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[연습문제 답안 이용 안내]

Chapter 01 연습문제 답안

1.1 (a) $\displaystyle\lim_{x\to 0}\frac{x^2+x+2}{x-2}=\frac{\displaystyle\lim_{x\to 0}(x^2+x+2)}{\displaystyle\lim_{x\to 0}(x-2)}=\frac{2}{-2}=-1$

(b) $\displaystyle\lim_{x\to 1}\left(\frac{x-1}{2x^2+x-1}+\frac{2x+1}{x+2}\right)=\lim_{x\to 1}\left(\frac{x-1}{2x^2+x-1}\right)+\lim_{x\to 1}\left(\frac{2x+1}{x+2}\right)=\frac{0}{2}+\frac{3}{3}=1$

(c) $\displaystyle\lim_{x\to 0}\frac{\cos 2x}{\cos x+\sin x}=\frac{\displaystyle\lim_{x\to 0}(\cos 2x)}{\displaystyle\lim_{x\to 0}(\cos x+\sin x)}=1$

(d) $\displaystyle\lim_{x\to 0}\left(\frac{e^x+1}{x+1}\right)\left(\frac{1}{e^{-x}-2}\right)=\lim_{x\to 0}\left(\frac{e^x+1}{x+1}\right)\lim_{x\to 0}\left(\frac{1}{e^{-x}-2}\right)=2\cdot(-1)=-2$

1.2 (a) $\displaystyle\lim_{x\to -2}\frac{x^3+2x^2-x-2}{x^2+x-2}=\lim_{x\to -2}\frac{x^2(x+2)-(x+2)}{(x+2)(x-1)}=\lim_{x\to -2}\frac{(x^2-1)(x+2)}{(x+2)(x-1)}=\lim_{x\to -2}\frac{(x^2-1)}{(x-1)}=\lim_{x\to -2}(x+1)=-1$

(b) $\displaystyle\lim_{x\to 1}\frac{x^2+x-2}{x-1}=\lim_{x\to 1}\frac{(x-1)(x+2)}{(x-1)}=\lim_{x\to 1}(x+2)=3$

(c) $\displaystyle\lim_{x\to 0}\frac{\sin x\cos^2 x+2\sin x}{\sin^2 x-\sin x\cos x}=\lim_{x\to 0}\frac{\sin x(\cos^2 x+2)}{\sin x(\sin x-\cos x)}=\lim_{x\to 0}\frac{\cos^2 x+2}{\sin x-\cos x}=-3$

(d) $\displaystyle\lim_{x\to 0}\frac{e^{2x}-1}{e^x-1}=\lim_{x\to 0}\frac{(e^x-1)(e^x+1)}{(e^x-1)}=\lim_{x\to 0}(e^x+1)=2$

1.3 (a) $\displaystyle\lim_{x\to 2}\frac{\sqrt{3x-1}-\sqrt{2x+1}}{\sqrt{x+3}-\sqrt{2x+1}}=\lim_{x\to 2}\frac{(\sqrt{3x-1}-\sqrt{2x+1})(\sqrt{x+3}+\sqrt{2x+1})}{(\sqrt{x+3}-\sqrt{2x+1})(\sqrt{x+3}+\sqrt{2x+1})}$

$\displaystyle=\lim_{x\to 2}\frac{(\sqrt{3x-1}-\sqrt{2x+1})(\sqrt{x+3}+\sqrt{2x+1})}{-x+2}=\infty$

(b) $\displaystyle\lim_{x\to 1}\frac{-x+1}{\sqrt{x}-\sqrt{2x-1}}=\lim_{x\to 1}\frac{(-x+1)(\sqrt{x}+\sqrt{2x-1})}{(\sqrt{x}-\sqrt{2x-1})(\sqrt{x}+\sqrt{2x-1})}=\lim_{x\to 1}\frac{(-x+1)(\sqrt{x}+\sqrt{2x-1})}{x-(2x-1)}$

$\displaystyle=\lim_{x\to 1}\frac{(-x+1)(\sqrt{x}+\sqrt{2x-1})}{-x+1}=1+1=2$

(c) $\displaystyle\lim_{x\to 1}\frac{\sqrt{3x-2}-\sqrt{2x-1}}{x-1}=\lim_{x\to 1}\frac{(\sqrt{3x-2}-\sqrt{2x-1})(\sqrt{3x-2}+\sqrt{2x-1})}{(x-1)(\sqrt{3x-2}+\sqrt{2x-1})}=\lim_{x\to 1}\frac{3x-2-(2x-1)}{(x-1)(\sqrt{3x-2}+\sqrt{2x-1})}$

$\displaystyle=\lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{3x-2}+\sqrt{2x-1})}=\frac{1}{2}$

(d) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+4}-\sqrt{2x+4}}{x} = \lim_{x\to 0}\frac{(x+4)-(2x+4)}{x\left(\sqrt{x+4}+\sqrt{2x+4}\right)} = \lim_{x\to 0}\frac{-1}{\sqrt{x+4}+\sqrt{2x+4}} = -\frac{1}{4}$

1.4 (a) $\displaystyle\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{3x}{\sin 3x}\cdot\frac{2}{3} = \frac{2}{3}$

(b) $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{1}{1+\cos x} = \frac{1}{2}$

1.5 (a) $y = (x+2)^2(x^2-1)^3$

$\quad y' = 2(x+2)(x^2-1)^3 + 3(x+2)^2(x^2-1)^2\cdot 2x = 2(x+2)(x^2-1)^2(x^2-1+3x^2+6x)$

$\quad\quad = 2(x+2)(x^2-1)^2(4x^2+6x-1)$

(b) $y = \sqrt[4]{x^2+2x+1} = (x^2+2x+1)^{\frac{1}{4}}$

$\quad y' = \dfrac{1}{4}(x^2+2x+1)^{-\frac{3}{4}}(2x+2) = \dfrac{x+1}{2\sqrt[4]{(x^2+2x+1)^3}}$

(c) $y = \dfrac{x^2-x+1}{x+2}$

$\quad y' = \dfrac{(2x-1)(x+2)-(x^2-x+1)}{(x+2)^2} = \dfrac{2x^2+3x-2-x^2+x-1}{(x+2)^2} = \dfrac{x^2+4x-3}{(x+2)^2}$

(d) $y = \dfrac{(x^2+1)^2}{x^2+2}$

$\quad y' = \dfrac{4(x^2+1)\cdot x(x^2+2)-(x^2+1)^2\cdot 2x}{(x^2+2)^2} = \dfrac{2x(x^2+1)(2x^2+4-x^2-1)}{(x^2+2)^2} = \dfrac{2x(x^2+1)(x^2+3)}{(x^2+2)^2}$

1.6 (a) $y = (\sin 2x+\cos x)^2$

$\quad y' = 2(\sin 2x+\cos x)(2\cos 2x-\sin x)$

(b) $y = \cos x(2+\sin 2x)$

$\quad y' = -\sin x(2+\sin 2x)+2\cos x\cos 2x$

(c) $y = \dfrac{1-\sin x}{1+\cos x}$

$\quad y' = \dfrac{-\cos x(1+\cos x)+(1-\sin x)\sin x}{(1+\cos x)^2} = \dfrac{\sin x-\cos x-(\sin^2 x+\cos^2 x)}{(1+\cos x)^2} = \dfrac{\sin x-\cos x-1}{(1+\cos x)^2}$

(d) $y = \dfrac{\sin x}{\tan x + 1}$

$y' = \dfrac{\cos x(\tan x + 1) - \sin x \cdot \sec^2 x}{(\tan x + 1)^2}$

(e) $y = \dfrac{e^x + 2}{e^{2x} - 1}$

$y' = \dfrac{e^x(e^{2x} - 1) - 2(e^x + 2)e^{2x}}{(e^{2x} - 1)^2} = \dfrac{-e^x(e^{2x} + 4e^x + 1)}{(e^{2x} - 1)^2}$

(f) $y = \dfrac{2^x \sin x}{e^x + 1}$

$y' = \dfrac{(2^x \ln 2 \sin x + 2^x \cos x)(e^x + 1) - 2^x \sin x \cdot e^x}{(e^x + 1)^2}$

(g) $y = \log_2(\tan x)$

$y' = \dfrac{1}{\tan x} \cdot \dfrac{1}{\ln^2}(\tan x)' = \dfrac{1}{\tan x} \cdot \dfrac{1}{\ln^2} \sec^2 x$

(h) $y = \sin x \ln 3x$

$y' = \cos x \ln 3x + \sin x \cdot \dfrac{1}{3x} \cdot 3 = \cos x \ln 3x + \dfrac{\sin x}{x}$

(i) $y = \dfrac{1}{\sqrt[3]{x}} + 2\dfrac{\sin x}{\cos 2x}$

$y' = (x^{-\frac{1}{3}})' + 2\dfrac{\cos x(\cos 2x) + 2\sin x \sin 2x}{(\cos 2x)^2} = \dfrac{-1}{3\sqrt[3]{x^4}} + \dfrac{2\cos x(\cos 2x) + 4\sin x \sin 2x}{(\cos 2x)^2}$

(j) $y = x\left(\dfrac{1}{x^3} - \dfrac{2}{x+2}\right)$

$y' = \left(\dfrac{1}{x^3} - \dfrac{2}{x+2}\right) + x\left(-\dfrac{3}{x^4} + \dfrac{2}{(x+2)^2}\right)$

(k) $y = \cos\sqrt[3]{x} - \sqrt[3]{\sin x} + \cos(\sin x) = \cos x^{\frac{1}{3}} - \sin x^{\frac{1}{3}} + \cos(\sin x)$

$y' = -\dfrac{1}{3\sqrt[3]{x^2}}\sin\sqrt[3]{x} - \dfrac{1}{3\sqrt[3]{x^2}}\cos\sqrt[3]{x} - \sin(\sin x) \cdot \cos x$

