\sqrt{2}}^{\sqrt{2}}=\frac{64\sqrt{2}\,\pi16}{15}$$

9.

풀이 (a) $y'=x^2,\ 1+(y')^2=1+x^2$

$$l=\int_0^1\sqrt{1+(y')^2}\,dx=\int_0^1\sqrt{1+x^2}\,dx=\int_0^{\pi/4}\sec^3\theta\,d\theta\quad(x=\tan\theta)$$

$$=\frac{1}{2}(\sec\theta\tan\theta+\ln|\sec\theta+\tan\theta|)\Big|_0^{\pi/4}=\frac{\sqrt{2}+\ln(1+\sqrt{2})}{2}$$

(b) $y'=(x-1)^{1/2},\ 1+(y')^2=1+(x-1)=x$

$$l=\int_1^4\sqrt{1+(y')^2}\,dx=\int_1^4\sqrt{x}\,dx=\frac{2}{3}x^{3/2}\Big|_1^4=\frac{14}{3}$$

(c) $\dfrac{dx}{dt}=4t,\ \dfrac{dy}{dt}=-2t,\ \left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2=(4t)^2+(-2t)^2=20t^2$

$$l=\int_1^2\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_1^3\sqrt{20t^2}\,dt$$

$$=\int_1^3 2\sqrt{5}\,t\,dt=2\sqrt{5}\left(\frac{1}{2}t^2\right)\Big|_1^3=8\sqrt{5}$$

(d) $\dfrac{dx}{dt}=6t,\ \dfrac{dy}{dt}=3t^2-3,\ \left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2=9(t^2+1)^2$

$$l=\int_0^4\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^4\sqrt{9(t^2+1)^2}\,dt=\int_0^4 3(t^2+1)\,dt$$

$$=3\left(\frac{1}{3}t^3+t\right)\Big|_0^4=76$$

제7장 연습문제

7.1

풀이 $\displaystyle\int \frac{x^3-2x+1}{x}dx = \int\left(x^2-2+\frac{1}{x}\right)dx = \frac{1}{3}x^3-2x+\ln|x|+C$

7.3

풀이 $u=x^3+1;\ ;\ \dfrac{1}{3}du=x^2dx$

$$\int \frac{2x^2}{(x^3+1)^2}dx=\frac{2}{3}\int\frac{1}{u^2}du=-\frac{2}{3}\frac{1}{u}+C=-\frac{2}{3(x^3+1)}+C$$

7.5

풀이 $u=x^3+3x+4;\ \dfrac{1}{3}du=(x^2+1)dx$

$$\int \frac{x^2+1}{\sqrt{x^3+3x+4}}dx=\frac{1}{3}\int u^{-1/2}du=\frac{2}{3}\sqrt{u}+C=\frac{2}{3}\sqrt{x^3+3x+4}+C$$

7.7

풀이 $u=(\ln x)^2 v'=1$이라 하면, $u'=\dfrac{2\ln x}{x}$, $v=x$이므로

$$\int(\ln x)^2dx=x(\ln x)^2-\int\frac{2\ln x}{x}\cdot x\,dx=x(\ln x)^2-2\int\ln x\,dx$$

$$=x(\ln x)^2-2x(-1+\ln x)+C=x\left[(\ln x)^2-2\ln x+2\right]+C$$

7.9

풀이 $\displaystyle\int x\sin 2x\,dx=-\frac{1}{2}x\cos 2x+\frac{1}{4}+C$

7.11

풀이 $$\int x(\ln x)^2\,dx = \frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx = \frac{1}{2}x^2(\ln x)^2 - \left(\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right)$$

$$= \frac{1}{2}x^2(\ln x)^2 - \left(\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right) = \frac{x^2}{4}\left[1 + 2(\ln x)^2 - 2\ln x\right]$$

7.13

풀이 $$\int \frac{x^2+x+1}{x^4-1}\,dx = \int\left(\frac{3/4}{x-1} + \frac{-1/4}{x+1} + \frac{-x/2}{x^2+1}\right)dx$$

$$= \frac{3}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{4}\int\frac{2x}{x^2+1}\,dx$$

$$= \frac{3}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + C$$

$$= \frac{1}{4}\ln\left|\frac{(x-1)^3}{(x+1)(x^2+1)}\right| + C$$

7.15

풀이 $$\int \frac{\sin^5 x}{\sqrt{\cos x}}\,dx = \int\frac{(1-\cos^2 x)^2}{\sqrt{\cos x}}\sin x\,dx = -\int(1-u^2)^2 u^{-1/2}\,dx$$

$$= -\frac{2}{45}\sqrt{\cos x}\,(45 - 18\cos^2 x + 5\cos^4 x) + C$$

7.17

풀이 $$\int \cos x\cos 2x\cos 3x\,dx = \frac{1}{2}\int(\cos 3x + \cos x)\cos 3x\,dx$$

$$= \frac{1}{2}\int(\cos^2 3x + \cos x\cos 3x)\,dx$$

$$= \frac{1}{2}\int\left(\frac{1+\cos 6x}{2} + \frac{\cos 4x + \cos 2x}{2}\right)dx$$

$$= \frac{1}{4}\int(1 + \cos 6x + \cos 4x + \cos 2x)\,dx$$

$$= \frac{1}{48}(12x + 2\sin 6x + 3\sin 4x + 6\sin 2x) + C$$

7.19

풀이 $\sqrt{\dfrac{x}{2-x}}=t$ 라 하면, $x=\dfrac{2t^2}{1+t^2}$, $dx=\dfrac{4t}{(1+t^2)^2}dt$

$$\int\sqrt{\frac{x}{2-x}}\,dx=4\int\frac{t^2}{(1+t^2)^2}dt=4\int\left(\frac{1}{1+t^2}-\frac{1}{(1+t^2)^2}\right)dt$$

한편 $t=\tan\theta$, $dt=\sec^2\theta\,d\theta$, $1+t^2=\sec^2\theta$이므로

$$\int\frac{1}{(1+t^2)^2}\,dt=\int\frac{1}{(\sec^2\theta)^2}\sec^2\theta\,d\theta=\int\cos^2\theta\,d\theta=\int\frac{1+\cos 2\theta}{2}d\theta$$

$$=\frac{1}{2}\left(\theta+\frac{1}{2}\sin 2\theta\right)=\frac{1}{2}(\theta+\sin\theta\cos\theta)$$

$$=\frac{1}{2}\left(\tan^{-1}t+\frac{t}{1+t^2}\right)$$

이고, 따라서

$$4\int\left(\frac{1}{1+t^2}-\frac{1}{(1+t^2)^2}\right)dt=2\left(\tan^{-1}t-\frac{t}{1+t^2}\right)+C$$

$$=2\left(\tan^{-1}\sqrt{\frac{x}{2-x}}-\sqrt{x(2-x)}\right)+C$$

7.21

풀이 $\displaystyle\int_0^1 x(x^2+4)\,dx=\int_4^5\frac{1}{2}u\,du=\frac{1}{4}u^2\Big|_4^5=\frac{9}{4}$

7.23

풀이 $\displaystyle\int_1^2\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx=\int_1^2\left(x-2+\frac{1}{x}\right)dx=\left(\frac{1}{2}x^2-2x+\ln x\right)\Big|_1^2$

$$=(-2+\ln 2)-\left(-\frac{3}{2}\right)=-\frac{1}{2}\ln 2$$

7.25

풀이 $\displaystyle\int_1^e\frac{1}{x(1+\ln x)^2}dx=\int_1^2\frac{1}{u^2}du=-\frac{1}{u}\Big|_1^2=-\frac{1}{2}+1=\frac{1}{2}$

7.27

풀이 $\int_0^{\pi} e^x \cos^2 x\, dx = \frac{1}{10} e^x (5 + \cos 2x + 2\sin 2x)\Big|_0^{\pi} = \frac{3}{5}(e^{\pi} - 1)$

7.29

풀이 생략

7.31

풀이 양변을 x에 관하여 미분하면,

$f(x) = f(x) + x f'(x) - 24x^3 + 18x^2;\ f'(x) = 24x^2 - 18x;$

$f(x) = \int (24x^2 - 18x)\, dx = 8x^3 - 9x^2 + C;\ f(1) = -1 + C = 0;$

$C = 1;\ f(x) = 8x^3 - 9x^2 + 1$

7.33

풀이 $f(x) = \int f'(x)\, dx = \int (2x - a)\, dx = x^2 - ax + C$이다. 한편 $\lim_{x \to 1} \frac{f(x)}{x-1} = a - 4$이고, $\lim_{x \to 1}(x-1) = 0$므로 $\lim_{x \to 1} f(x) = 0$이다. 따라서 $\lim_{x \to 1}(x^2 - ax + C) = 1 - a + C = 0;$