$= -\dfrac{1}{3\sqrt[3]{x^2}}(\sin\sqrt[3]{x} + \cos\sqrt[3]{x}) - \cos x \sin(\sin x)$

(l) $y = \dfrac{(1 + 2\ln x)^2}{2x}$

$$y' = \frac{2(1+2\ln x) \cdot \frac{1}{x} - 2(1+2\ln x)^2}{2x^2} = \frac{(1+2\ln x) - x(1+2\ln x)^2}{x^3} = \frac{(1+2\ln x)(1-x-2x\ln x)}{x^3}$$

1.7 (a) $\displaystyle\lim_{x\to 1}\frac{2^x-2}{x-1} = \lim_{x\to 1}\frac{(2^x-2)'}{(x-1)'} = \lim_{x\to 1}\frac{2^x \cdot \ln 2}{1} = 2\ln 2$

(b) $\displaystyle\lim_{x\to 0}\frac{e^x-1}{x^2} = \lim_{x\to 0}\frac{e^x}{2x} = \infty$

(c) $\displaystyle\lim_{x\to 0}\frac{\sin x}{e^x-1} = \lim_{x\to 0}\frac{\cos x}{e^x} = 1$

(d) $\displaystyle\lim_{x\to 0}\frac{e^{2x}-e^{-2x}}{\sin x} = \lim_{x\to 0}\frac{2e^{2x}+2e^{-2x}}{\cos x} = 4$

(e) $\displaystyle\lim_{x\to 0}\frac{(1-\cos x)^2}{x} = \lim_{x\to 0}\frac{2(1-\cos x)\sin x}{1} = 0$

(f) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2} = \lim_{x\to 2}\frac{\frac{1}{2}(x+2)^{-\frac{1}{2}}}{1} = \frac{1}{4}$

1.8 (a) $\displaystyle\int\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)dx = \int(x^{\frac{1}{2}}+2x^{-\frac{1}{2}})dx = \frac{2}{3}x^{\frac{3}{2}}+4x^{\frac{1}{2}}+c = \frac{2}{3}x\sqrt{x}+4\sqrt{x}+c$

check : $\dfrac{d}{dx}(\dfrac{2}{3}x^{\frac{3}{2}}+4x^{\frac{1}{2}}+c) = x^{\frac{1}{2}}+2x^{-\frac{1}{2}} = \sqrt{x}+\dfrac{2}{\sqrt{x}}$

(b) $\displaystyle\int\frac{x^3-2x^2+x-2}{x-2}dx = \int\frac{x^2(x-2)+x-2}{x-2}dx = \int(x^2+1)dx = \frac{1}{3}x^3+x+c$

check : $\dfrac{d}{dx}\left(\dfrac{1}{3}x^3+x+c\right) = x^2+1$

(c) $\displaystyle\int(\sec^2 x - 2\cos x)dx = \tan x - 2\sin x + c$

check : $\dfrac{d}{dx}(\tan x - 2\sin x + c) = \sec^2 x + 2\cos x$

(d) $\displaystyle\int\frac{2^x-3^x-1}{2^x}dx = \int\left[1-\left(\frac{3}{2}\right)^x - 2^{-x}\right]dx = x - \frac{\left(\frac{3}{2}\right)^x}{\ln\frac{3}{2}} + \frac{2^{-x}}{\ln 2} + c$

check : $\dfrac{d}{dx}[x - \dfrac{\left(\frac{3}{2}\right)^x}{\ln\frac{3}{2}} + \dfrac{2^{-x}}{\ln 2} + c] = 1 - \dfrac{1}{\ln\frac{3}{2}}\left(\dfrac{3}{2}\right)^x\ln\dfrac{3}{2} - \dfrac{1}{\ln 2}2^{-x}\ln 2 = 1 - \left(\dfrac{3}{2}\right)^x - 2^{-x}$

1.9 (a) $\int (2x+3)dx = x^2+3x+c$

(b) $\int (2\sqrt{x}+3x^2)dx = \dfrac{4}{3}x^{\frac{3}{2}}+x^3+c$

(c) $\int (2\sin x+3\cos x)dx = -2\cos x+3\sin x+c$

(d) $\int \dfrac{1+\cos x-\sin^2 x}{\cos x}\,dx = \int (\dfrac{1+\cos x-1+\cos^2 x}{\cos x})dx = \int (1+\cos x)dx = x+\sin x+c$

1.10 (a) $\int \dfrac{2x}{\sqrt{x^2-2}}\,dx$ $x^2-2=t$라 치환. $2x\,dx=dt$

$\int \dfrac{2x}{\sqrt{x^2-2}}\,dx = \int \dfrac{1}{\sqrt{t}}\,dt = 2\sqrt{t}+c = 2\sqrt{x^2-2}+c$

(b) $\int 2x\cos x^2\,dx$ $x^2=t$라 치환. $2x\,dx=dt$

$\int 2x\cos x^2\,dx = \int \cos t\,dt = \sin t = \sin x^2$

(c) $\int \cos 2x\sin 2x\,dx$ $\sin 2x=t$라 치환. $2\cos 2x\,dx=dt$

$\int \cos 2x\sin 2x\,dx = \dfrac{1}{2}\int t\,dt = \dfrac{1}{4}t^2+c = \dfrac{1}{4}(\sin 2x)^2+c$

(d) $\int \dfrac{3\ln x}{x}\,dx$ $\ln x=t$라 치환. $\dfrac{1}{x}\,dx=dt$

$\int \dfrac{3\ln x}{x}\,dx = 3\int t\,dt = \dfrac{3}{2}t^2+c = \dfrac{3}{2}(\ln x)^2+c$

(e) $\int 2x\,e^{x^2}dx$ $x^2=t$라 치환. $2x\,dx=dt$

$\int 2x\,e^{x^2}dx = \int e^t\,dt = e^t+c = e^{x^2}+c$

(f) $\int \dfrac{2x+1}{\sqrt{x^2+x+1}}\,dx$ $x^2+x+1=t$라 치환. $(2x+1)\,dx=dt$

$\int \dfrac{2x+1}{\sqrt{x^2+x+1}}\,dx = \int \dfrac{1}{\sqrt{t}}\,dt = 2t^{\frac{1}{2}}+c = 2\sqrt{x^2+x+1}+c$

1.11 (a) $\int (x-2)e^x \, dx$

$$u' = e^x \qquad v = (x-2)$$
$$u = e^x \qquad v' = 1$$
$$\int (x-2)e^x \, dx = e^x(x-2) - \int e^x \, dx + c = xe^x - 3e^x + c$$

(b) $\int e^x \cos x \, dx$

$$u' = e^x \qquad v = \cos x$$
$$u = e^x \qquad v' = -\sin x$$
$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$$

예제 1-28의 결과를 사용하면

$$\int e^x \sin x \, dx = \frac{1}{2}e^x(\sin x - \cos x) + c \text{ 이므로}$$
$$\int e^x \cos x \, dx = e^x \cos x + \frac{1}{2}e^x(\sin x - \cos x) + c = \frac{1}{2}e^x(\sin x + \cos x) + c$$

(c) $\int e^x(\sin x - x) \, dx = \int (e^x \sin x - xe^x) \, dx$

예제 1-27과 예제1-28의 결과를 사용하면

$$\int (e^x \sin x) \, dx - \int (xe^x) \, dx = \frac{1}{2}e^x(\sin x - \cos x) - e^x(x-1) + c$$

(d) $\int x \ln x \, dx$

$$u' = x \qquad v = \ln x$$
$$u = \frac{1}{2}x^2 \qquad v' = \frac{1}{x}$$
$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

1.12 (a) $\displaystyle\int_1^2 \frac{x^2+x-2}{2x} \, dx = \int_1^2 \left(\frac{x}{2} + \frac{1}{2} - x^{-1}\right) dx = \frac{1}{4}x^2 + \frac{1}{2}x - \ln x \Big|_1^2$

$$= 1 + 1 - \ln 2 - \left(\frac{1}{4} + \frac{1}{2} + 0\right) = \frac{5}{4} - \ln 2$$

(b) $\displaystyle\int_0^2 \frac{1}{\sqrt{2x-1}-\sqrt{2x}}\,dx = \int_0^2 \frac{\sqrt{2x-1}+\sqrt{2x}}{(\sqrt{2x-1}-\sqrt{2x})(\sqrt{2x-1}+\sqrt{2x})}\,dx$

$\displaystyle\qquad\qquad\qquad\qquad\qquad = -\int_0^2 (\sqrt{2x-1}+\sqrt{2x})\,dx$

$\displaystyle\qquad\qquad\qquad\qquad\qquad = -\left[\frac{1}{3}(2x-1)^{\frac{3}{2}}+\frac{1}{3}(2x)^{\frac{3}{2}}\right]_0^2$

$\displaystyle\qquad\qquad\qquad\qquad\qquad = -\frac{1}{3}(3)^{\frac{3}{2}}-\frac{1}{3}(2)^3+\frac{1}{3}(-1)^{\frac{3}{2}}$

(c) $\displaystyle\int_0^1 \frac{\cos x}{1-\cos^2 x}\,dx = \int_0^1 \frac{\cos x}{\sin^2 x}\,dx$　　$\sin x = t$ 라 치환.　　$\cos x\,dx = dt$

$\displaystyle\int_0^1 \frac{\cos x}{\sin^2 x}\,dx = \int_0^{\sin 1}\frac{1}{t^2}\,dt = -\frac{1}{t}\bigg|_0^{\sin 1} = -\frac{1}{\sin 1}+\frac{1}{0}=\infty$

(d) $\displaystyle\int_0^2 \sin x \cos^3 x\,dx$　　$\cos x = t$ 라 치환.　　$-\sin x\,dx = dt$

$\displaystyle\int_0^2 \sin x \cos^3 x\,dx = -\int_1^{\cos 2} t^3\,dt = -\frac{1}{4}t^4\bigg|_1^{\cos 2} = -\frac{1}{4}\cos^4 2+\frac{1}{4}$

(e) $\displaystyle\int_0^{\pi} x^2 \cos x\,dx$　　부분적분 이용

$u' = \cos x$　　$v = x^2$

$u = \sin x$　　$v' = 2x$

$\displaystyle\int_0^{\pi} x^2 \cos x\,dx = x^2 \sin x\bigg|_0^{\pi} - 2\int_0^{\pi} x\sin x\,dx$

$\displaystyle\int_0^{\pi} x\sin x\,dx$ 을 구하기 위해 부분적분 이용

$u' = \sin x$　　$v = x$

$u = -\cos x$　　$v' = 1$

$\displaystyle\int_0^{\pi} x\sin x\,dx = -x\cos x\bigg|_0^{\pi} + \int_0^{\pi}\cos x\,dx = \pi + \sin x\bigg|_0^{\pi} = \pi$

$\displaystyle\therefore \int_0^{\pi} x^2 \cos x\,dx = -2\int_0^{\pi} x\sin x\,dx = -2\pi$

Chapter 02 연습문제 답안

2.1 (a) $\dfrac{dy}{dx} + (x+1)y^2 = 0$, $\dfrac{dy}{dx} = -(x+1)y^2$, $\dfrac{dy}{y^2} = -(x+1)dx$ 양변적분하면,

$$-\frac{1}{y} = -(\frac{1}{2}x^2 + x + c) \quad y = \frac{1}{\frac{1}{2}x^2 + x + c}$$

(b) $\dfrac{dy}{dx} - x(1+y^2) = 0$, $\dfrac{dy}{dx} = x(1+y^2)$, $\dfrac{1}{1+y^2}dy = x\,dx$, $\tan^{-1}y = \frac{1}{2}x^2 + c$,

$$y = \tan\left(\frac{1}{2}x^2 + c\right)$$

(c) $\dfrac{dy}{dx} - ye^x = 0$, $\dfrac{dy}{dx} = ye^x$, $\dfrac{1}{y}dy = e^x dx$, $\ln y = e^x + c_1$, $y = ce$

(d) $\dfrac{dy}{dx} + \dfrac{y}{x} = 0$, $\dfrac{1}{y}dy = -\dfrac{1}{x}dx$, $\ln y = -\ln x + c_1$, $y = c\dfrac{1}{x}$

=> (한빛아카데미 판) 본문 문제 수정됨

(e) $\dfrac{dy}{dx} + 2y - xy = 0$, $\dfrac{dy}{dx} = y(x-2)$, $\dfrac{dy}{y} = (x-2)dx$, $\ln y = \frac{1}{2}x^2 - 2x + c_1$,

$$y = e^{\frac{1}{2}x^2 - 2x + c_1} = c\,e^{\frac{1}{2}x^2 - 2x}$$

(f) $\dfrac{x}{y}\dfrac{dy}{dx} + x - 2 = 0$, $\dfrac{dy}{y} = (1 - \dfrac{2}{x})dx$, $\ln y = x - 2\ln x + c_1$, $y = e^{x - 2\ln x + c_1} = c\dfrac{1}{x^2}e^x$

(g) $\dfrac{1}{y}\dfrac{dy}{dx} + 2\cos x = 0$, $\dfrac{dy}{y} = -2\cos x\,dx$, $\ln y = -2\sin x + c_1$, $y = ce^{-2\sin x}$

(h) $\dfrac{dy}{dx} + \dfrac{2x}{y+1} - \dfrac{2x}{y} = 0$, $\dfrac{dy}{dx} = (\dfrac{1}{y} - \dfrac{1}{y+1})2x$, $\dfrac{dy}{dx} = \dfrac{y+1-y}{y(y+1)}2x$, $\dfrac{dy}{dx} = \dfrac{1}{y(y+1)}2x$,

$y(y+1)dy = 2x\,dx$, $\dfrac{1}{3}y^3 + \dfrac{1}{2}y^2 = x^2 + c$

(i) $\dfrac{dy}{dx} = \dfrac{y}{x}\ln x$, $\dfrac{dy}{y} = \dfrac{1}{x}\ln x\,dx$, $\displaystyle\int \dfrac{dy}{y} = \int \dfrac{1}{x}\ln x\,dx$, $\ln x = u$와 치환, $\dfrac{1}{x}dx = du$,

$\displaystyle\int \dfrac{1}{x}\ln x\,dx = \int u\,du = \dfrac{1}{2}u^2 + c_1$, $\displaystyle\int \dfrac{dy}{y} = \dfrac{1}{2}(\ln x)^2 + c_1$,

$\ln y = \dfrac{1}{2}(\ln x)^2 + c_1,\ \ y = ce^{\frac{1}{2}(\ln x)^2}$

(j) $\dfrac{dy}{dx} = y\tan x,\ \ \dfrac{dy}{y} = \tan x\,dx,\ \ \ln y = -\ln\cos x + c_1,\ \ y = c\sec x$

(k) $\dfrac{dy}{dx} - \dfrac{x^2 y^2}{1+x^2} = 0,\ \ \dfrac{dy}{y^2} = \dfrac{x^2}{1+x^2}dx,\ \ y = -\dfrac{1}{x - \tan^{-1}x + c}$

=> (한빛아카데미 판) 본문 문제 수정됨

(l) $\dfrac{dy}{dx} + \dfrac{1}{y}\cos x = 0,\ \ \dfrac{dy}{dx} = -\dfrac{1}{y}\cos x,\ \ y\,dy = -\cos x\,dx,$

$\dfrac{1}{2}y^2 = -\sin x + c_1,\ \ y^2 = -2\sin x + c$

(m) $\dfrac{1}{x}\dfrac{dy}{dx} = y^2 - y,\ \ y(1) = 2$

$\dfrac{1}{y^2 - y}dy = x\,dx,\ \ \dfrac{1}{y(y-1)}dy = x\,dx,\ \ (-\dfrac{1}{y} + \dfrac{1}{y-1})dy = x\,dx,$

$-\ln y + \ln(y-1) = \dfrac{1}{2}x^2 + c_1,$

$\ln\dfrac{y-1}{y} = \dfrac{1}{2}x^2 + c_1,\ \ \dfrac{y-1}{y} = ce^{\frac{1}{2}x^2},\ \ y(1) = 2$이므로 $\dfrac{2-1}{2} = ce^{\frac{1}{2}},\ \ c = \dfrac{1}{2}e^{-\frac{1}{2}},$

$\dfrac{y-1}{y} = \dfrac{1}{2\sqrt{e}}e^{-\frac{1}{2}x^2}$

(n) $\dfrac{dy}{dx} + \dfrac{x}{y} = 0,\ \ y(0) = -2$

$\dfrac{dy}{dx} = -\dfrac{x}{y},\ \ y\,dy = -x\,dx,\ \ \dfrac{1}{2}y^2 = -\dfrac{1}{2}x^2 + c_1,\ \ y^2 = -x^2 + c,\ \ y(0) = -2$이므로

$(-2)^2 = c,\ \ y^2 = -x^2 + 4$

(o) $y\dfrac{dy}{dx} + x - 1 = 0,\ \ y(1) = 1$

$y\dfrac{dy}{dx} = -(x-1),\ \ y\,dy = -(x-1)dx,\ \ \dfrac{1}{2}y^2 = -(\dfrac{1}{2}x^2 - x) + c,$

$y(1) = 1$이므로 $\dfrac{1}{2} = \dfrac{1}{2} + c,\ \ c = 0$

$\dfrac{1}{2}y^2 = -\dfrac{1}{2}x^2 + x,\ \ y^2 = -x^2 + 2x$

(p) $\dfrac{dy}{dx} = e^{-x} - 1,\ \ y(0) = 1$

$dy = (e^{-x} - 1)dx,\ \ y = -e^{-x} - x + c,\ \ y(0) = 1$이므로 $1 = -1 + c,\ \ c = 2$

$$\therefore y = -e^{-x} - x + 2$$

(q) $\dfrac{dy}{dx} = -\dfrac{y^2}{x^2}, \ y(1) = 2$

$\dfrac{dy}{y^2} = -\dfrac{dx}{x^2}, \ -\dfrac{1}{y} = \dfrac{1}{x} + c, \ y(1) = 2$이므로 $\ -\dfrac{1}{2} = 1 + c, \ c = -\dfrac{3}{2}, \ -\dfrac{1}{y} = \dfrac{1}{x} - \dfrac{3}{2},$

$\dfrac{1}{y} = \dfrac{3x-2}{2x}, \ y = \dfrac{2x}{3x-2}$

(r) $\dfrac{dy}{dx} = \dfrac{y}{2+x}, \ y(0) = 1, \ \dfrac{1}{y}dy = \dfrac{1}{x+2}dx, \ \ln y = \ln(x+2) + \ln c, \ y = c(x+2),$

$y(0) = 1$이므로 $\ 1 = 2c, \ c = \dfrac{1}{2}$

$y = \dfrac{1}{2}(x+2)$

2.2 $\ v(t) = -gt + 10$

$v(1) = -g + 10 = -9.8 + 10 = 0.2(m/s)$

2.3 $\ w(t) = w_0 e^{-\frac{c}{J}t}$

2.4 (a) $y = -x - 1 + 2e^x$

(b) $y = -\dfrac{1}{2} + \dfrac{3}{2}e^{2x}$

(c) $y = x[\ln x + 1]$ => (한빛아카데미 판) 본문 문제 수정됨

(d) $y = -x - 3 + 5e^x$

(e) $y = e^x\left[\displaystyle\int e^x e^{-x} dx + c\right] = e^x(x + c)$

(f) $y = \dfrac{1}{2}(\cos x + \sin x) + ce^{-x}$

(g) $y = ce^{-\frac{1}{2}\ln(x^2-4)} = ce^{\ln\frac{1}{\sqrt{x^2-4}}} = c\dfrac{1}{\sqrt{x^2-4}}$