$C = a - 1;\ f(x) = x^2 - ax + a - 1;$

$$\lim_{x \to 1} \frac{f(x)}{x-1} = \lim_{x \to 1} \frac{x^2 - ax + a - 1}{x - 1} = \lim_{x \to 1}(x - a + 1) = 2 - a = a - 4$$

$$a = 3,\ C = 2;\ f(x) = x^2 - 3x + 2$$

7.35

풀이 $\int_0^4 (2e^x + 1)\, dx = (2e^4 + 4) - 2 = 2(e^4 + 1) = 4(2e^c + 1);\ e^c = \frac{e^4 - 1}{4};\ c = \ln \frac{e^4 - 1}{4}$

7.37

풀이 $f(x) = \int_0^x t(t-2)(t-1)\, dt = \frac{1}{4}x^4 - x^3 + x^2$이고, $f'(x) = x(x-2)(x-1) = 0$을 만족하는 x는 $x = 0,\ x = 1,\ x = 2$이다. 그러므로 극값 판정표로부터 최댓값은 $f(-2) = 16$ 이고, 최솟값은 $f(1) = \frac{1}{4}$이다.

x	-2		0		1		2
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$	16		극소 0		극대 $\frac{1}{4}$		0

7.39

풀이 $k=\int_1^e f(x)\,dx$라 하면, $f(x)=\frac{1}{x}\ln x-k$이고 따라서

$$k=\int_1^e\left(\frac{1}{x}\ln x-k\right)dx=\int_0^1 u\,du-k(e-1)=\frac{1}{2}-k(e-1)\ ;\ k=\frac{1}{2e}$$

$$f(x)=\frac{1}{x}\ln x-\frac{1}{2e}\ ;\ f(1)=-\frac{1}{2e};\ f(e)=\frac{1}{2e}$$

7.41

풀이 $$\frac{d}{dx}\int_{-x}^{x^2}(x-t)^2(t+2)\,dt=(x-x^2)^2(x^2+2)\cdot(2x)-(x-(-x))^2(-x+2)\cdot(-1)$$

$$=2x^3(x-1)^2(x^2+2)-4x^2(x-2)$$

$$=2x^7-\frac{14}{3}x^6+7x^5-10x^4+\frac{7}{3}x^3+14x^2$$

7.43

풀이 $$\frac{d}{dx}\int_x^{\sqrt{x}}\sqrt{t+1}\,dt=\frac{\sqrt{1+\sqrt{x}}}{2\sqrt{x}}-\sqrt{x+1}$$

7.45

풀이 $$\frac{d}{dx}\int_x^{x+2}e^t(\sin t+\cos t)\,dt=e^{x+2}[\sin(x+2)+\cos(x+2)]-e^x(\sin x+\cos x)$$

7.47

풀이 $$\frac{d}{dx}\int_x^{2x}(t-\ln t)\,dt=(2x-\ln 2x)\cdot 2-(x-\ln x)=3x-\ln 4x$$

7.49

풀이 $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2+k^2}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^2}{n^2+k^2}\frac{1}{n}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{1+\left(\frac{k}{n}\right)^2}\frac{1}{n}=\int_0^1\frac{1}{1+x^2}dx$

$=\tan^{-1}x\Big|_0^1=\frac{\pi}{4}$

7.51

풀이 $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right)=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n+k}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n+k}\frac{1}{n}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{1+\frac{k}{n}}\frac{1}{n}=\int_0^1\frac{1}{1+x}dx$

$=\ln|x+1|\Big|_0^1=\ln 2$

7.53

풀이 $-y^2+1=y^2-1$, $y^2-1=0$이므로 교점의 y좌표는 $y=-1$, $y=1$

$$A=\int_{-1}^{1}[(-y^2+1)-(y^2-1)]\,dy=\int_{-1}^{1}(-2y^2+2)\,dy=\left(-\frac{2}{3}y^3+2y\right)\Big|_{-1}^{1}=\frac{8}{3}$$