(h) $y = \dfrac{1}{x}[-xe^{-x} - e^{-x} + c] = -e^{-x} - \dfrac{1}{x}e^{-x} + \dfrac{c}{x}$

(i) $y = \dfrac{1}{-\dfrac{1}{2}x^3 + cx} = \dfrac{2}{2cx - x^3}$

(j) $y = \dfrac{1}{\dfrac{1}{2}e^{-x} + ce^x}$

2.5 (a) $f(x,y) = x^2 + y^2x + c_1 = c$ $c - c_1 = c$라 놓으면 $x^2 + y^2x = c$

(b) $f(x,y) = x^2y + xy + c_1 = c$, $c - c_1 = c$라 놓으면 $x^2y + xy = c$

=> (한빛아카데미 판) 본문 문제 수정됨

(c) $f(x,y) = \dfrac{1}{2}x^2 - x + \dfrac{1}{2}y^2 + y + c_1 = c$, $\dfrac{1}{2}x^2 - x + \dfrac{1}{2}y^2 + y = c$

(d) $f(x,y) = 2x^2 + xy - 2y^2 + c_1 = c$, $2x^2 + xy - 2y^2 = c$

(e) $f(x,y) = 2e^x + 2xy - ye^{-y} - e^{-y} = c$

(f) $f(x,y) = \dfrac{1}{3}x^3 + y^2x + y = c$

(g) $f(x,y) = -xy + \dfrac{1}{3}y^3 = c$, $y(0) = 1$이므로 $f(0,1) = \dfrac{1}{3} = c$

$\quad -xy + \dfrac{1}{3}y^3 = \dfrac{1}{3}$

(h) $f(x,y) = y^2x + 2y = c$, $y(0) = 1$이므로 $f(0,1) = 2 = c$

$\quad y^2x + 2y = 2$

2.6 (a) $f(x,y) = xy^3 - 3x^2y^2 = C$

(b) $f(x,y) = 2xye^{\frac{1}{2}x} = c$

(c) $f(x,y) = -\dfrac{y+1}{x+1} = c$

(d) $f(x,y) = x^2y = c$

(e) $\dfrac{1}{2}x^2y^2 + \dfrac{1}{2}y^4 = c$

(f) $y = \dfrac{c}{\sqrt{x}}$

(g) $y = \dfrac{1}{2}(-2 + 4x - 4x^2 + 18e^{-2x})^{\frac{1}{2}}$

(h) $y = (4 - \ln(x^2+1))^{\frac{1}{2}}$

2.7 $q(t) = -5e^{-2t} + 5$, $q(0.2) = -5e^{-2(0.2)} + 5 = 5 - 5e^{-0.4}$

Chapter 03 연습문제 답안

3.1 (a) 특성방정식 : $\lambda^2 - 3\lambda - 4 = 0$,

특성값 : $(\lambda - 4)(\lambda + 1) = 0$, $\quad \lambda_1 = 4, \lambda_2 = -1$

기본해 : $y_1 = e^{4x}, y_2 = e^{-x}$

일반해 : $y = c_1 e^{4x} + c_2 e^{-x}$

(b) 특성방정식 : $\lambda^2 - 4 = 0$

특성값 : $\lambda_1 = 2, \lambda_2 = -2$

일반해 : $y = c_1 e^{2x} + c_2 e^{-2x}$

(c) 특성방정식 : $\lambda^2 + 4 = 0$

특성값 : $\lambda_1 = 2i, \lambda_2 = -2i$

일반해 : $y = c_1 \cos 2x + c_2 \sin 2x$

(d) 특성방정식 : $\lambda^2 - 1 = 0$

특성값 : $\lambda_1 = 1, \lambda_2 = -1$

일반해 : $y = c_1 e^x + c_2 e^{-x}$

(e) 특성방정식 : $\lambda^2 - \lambda - 2 = 0$

특성값 : $\lambda_1 = 2, \lambda_2 = -1$

일반해 : $y = c_1 e^{2x} + c_2 e^{-x}$

(f) 특성방정식 : $\lambda^2 + 4\lambda + 3 = 0$

특성값 : $\lambda_1 = -1, \lambda_2 = -3$

일반해 : $y = c_1 e^{-x} + c_2 e^{-3x}$

(g) 특성방정식 : $\lambda^2 - \lambda - 6 = 0$

특성값 : $\lambda_1 = 3, \lambda_2 = -2$

일반해 : $y = c_1 e^{3x} + c_2 e^{-2x}$

(h) 특성방정식 : $\lambda^2 + 3\lambda - 10 = 0$

특성값 : $\lambda_1 = 2, \lambda_2 = -5$

일반해 : $y = c_1 e^{2x} + c_2 e^{-5x}$

(i) 특성방정식 : $\lambda^2 + 2\lambda + 1 = 0$

　 특성값 : $\lambda_1, \lambda_2 = -1$

　 일반해 : $y = c_1 e^{-x} + c_2 x e^{-x}$

(j) 특성방정식 : $\lambda^2 - 6\lambda + 9 = 0$

　 특성값 : $\lambda_1, \lambda_2 = 3$

　 일반해 : $y = c_1 e^{3x} + c_2 x e^{3x}$

(k) 특성방정식 : $\lambda^2 - 4\lambda + 4 = 0$

　 특성값 : $\lambda_1, \lambda_2 = 2$

　 일반해 : $y = c_1 e^{2x} + c_2 x e^{2x}$

(l) 특성방정식 : $\lambda^2 + 6\lambda + 9 = 0$

　 특성값 : $\lambda_1, \lambda_2 = -3$

　 일반해 : $y = c_1 e^{-3x} + c_2 x e^{-3x}$

(m) 특성방정식 : $\lambda^2 + 8\lambda + 16 = 0$

　 특성값 : $\lambda_1, \lambda_2 = -4$

　 일반해 : $y = c_1 e^{-4x} + c_2 x e^{-4x}$

(o) 특성방정식 : $\lambda^2 - 2\lambda + 3 = 0$

　 특성값 : $\lambda_1 = 1 + \sqrt{2}\,i, \quad \lambda_2 = 1 - \sqrt{2}\,i$

　 일반해 : $y = e^x (c_1 \cos \sqrt{2}\,x + c_2 \sin \sqrt{2}\,x)$

(p) 특성방정식 : $\lambda^2 - 2\lambda + 2 = 0$

　 특성값 : $\lambda_1 = 1 + i, \quad \lambda_2 = 1 - i$

　 일반해 : $y = e^x (c_1 \cos x + c_2 \sin x)$

(q) 특성방정식 : $\lambda^2 - 3\lambda + 3 = 0$

　 특성값 : $\lambda_1 = \dfrac{3 + \sqrt{3}\,i}{2}, \quad \lambda_2 = \dfrac{3 - \sqrt{3}\,i}{2}$

　 일반해 : $y = e^{\frac{3}{2}x} \left(c_1 \cos \dfrac{\sqrt{3}}{2} x + c_2 \sin \dfrac{\sqrt{3}}{2} x \right)$

(r) 특성방정식 : $\lambda^2 - \lambda + 3 = 0$

　 특성값 : $\lambda_1 = \dfrac{1 + \sqrt{11}\,i}{2}, \quad \lambda_2 = \dfrac{1 - \sqrt{11}\,i}{2}$

일반해 : $y = e^{\frac{1}{2}x}(c_1\cos\frac{\sqrt{11}}{2}x + c_2\sin\frac{\sqrt{11}}{2}x)$

3.2　(a)　$y = \frac{2}{3}e^{3x} + \frac{1}{3}e^{-3x}$

　　　(b)　$y = -e^{-x}$

　　　(c)　$y = -\frac{1}{2}e^{4x} - \frac{1}{2}e^{-2x}$

　　　(d)　$y = -\frac{1}{3}e^{x} + \frac{1}{3}e^{-2x}$

　　　(e)　$y = 2e^{-3x} + 5xe^{-3x}$

　　　(f)　$y = 2e^{2x} - 4xe^{2x}$

　　　(g)　$y = e^{x}(2\cos2x - 2\sin2x)$

　　　(h)　$y = e^{\frac{1}{2}x}(-2\cos\frac{\sqrt{15}}{2}x + \frac{4}{\sqrt{15}}sin\frac{\sqrt{15}}{2}x)$

3.3　(a)　$y = c_1e^{3x} + c_2e^{-2x} - \frac{1}{4}e^{-x}$

　　　(b)　$y = y_c + y_p = c_1e^{2x} + c_2e^{-4x} - \frac{1}{8}x^2 + \frac{1}{16}x - \frac{1}{64}$

　　　(c)　$y = y_c + y_p = c_1e^{-x} + c_2e^{-2x} + \frac{1}{10}\cos x + \frac{3}{10}\sin x$

　　　(d)　$y = c_1e^{x} + c_2e^{-3x} - \frac{1}{5}\cos x - \frac{2}{5}\sin x$

　　　(e)　$y = c_1e^{-2x} + c_2 xe^{-2x} + \frac{1}{9}e^{x} + \frac{1}{4}x - \frac{1}{4}$

　　　(f)　$y = c_1e^{-3x} + c_2xe^{-3x} + \frac{15}{289}e^{x}\cos x + \frac{8}{289}e^{x}\sin x$

　　　(g)　$y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{2}xe^{2x}$

　　　(h)　$y = c_1e^{-2x} + c_2xe^{-2x} + \frac{1}{2}x^2e^{-2x}$

3.4　(a)　$y = -\frac{7}{25}e^{x} + \frac{7}{25}e^{-4x} + \frac{2}{5}xe^{x}$

　　　(b)　$y = \frac{1}{2}e^{2x} + \frac{10}{17}e^{-3x} - \frac{1}{34}(3\cos x + 5\sin x)$

　　　(c)　$y = \frac{7}{4}e^{x} + \frac{3}{20}e^{3x} + \frac{1}{10}\cos x - \frac{1}{5}\sin x - e^{-2x}$

　　　(d)　$y = \frac{5}{2}e^{-x} - \frac{75}{272}e^{-4x} + \frac{1}{4}x - \frac{5}{16} + \frac{3}{34}\cos x + \frac{5}{34}\sin x$

　　　(e)　$y = -\frac{1}{4}e^{2x} - \frac{5}{2}xe^{x} + \frac{1}{2}x^2 + x + \frac{5}{4}$

(f) $y = \dfrac{35}{54}e^{3x} - \dfrac{14}{9}xe^{3x} + \dfrac{1}{2}e^{x} + \dfrac{1}{9}x - \dfrac{4}{27}$

(g) $y = \dfrac{31}{45}e^{3x} + \dfrac{37}{90}e^{-3x} + \dfrac{1}{6}xe^{3x} - \dfrac{1}{10}\cos x$

(h) $y = 2e^{x}\sin x + 1$

3.5 (a) $y = c_1 e^{x} + c_2 e^{-4x} + \dfrac{5}{13}e^{x}\sin x - \dfrac{1}{13}e^{x}\cos x$

(b) $y = c_1 e^{-x} + c_2 e^{2x} - x^2 + x - 1$

(c) $y = c_1 e^{x} + c_2 e^{3x} - e^{2x} + \dfrac{1}{10}\cos x - \dfrac{1}{5}\sin x$

(d) $y = c_1 e^{-x} + c_2 e^{-4x} + \dfrac{1}{4}x - \dfrac{5}{16} + \dfrac{1}{34}(3\cos x + 5\sin x)$

(e) $y = c_1 e^{2x} + c_2 xe^{2x} + \left(-\dfrac{8}{25}x - \dfrac{44}{125}\right)\sin x + \left(\dfrac{6}{25}x + \dfrac{8}{125}\right)\cos x$

(f) $y = c_1 e^{2x} + c_2 xe^{2x} - \dfrac{1}{32}e^{x} + \dfrac{1}{16}xe^{x}$

(g) $y = \dfrac{26}{45}e^{2x} + \dfrac{1}{5}e^{-2x} - \dfrac{1}{3}xe^{-x} + \dfrac{2}{9}e^{-x} - \dfrac{1}{5}\sin x$

(h) $y = \dfrac{1}{5}e^{x}(3\sin x - \cos x) - \dfrac{2}{5}\sin x + \dfrac{1}{5}\cos x + x + 1$

3.6 (a) $y = c_1 x^{\lambda_1} + c_2 x^{\lambda_2}$

(b) $y = c_1 x^{-\frac{1}{2}} + c_2 x^{-1} = c_1 \dfrac{1}{\sqrt{x}} + c_2 \dfrac{1}{x}$

(c) $y = x^{-2}(c_1 + c_2 \ln|x|) = \dfrac{1}{x^2}(c_1 + c_2 \ln|x|)$

(d) $y = x^{-\frac{1}{2}}\left[c_1 \cos\left(\dfrac{1}{2}ln|x|\right) + c_2 \sin\left(\dfrac{1}{2}ln|x|\right)\right]$

3.7 (a) $y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x}$

(b) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-2x}$

(c) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

(d) $y = c_1 e^{\frac{1}{2}x} + c_2 e^{-x} + c_3 e^{-2x}$

(e) $y = c_1 e^{-x} + c_2 x e^{-x} + c_3 e^{2x}$

(f) $y = c_1 e^{x} + e^{-2x}(c_2 + c_3 x)$

(g) $y = e^{-x}(c_1 + c_2 x + c_3 x^2) + c_4 e^{2x}$

(h) $y = c_1 e^{5.1349x} + e^{0.4325x}(c_2 \cos 1.1708x + c_3 \sin 1.1708x)$

(i) $y = c_1 e^{x} + c_2 \cos x + c_3 \sin x$

(j) $y = c_1 \cos x + c_2 \sin x + c_3 \sin 2x + c_4 \cos 2x$

(k) $y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$

(l) $y = c_1 e^x + c_2 \cos x + c_3 \sin x + c_4 x \cos x + c_5 x \sin x$

3.8 (a) $-2c_1 e^{-2t} - 4c_2 e^{-4t} = -3(c_1 e^{-2t} + c_2 e^{-4t}) + y, \quad y = c_1 e^{-2t} - c_2 e^{-4t}$

(b) $c_1 e^t - 2c_2 e^{-2t} = -2(c_1 e^t + c_2 e^{-2t}) - 4y, \quad y = \frac{1}{4} c_1 e^t + c_2 e^{-2t}$

(c) $c_1 \cos t - c_2 \sin t = y$

(d) $\frac{dx}{dt} = x + c_1 e^t$ 　　비동차형 1계 선형미분방정식

$x = (c_1 + c_2 t) e^t$

(e) $x = c_1 e^{3t} + c_2 e^{-t}$

$y = -c_1 e^{3t} + c_2 e^{-t}$

(f) $x = e^t(c_1 \cos 2t - c_2 \sin 2t)$

$y = e^t(c_1 \sin 2t + c_2 \cos 2t)$

(g) $x = c_1 e^t - \frac{1}{5} c_2 e^{-4t}$

$y = c_2 e^{-4t}$

(h) $x = -e^{2t}(c_1 + c_2 + c_2 t)$

$y = e^{2t}(c_1 + c_2 t)$

(i) $x = c_1 e^{5t} + c_2 e^{-2t} + \frac{1}{5} t - \frac{23}{50}$

$y = \frac{4}{3} c_1 e^{5t} - c_2 e^{-2t} - \frac{2}{5} t + \frac{11}{50}$

(j) $x = c_1 e^{-5t} + c_2 e^{-2t} + \frac{7}{4} e^{-t}$

$y = -c_1 e^{-5t} + \frac{1}{2} c_2 e^{-2t} + \frac{5}{4} e^{-t}$

(k) $x = -2c_1 e^{-3t} - \frac{1}{2} c_2 e^{3t} + \frac{5}{9} t + \frac{7}{9}$

$y = c_1 e^{-3t} + c_2 e^{3t} - \frac{4}{9} t - \frac{10}{9}$

(l) $x = c_1 e^{3t} + c_2 e^{5t} - \frac{61}{130} \cos t + \frac{7}{130} \sin t$

$y = -c_1 e^{3t} - 2c_2 e^{5t} - \frac{27}{130} \sin t + \frac{31}{130} \cos t$

3.9 (a) $x = e^t, \ y = 0$ => (한빛아카데미 판) 본문 문제 수정됨

(b) $x = 3e^{-2t} - e^{-4t}, \ y = 2e^{-2t} - e^{-4t}$

(c) $x = -2\sin t - \cos t, \ y = -2\cos t + \sin t$

(d) $x = e^t(2t + 1), \ y = 2e^t$

(e) $x = \dfrac{8}{7}e^{5t} + \dfrac{13}{7}e^{-2t}, \ y = \dfrac{6}{7}e^{5t} - \dfrac{13}{7}e^{-2t}$

(f) $x = e^{-t} - 4e^{-4t}, \ y = 4e^{-t} - 4e^{-4t}$

(g) $x = \dfrac{2}{5}e^{-4t} - \dfrac{12}{5}e^{t}, \ y = -2e^{-4t}$

(h) $x = 2te^{2t}, \ y = -e^{2t}(2t-2)$

(i) $x = \dfrac{11}{9}e^{-3t} - \dfrac{29}{18}e^{3t} + \dfrac{1}{2}e^{t} + \dfrac{8}{9}$

$\quad y = -\dfrac{11}{18}e^{-3t} + \dfrac{29}{9}e^{3t} - \dfrac{1}{2}e^{t} - \dfrac{10}{9}$

(j) $x = -\dfrac{207}{130}e^{5t} + \dfrac{79}{30}e^{3t} - \dfrac{14}{15} - \dfrac{7}{65}\cos t + \dfrac{4}{65}\sin t$

$\quad y = \dfrac{207}{65}e^{5t} - \dfrac{79}{30}e^{3t} - \dfrac{3}{130}\sin t - \dfrac{11}{130}\cos t + \dfrac{8}{15}$

3.10 $\quad x = 0.2\cos t$

3.11 $\quad x = -\dfrac{7}{30}\cos 2t + \dfrac{1}{10}\sin 2t + \dfrac{1}{3}\cos t$

3.12 $\quad q = -\dfrac{5}{2}\cos\sqrt{2}\,t + \dfrac{5}{2}$

3.13 $\quad x(t) = e^{-t}(0.1\cos t + 0.3\sin t)$

3.14 $\quad x(t) = (0.2 + 0.5t)e^{-2t}$

3.15 $\quad x = 0.5e^{-t} - 0.2e^{-3t}$

3.16 $\quad q(t) = e^{-t}(-0.2\cos 3t + 0.1\sin 3t) + 0.5$

4.1 (a) $R = 2$

    (b) $R = -1$

4.2 (a) $y = b_0 + b_0 x + \dfrac{b_0}{2!} x^2 + \dfrac{b_0}{3!} x^3 + \cdots = b_0 (1 + x + \dfrac{1}{2!} x^2 + \dfrac{1}{3!} x^3 + \cdots) = b_0 e^x$