7.55

풀이 곡선 $y=\sqrt{x}$ 위의 임의의 점 $P(a,\sqrt{a})$에서 그은 접선의 기울기는 $f'(a)=\frac{1}{2\sqrt{a}}$이므로 $P(a,\sqrt{a})$에서의 접선의 방정식은 $y-\sqrt{a}=\frac{1}{2\sqrt{a}}(x-a)$이고, 이 직선이 점 $(0,1)$을 지나므로, $1-\sqrt{a}=\frac{1}{2\sqrt{a}}(-a)$ 즉, $a=4$이다. 따라서 접선의 방정식은 $y=\frac{1}{4}x+1$이다. 그러므로 접선과 곡선 그리고 y축으로 둘러싸인 부분의 넓이는 다음과 같다.

$$S=\int_0^4\left[\left(\frac{1}{4}x+1\right)-\sqrt{x}\right]dx=\left(\frac{1}{8}x^2+x-\frac{2}{3}x^{3/2}\right)\Big|_0^4=\frac{2}{3}$$

7.57

풀이 $V=\pi\int_{-3}^{3}y^2\,dx=\pi\int_{-3}^{3}\left(4-\frac{x^2}{4}\right)dx=\frac{39}{2}\pi$

7.59

풀이 기름 탱크의 부피를 V라 하면, V는 함수 $f(x)$를 x축을 중심으로 회전한 회전체의 부피이다. 따라서 탱크의 부피는 다음과 같다.

$$V=\pi\int_0^2[f(x)]^2\,dx=\pi\int_0^2\frac{x^4}{64}(2-x)\,dx=\frac{\pi}{64}\left[\frac{2}{5}x^5-\frac{1}{6}x^6\right]_0^2=\frac{\pi}{64}\frac{32}{15}=\frac{\pi}{30}$$

7.61

풀이 생략

7.63

풀이 $\frac{dx}{dt}=a(1-\cos t)$, $\frac{dy}{dt}=-a\sin t$, $\left(\frac{dx}{dt}\right)^2=a^2(1-\cos t)^2$, $\left(\frac{dy}{dt}\right)^2=a^2\sin^2 t$ 이므로 $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=a^2(1-\cos t)^2+a^2\sin^2 t=2a^2(1-\cos t)$이고, 따라서 곡선의 길이는 다음과 같다.

$$s=\int_0^{2\pi}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=2\sqrt{2}\,a\int_0^{\pi}\sqrt{1-\cos t}\,dt$$

$$=2\sqrt{2}\,a\int_0^{\pi}\sqrt{2}\cos\frac{t}{2}\,dt=4\,a\int_0^{\pi}\cos\frac{t}{2}\,dt$$

$$=4\,a\int_0^{\pi/2}2\cos u\,du\ (u=t/2)=8a\sin u\Big|_0^{\pi/2}=8a$$

7.65

풀이 (a) 전구의 부피를 V라 하면, V는 함수 $f(x)$를 x축을 중심으로 회전한 회전체의 부피이다. 따라서 전구의 부피는 다음과 같다.

$$V = \pi\int_0^{1/3}[f(x)]^2\,dx = \pi\int_0^{1/3}\frac{1}{9}x(1-3x)^2\,dx$$

$$= \frac{\pi}{9}\int_0^{1/3}(9x^3-6x^2+x)\,dx = \frac{\pi}{9}\left[\frac{9}{4}x^4-2x^3+\frac{1}{2}x^2\right]_0^{1/3} = \frac{\pi}{972}$$

(b) $f'(x) = \dfrac{1-9x}{6\sqrt{x}}$ 이므로 $1+[f'(x)]^2 = \dfrac{(1+9x)^2}{36x}$ 이고, 따라서

$$f(x)\sqrt{1+[f'(x)]^2} = \frac{1}{3}\sqrt{x}(1-3x)\frac{(1+9x)}{6\sqrt{x}} = \frac{1}{18}(1-3x)(1+9x)$$

이다. 그러므로 구하고자 하는 겉넓이는 다음과 같다.

$$A = 2\pi\int_0^{1/3}f(x)\sqrt{1+[f'(x)]^2}\,dx = \frac{\pi}{9}\int_0^{1/3}(1-3x)(1+9x)\,dx$$

$$= \frac{\pi}{9}\left[-9x^3+3x^2+x\right]_0^{1/3} = \frac{\pi}{27}$$