    (b) $y = b_0 - b_0 x + \dfrac{b_0}{2!} x^2 - \dfrac{b_0}{3!} x^3 + \cdots = b_0 e^{-x}$

    (c) $y = b_0 + b_1 x + \dfrac{1}{2!} b_1 x^2 + \dfrac{1}{3!} b_1 x^3 + \cdots$

$$= b_0 - b_1 + (b_1 + b_1 x + \dfrac{1}{2!} b_1 x^2 + \dfrac{1}{3!} b_1 x^3 + \cdots) = b + b_1 e^x$$

    (d) $y = c_1 e^{2x} + c_2 e^{-2x}$

    (e) $y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + \cdots$

$$= (b_0 + b_2 x^2 + b_4 x^4 + b_6 x^6 + \cdots) + (b_1 x + b_3 x^3 + b_5 x^5 + \cdots)$$

$$= [b_0 - \dfrac{b_0}{2!} (3x)^2 + \dfrac{b_0}{4!} (3x)^4 - \dfrac{b_0}{6!} (3x)^6 + \cdots] + \dfrac{1}{3} [b_1 (3x) - \dfrac{b_1}{3!} (3x)^3 + \dfrac{b_1}{5!} (3x)^5 - \cdots]$$

$\dfrac{b_1}{3} = b$ 라 놓으면 $= b_0 \cos 3x + b \sin 3x$

    (f) $y = c_1 \cos x + c_2 \sin x$

4.3 (a) $y = e^{2x}$

    (b) $y = e^{-3x}$

    (c) $y = e^x$

    (d) $y = \dfrac{1}{2} e^{2x} + \dfrac{1}{2} e^{-2x}$

4.4 (a) $y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots = b_0 + b_0 x^2 + \dfrac{b_0}{2!} x^4 + \dfrac{b_0}{3!} x^6 + \cdots$

$$= b_0 (1 + x^2 + \dfrac{1}{2!} x^4 + \dfrac{1}{3!} x^6 + \cdots) = b e^{x^2}$$

    (b) $y = b_0 + b_0 x + \dfrac{1+b_0}{2!} x^2 + \dfrac{1+b_0}{3!} x^3 + \dfrac{1+b_0}{4!} x^4 + \cdots$

$$= b_0 (1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots) + (\dfrac{1}{2!} x^2 + \dfrac{1}{3!} x^3 + \dfrac{1}{4!} x^4 + \cdots)$$

$$= b_0 (1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots) + (1 + x + \dfrac{1}{2!} x^2 + \dfrac{1}{3!} x^3 + \dfrac{1}{4!} x^4 + \cdots) - 1 - x$$

$$= b_0 e^x + e^x - x - 1 = (b_0 + 1) e^x - x - 1$$

$$= c\, e^x - x - 1$$

(c) $y = c_1\left(1 - \dfrac{1}{2}x^2 - \dfrac{1}{24}x^4 - \dfrac{1}{240}x^6 - \cdots\right) + c_2 x$

(d) $b_2 = -\dfrac{1}{2}b_0,\ b_3 = -\dfrac{1}{3}b_1,\ b_4 = \dfrac{1}{8}b_0,\ b_5 = \dfrac{1}{15}b_1,\ b_6 = -\dfrac{1}{48}b_0,\ \cdots$

$\quad y = b_0\left(1 - \dfrac{1}{2}x^2 + \dfrac{1}{8}x^4 - \dfrac{1}{48}x^6 + \cdots\right) + b_1\left(x - \dfrac{1}{3}x^3 + \dfrac{1}{15}x^5 - \cdots\right)$

(e) $y = c_1\left(1 + x^2 + \dfrac{1}{2}x^4 + \dfrac{1}{6}x^6 + \cdots\right) + c_2\left(x + \dfrac{2}{3}x^3 + \dfrac{4}{15}x^5 + \dfrac{8}{105}x^7 + \cdots\right)$

(f) $y = c_1 y_1 + c_2 y_2 = c_1\left(1 + \dfrac{1}{6}x^3 + \dfrac{1}{180}x^6 + \cdots\right) + c_2\left(x + \dfrac{1}{12}x^4 + \dfrac{1}{504}x^7 + \cdots\right)$

(g) $y_1 = b_0\left(1 - \dfrac{1}{6}x^3 + \dfrac{1}{180}x^6 - \cdots\right),\ y_2 = b_1\left(x - \dfrac{1}{12}x^4 + \dfrac{1}{504}x^7 - \cdots\right)$

$\quad y = b_0 y_1 + b_1 y_2$ 초깃값을 대입하여 b_0와 b_1을 구하면 된다.

(h) $y = c_1 y_1 + c_2 y_2 = c_1\left(x + \dfrac{1}{3}x^3 + \dfrac{1}{15}x^5 + \dfrac{1}{105}x^7 + \cdots\right) + c_2\left(1 + \dfrac{1}{2}x^2 + \dfrac{1}{8}x^4 + \dfrac{1}{48}x^6 + \cdots\right)$

$\quad y(0) = 0 \rightarrow c_2 = 0,\ y'(0) = 1 \rightarrow c_1 = 1$

$\quad y = x + \dfrac{1}{3}x^3 + \dfrac{1}{15}x^5 + \dfrac{1}{105}x^7 + \cdots$

4.5 (a) $y = y_1 + y_2 = c_1 x + c_2 \sqrt{x}$

(b) $y = y_1 + y_2 = c_1 x + \dfrac{c_2}{x^2}$

(c) $y = c\left(1 + \dfrac{1}{4}x^2 + \dfrac{1}{64}x^4 + \cdots\right)$

(d) $y = c_1\left(1 + 2x - 2x^2 + \dfrac{4}{9}x^3 + \cdots\right) + c_2 x^{\frac{3}{2}}\left(1 - \dfrac{2}{5}x + \dfrac{2}{35}x^2 - \dfrac{4}{945}x^3 + \cdots\right)$

(e) $y = c_1\left(1 + \dfrac{1}{2}x + \dfrac{1}{24}x^2 + \dfrac{1}{720}x^3 + \cdots\right) + c_2 \sqrt{x}\left(1 + \dfrac{1}{6}x + \dfrac{1}{120}x^2 + \dfrac{1}{5040}x^3 + \cdots\right)$

(f) $y = c_1 y_1 + c_2 y_2$

4.6 $y = c_1\left(1 - 3x^2\right) + c_2\left(x - \dfrac{1}{2}x^3 - \dfrac{24}{5!}x^5 - \cdots\right)$

4.7 (a) $y = c_1 J_{1/4}(x) + c_2 Y_{1/4}(x)$

(b) $y = c_1 J_0(4\sqrt{x}) + c_2 Y_0(4\sqrt{x})$

(c) $y = c_1 J_2(3x) + c_2 Y_2(3x)$

Chapter 05 연습문제 답안

5.1 (a) $\mathcal{L}\{5t^2 + 2\} = 5\mathcal{L}\{t^2\} + 2\mathcal{L}\{1\} = 5\dfrac{2}{s^3} + 2\dfrac{1}{5}$

(b) $\mathcal{L}\{(2t+1)^2\} = \mathcal{L}\{4t^2 + 4t + 1\} = 4\mathcal{L}\{t^2\} + 4\mathcal{L}\{t\} + \mathcal{L}\{1\} = 4\dfrac{2}{s^3} + 4\dfrac{1}{s^2} + \dfrac{1}{s}$

(c) $\mathcal{L}\{\cos 2t\} = \dfrac{\dfrac{1}{s}}{1 + \dfrac{4}{s^2}} = \dfrac{\dfrac{1}{s}}{\dfrac{(s^2+4)}{s^2}} = \dfrac{s^2}{s(s^2+4)} = \dfrac{s}{s^2+4}$

(d) $\mathcal{L}\{\sin 3t\} = \dfrac{\dfrac{3}{s^2}}{1 + \dfrac{9}{s^2}} = \dfrac{\dfrac{3}{s^2}}{\dfrac{(s^2+9)}{s^2}} = \dfrac{3}{s^2+9}$

(e) $\mathcal{L}\{e^{2t-2}\} = \mathcal{L}\{e^{2t}e^{-2}\} = e^{-2}\mathcal{L}\{e^{2t}\} = e^{-2}\dfrac{1}{s-2} = \dfrac{e^{-2}}{s-2}$

(f) $\mathcal{L}\{te^{-2t}\} = \dfrac{1}{(s+2)^2}$

(g) $\mathcal{L}\{t^2 e^{2t}\} = \dfrac{2}{(s-2)^3}$

(h) $\mathcal{L}\{e^{-2t}\cos 2t\} = \dfrac{(s+2)}{(s+2)^2 + 2^2}$

(i) $\mathcal{L}\{2t\sin t\} = 2\mathcal{L}\{t\sin t\} = 2(-1)\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right) = -2\left(\dfrac{-2s}{(s^2+1)^2}\right) = \dfrac{4s}{(s^2+1)^2}$

(j) $\mathcal{L}\{t\cos 2t\} = (-1)\dfrac{d}{ds}\mathcal{L}\{\cos 2t\} = -\dfrac{d}{ds}\left(\dfrac{s}{s^2+2^2}\right)$

$\qquad = -\dfrac{1(s^2+2^2) - s2s}{(s^2+2^2)^2}$

$\qquad = \dfrac{-s^2 - 2^2 + 2s^2}{(s^2+2^2)^2} = \dfrac{s^2 - 2^2}{(s^2+2^2)^2}$

(k) $\mathcal{L}\{\sin4t\cos4t\}= \mathcal{L}\left\{\dfrac{\sin8t}{2}\right\}= \dfrac{1}{2}\left(\dfrac{8}{s^2+8^2}\right)= \dfrac{4}{s^2+64}$

(l) $\mathcal{L}\{\cos^2 at\}= \mathcal{L}\left\{\dfrac{1}{2}+\dfrac{1}{2}\cos2at\right\}= \dfrac{1}{2s}+\dfrac{s}{2(s^2+4a^2)}$

(m) $\mathcal{L}\{f(t)\}= \displaystyle\int_0^\infty e^{-st}f(t)dt= \int_0^2 e^{-st}2\,dt+\int_2^\infty e^{-st}(-2)\,dt$

$\qquad\qquad = \left[-\dfrac{2}{s}e^{-st}\right]_0^2+\left[\dfrac{2}{s}e^{-st}\right]_2^\infty$

$\qquad\qquad = \left[-\dfrac{2}{s}e^{-2s}+\dfrac{2}{s}\right]+\left[-\dfrac{2}{s}e^{-2s}\right]=-\dfrac{4}{s}e^{-2s}+\dfrac{2}{s}$

(n) 문제 (m)과 동일

(o) $\mathcal{L}\{f(t)\}= \displaystyle\int_0^1 e^{-st}\,dt= \left[-\dfrac{1}{s}e^{-st}\right]_0^1 =-\dfrac{1}{s}\left[e^{-s}-1\right]$

(p) $\mathcal{L}\{f(t)\}= \displaystyle\int_0^1 e^{-st}2e^{-t}\,dt+\int_1^\infty e^{-st}\cdot0\cdot dt= \int_0^1 2e^{-(s+1)t}\,dt$

$\qquad\qquad = \left[\dfrac{2}{-(s+1)}e^{-(s+1)t}\right]= \dfrac{2}{-(s+1)}\left[e^{-(s+1)}-1\right]$

(q) $\mathcal{L}\{f(t)\}= \displaystyle\int_0^1 e^{-st}(2)\,dt+\int_1^\infty e^{-st}\cdot(0)\cdot dt= \left[-\dfrac{2}{3}e^{-st}\right]_0^1 =-\dfrac{2}{s}\left[e^{-s}-1\right]$

(r) $\mathcal{L}\{f(t)\}= \displaystyle\int_1^2 e^{-st}\cdot(1)\cdot dt= \left[-\dfrac{1}{s}e^{-st}\right]_1^2 =-\dfrac{1}{s}\left[e^{-2s}-e^{-s}\right]$

(s) $\mathcal{L}\{f(t)\}= \displaystyle\int_1^\infty e^{-st}(t-1)dt= \left[-\dfrac{1}{s}e^{-st}(t-1)\right]_1^\infty-\int_1^\infty\left(-\dfrac{1}{s}\right)e^{-st}\cdot1\cdot dt$

$\qquad\qquad = [0-0]+\left(\dfrac{1}{s}\right)\left[-\dfrac{1}{s}e^{-st}\right]_1^\infty =-\dfrac{1}{s^2}[0-e^{-s}]= \dfrac{e^{-s}}{s^2}$

(t) $\mathcal{L}\{f(t)\}= \displaystyle\int_1^2 e^{-st}(-t+2)dt= \left[-\dfrac{1}{s}e^{-st}(-t+2)\right]_1^2-\int_1^2\left(-\dfrac{1}{s}\right)e^{-st}(-1)dt$

$\qquad\qquad =-\dfrac{1}{s}[0-e^{-s}]-\dfrac{1}{s}\left[-\dfrac{1}{s}e^{-st}\right]_1^2 = \dfrac{e^{-s}}{s}+\dfrac{1}{s^2}\left[e^{-2s}-e^{-s}\right]$

5.2 (a) $\mathcal{L}\{f(t)\}=\dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st}f(t)\,dt=\dfrac{1}{1-e^{-2s}}\left[\int_0^1 e^{-st}\cdot(0)\cdot dt+\int_1^2 e^{-st}\cdot(1)\cdot dt\right]$

$\qquad\qquad =\dfrac{1}{1-e^{-2s}}\left[-\dfrac{1}{s}e^{-st}\right]_1^2=\dfrac{1}{1-e^{-2s}}\left[-\dfrac{1}{s}e^{-2s}+\dfrac{1}{s}e^{-s}\right]$

(b) $\mathcal{L}\{f(t)\}=\dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st}f(t)dt=\dfrac{1}{1-e^{-2s}}\left[\int_0^1 e^{-st}(-1)dt+\int_1^2 e^{-st}(1)dt\right]$

$\qquad =\dfrac{1}{1-e^{-2s}}\left\{\left[\dfrac{1}{s}e^{-st}\right]_0^1+\left[-\dfrac{1}{s}e^{-st}\right]_1^2\right\}=\dfrac{1}{1-e^{-2s}}\left[\dfrac{1}{s}e^{-s}-\dfrac{1}{s}-\dfrac{1}{s}e^{-2s}+\dfrac{1}{s}e^{-s}\right]$

$\qquad =\dfrac{1}{1-e^{-2s}}\left[\dfrac{2}{s}e^{-s}-\dfrac{1}{s}-\dfrac{1}{s}e^{-2s}\right]$

(c) $\mathcal{L}\{f(t)\}=\dfrac{1}{1-e^{-3s}}\left[\displaystyle\int_0^1 e^{-st}dt+\int_1^2(-e^{-st})dt\right]$

$\qquad =\dfrac{1}{1-e^{-3s}}\left\{\left[-\dfrac{1}{s}e^{-st}\right]_0^1+\left[\dfrac{1}{s}e^{-st}\right]_1^2\right\}$

$\qquad =\dfrac{1}{1-e^{-3s}}\left[-\dfrac{1}{s}e^{-s}+\dfrac{1}{s}+\dfrac{1}{s}e^{-2s}-\dfrac{1}{s}e^{-s}\right]$

$\qquad =\dfrac{1}{1-e^{-3s}}\left[-\dfrac{2}{s}e^{-s}+\dfrac{1}{s}+\dfrac{1}{s}e^{-2s}\right]$

(d) $\mathcal{L}\{f(t)\}=\dfrac{1}{1-e^{-2s}}\left[\displaystyle\int_0^1 e^{-st}t\,dt\right]=\dfrac{1}{1-e^{-2s}}\left\{\left[-\dfrac{1}{s}e^{-st}\cdot t\right]_0^1-\int_0^1 -se^{-st}\cdot1\cdot dt\right\}$

$\qquad =\dfrac{1}{1-e^{-2s}}\left\{-\dfrac{1}{s}e^{-s}+\dfrac{1}{s}\left[-\dfrac{1}{s}e^{-st}\right]_0^1\right\}=\dfrac{1}{1-e^{-2s}}\left\{-\dfrac{1}{s}e^{-s}-\dfrac{1}{s^2}\left[e^{-s}-1\right]\right\}$

5.3 (a) $\mathcal{L}\{f(t)\}=\displaystyle\int_0^\infty e^{-st}e^t(\cos2t+\sin2t)dt=\int_0^\infty e^{-(s+1)t}(\cos2t+\sin2t)dt$

$\qquad =\dfrac{s+1}{(s+1)^2+2^2}+\dfrac{2}{(s+1)^2+2^2}=\dfrac{s+3}{(s+1)^2+2^2}$

(b) $\mathcal{L}\{e^{-2t}\sin(2t+\theta)\}=\dfrac{2\cos\theta+(s+2)\sin\theta}{(s+2)^2+2^2}$

(c) $\mathcal{L}\{(t-2)u(t-2)\}=e^{-2s}\dfrac{1}{s^2}$

(d) $\mathcal{L}\{2t\cdot u(t-1)\}=2\mathcal{L}\{(t-1+1)\cdot u(t-1)\}$

$\qquad\qquad =2\mathcal{L}\{(t-1)u(t-1)\}+2\mathcal{L}\{u(t-1)\}=2e^{-s}\dfrac{1}{s^2}+2\dfrac{e^{-s}}{s}$

(e) $\mathcal{L}\left\{e^{2t}u(t-1)\right\}=\dfrac{e^{-(s-2)}}{s-2}$

(f) $\mathcal{L}\left\{u(t-\pi)\cos2t\right\}=\mathcal{L}\left\{u(t-\pi)\cos2(t-\pi)\right\}=e^{-\pi s}\dfrac{s}{s^2+2^2}$

5.4   (a) $\mathcal{L}\left[f(t)\right]=(-1)^1\dfrac{d}{ds}\left(\dfrac{1}{s+2}\right)=\dfrac{1}{(s+2)^2}$

(b) $\mathcal{L}\left[f(t)\right]=(-1)^2\dfrac{d^2}{ds^2}\left(\dfrac{1}{s-2}\right)=\dfrac{2}{(s-2)^3}$

(c) $\mathcal{L}\left[f(t)\right]=(-1)^1\dfrac{d}{ds}\left(\dfrac{s}{s^2+4}\right)=\dfrac{s^2-4}{(s^2+4)^2}$

(d) $\mathcal{L}\left[f(t)\right]=(-1)^2\dfrac{d^2}{ds^2}\left(\dfrac{2}{s^2+4}\right)=\dfrac{4(3s^2-4)}{(s^2+4)^3}$

(e) $\mathcal{L}\left[f(t)\right]=(-1)\dfrac{d}{ds}\left[\dfrac{2}{(s+1)^2+2^2}\right]=\dfrac{4(s+1)}{[(s+1)^2+2^2]^2}$

(f) $\mathcal{L}\left[f(t)\right]=\dfrac{s^2-2s-3}{[(s-1)^2+2^2]^2}$

5.5   (a) $\mathcal{L}\left[f(t)\right]=\mathcal{L}\left[2\right]\mathcal{L}\left[t^2\right]=\dfrac{2}{s}\dfrac{2}{s^3}=\dfrac{4}{s^4}$

(b) $\mathcal{L}\left[f(t)\right]=\mathcal{L}\left[e^{-t}\right]\mathcal{L}\left[\cos t\right]=\dfrac{1}{s+1}\dfrac{s}{s^2+1}=\dfrac{s}{(s+1)(s^2+1)}$

(c) $\mathcal{L}\left[f(t)\right]=\dfrac{1}{s+1}\dfrac{1}{s-1}=\dfrac{1}{s^2-1}$

(d) $\mathcal{L}\left[f(t)\right]=\dfrac{2}{s^2}\dfrac{1}{s-2}=\dfrac{2}{s^3-2s^2}$

(e) $\mathcal{L}\left[f(t)\right]=\dfrac{s}{s^2+1}\dfrac{s}{s^2+2^2}=\dfrac{s^2}{(s^2+1)(s^2+4)}$

(f) $\mathcal{L}\left[f(t)\right]=\dfrac{1}{s}\dfrac{s}{s^2+1}=\dfrac{1}{s^2+1}$

5.6   (a) $\mathcal{L}\left[\displaystyle\int_0^t e^{-2\tau}d\tau\right]=\dfrac{1}{s}F(s)=\dfrac{1}{s(s+2)}$

(b) $\mathcal{L}\left[\displaystyle\int_0^t \sin 2\tau d\tau\right] = \dfrac{1}{s}\dfrac{2}{s^2+4} = \dfrac{2}{s^3+4s}$

5.7 (a) $\mathcal{L}^{-1}[F(s)] = f(t) = e^{2t}$

(b) $\mathcal{L}^{-1}[F(s)] = \cos 2t + \dfrac{3}{2}\sin 2t$

(c) $\mathcal{L}^{-1}[F(s)] = u(t) + 2t$

(d) $\mathcal{L}^{-1}[F(s)] = 2u(t) - t + e^{3t}$

(e) $\mathcal{L}^{-1}[F(s)] = -\dfrac{2}{9}u(t) + \dfrac{1}{9}e^{3t} + \dfrac{1}{9}e^{-3t}$

(f) $\mathcal{L}^{-1}[F(s)] = -\dfrac{1}{3}e^{-2t} + \dfrac{1}{3}e^{t}$

(g) $\mathcal{L}^{-1}[F(s)] = \dfrac{1}{2}e^{t} - \dfrac{1}{2}(\cos t + \sin t)$

(h) $\mathcal{L}^{-1}[F(s)] = -\dfrac{1}{20}e^{-2t} + \dfrac{1}{20}e^{2t} - \dfrac{1}{5}\sin t$

(i) $\mathcal{L}^{-1}[F(s)] = -\dfrac{3}{2}u(t) + \dfrac{1}{6}e^{-2t} + \dfrac{4}{3}e^{t}$

(j) $\mathcal{L}^{-1}[F(s)] = -\dfrac{5}{2}e^{t} + \dfrac{1}{6}e^{-t} + \dfrac{7}{3}e^{2t}$

(k) $\mathcal{L}^{-1}[F(s)] = -\dfrac{1}{3}u(t) - \dfrac{4}{5}e^{-2t} + \dfrac{23}{60}e^{3t} + \dfrac{3}{4}e^{-t}$

(l) $\mathcal{L}^{-1}[F(s)] = \dfrac{3}{32}e^{2t} + \dfrac{1}{32}e^{-2t} - \dfrac{1}{16}\sin 2t - \dfrac{1}{8}\cos 2t$

5.8 (a) $f(t) = \mathcal{L}^{-1}[F(s)] = \dfrac{1}{2}t^2 e^{-t}$

(b) $f(t) = 2e^{2t}\cos t + 7e^{2t}\sin t$

(c) $f(t) = e^{-t}\cos 2t + e^{-t}\sin 2t$

(d) $f(t) = 2e^{-3t}\cos 2t - \dfrac{5}{2}e^{-3t}\sin 2t$

(e) $f(t) = 3e^{-2t} - 5te^{-2t}$

(f) $f(t) = 2t - 3u(t) + te^{-t} + 3e^{-t}$

(g) $f(t) = (t-1)u(t-1)$

(h) $f(t) = 2e^{-t} + e^{-(t-2)}u(t-2)$

(i) $f(t) = \dfrac{1}{2}\sin 2(t-2)u(t-2)$

(j) $f(t) = \dfrac{1}{4}e^{t+2} - \dfrac{1}{4}e^{-3(t+2)}$

(k) $f(t) = \dfrac{1}{3}e^{-(t-1)}u(t-1) + \dfrac{2}{3}e^{2(t-1)}u(t-1)$

(l) $f(t) = -\dfrac{1}{5}e^{t-2}u(t-2) + \dfrac{1}{5}\cos 2(t-2)u(t-2) + \dfrac{3}{5}\sin 2(t-2)u(t-2)$

5.9   (a) $sY(s) - y(0) + 2Y(s) = 0$, $(s+2)Y(s) = 1$, $Y(s) = \dfrac{1}{s+2}$, $y(t) = e^{-2t}$

     (b) $2[sY(s) - y(0)] + Y(s) = 0$, $(2s+1)Y(s) = 4$, $Y(s) = \dfrac{4}{2s+1}$, $y(t) = 2e^{-\frac{1}{2}t}$

     (c) $sY(s) - y(0) - 2Y(s) = \dfrac{1}{s-1}$, $(s-2)Y(s) = \dfrac{1}{s-1} + 2$,

        $Y(s) = \dfrac{2s-1}{(s-2)(s-1)} = \dfrac{3}{s-2} - \dfrac{1}{s-1}$, $y(t) = 3e^{2t} - e^t$

     (d) $Y(s) = \dfrac{2}{5}\dfrac{1}{s-2} + \dfrac{1}{5}\dfrac{1}{s^2+1} - \dfrac{2}{5}\dfrac{s}{s^2+1}$, $y(t) = \dfrac{2}{5}e^{2t} + \dfrac{1}{5}\sin t - \dfrac{2}{5}\cos t$

     (e) $s^2Y(s) - sy(0) - y'(0) - Y(s) = 0$, $(s^2-1)Y(s) = s+1$, $Y(s) = \dfrac{1}{s-1}$, $y(t) = e^t$

     (f) $s^2Y(s) - sy(0) - y'(0) - 2Y(s) - \dfrac{3}{s} = 0$, $(s^2-2)Y(s) = -s + \dfrac{3}{s} + 1$

        $Y(s) = \dfrac{-s^2+s+3}{s(s+\sqrt{2})(s-\sqrt{2})} = -\dfrac{3}{2}\dfrac{1}{s} + \dfrac{1}{4}(1+\sqrt{2})\dfrac{1}{s-\sqrt{2}} + \dfrac{1}{4}(1-\sqrt{2})\dfrac{1}{s+\sqrt{2}}$

        $y(t) = -\dfrac{3}{2} + \dfrac{1}{4}(1+\sqrt{2})e^{\sqrt{2}t} + \dfrac{1}{4}(1-\sqrt{2})e^{-\sqrt{2}t}$

     (g) $s^2Y(s) - sy(0) - y'(0) + 4Y(s) = \dfrac{1}{s+2}$, $(s^2+4)Y(s) = \dfrac{1}{s+2}$

        $Y(s) = \dfrac{1}{(s+2)(s^2+4)} = \dfrac{1}{8}\dfrac{1}{(s+2)} + \dfrac{1}{8}\dfrac{2}{s^2+4} - \dfrac{1}{8}\dfrac{s}{s^2+4}$

        $y(t) = \dfrac{1}{8}e^{-2t} + \dfrac{1}{8}\sin 2t - \dfrac{1}{8}\cos 2t$

(h) $s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) - 3[s Y(s) - y(0)] + 2 Y(s) = \dfrac{1}{s+2}$

$(s^3 - 3s + 2) Y(s) = \dfrac{1}{s+2}$

$Y(s) = \dfrac{1}{(s-1)^2 (s+2)^2} = -\dfrac{2}{27} \dfrac{1}{s-1} + \dfrac{1}{9} \dfrac{1}{(s-1)^2} + \dfrac{2}{27} \dfrac{1}{s+2} + \dfrac{1}{9} \dfrac{1}{(s+2)^2}$

$y(t) = -\dfrac{2}{27} e^t + \dfrac{1}{9} t e^t + \dfrac{2}{27} e^{-2t} + \dfrac{1}{9} t e^{-2t}$

(i) $s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 Y(s) - s y(0) - y'(0) - 2[s Y(s) - y(0)] = \dfrac{2}{s}$

$(s^3 + s^2 - 2s) Y(s) = \dfrac{2}{s} + s + 2$

$Y(s) = \dfrac{s^2 + 2s + 2}{s^2 (s-1)(s+2)} = -\dfrac{3}{2} \dfrac{1}{s} - \dfrac{1}{s^2} + \dfrac{5}{3} \dfrac{1}{s-1} - \dfrac{1}{6} \dfrac{1}{s+2}$

$y(t) = -\dfrac{3}{2} u(t) - t + \dfrac{5}{3} e^t - \dfrac{1}{6} e^{-2t}$

5.10 (a) $s Y(s) - y(0) - 2 Y(s) = \dfrac{1}{s-2}, \quad (s-2) Y(s) = \dfrac{s-1}{s-2},$

$Y(s) = \dfrac{s-1}{(s-2)^2} = \dfrac{1}{s-2} + \dfrac{1}{(s-2)^2}$

$y(t) = e^{2t} + t e^{2t}$

(b) $s Y(s) - y(0) + Y(s) = \dfrac{1}{s+1}, \quad (s+1) Y(s) = \dfrac{1}{s+1} + 1, \quad Y(s) = \dfrac{1}{(s+1)^2} + \dfrac{1}{s+1}$

$y(t) = t e^{-t} + e^{-t}$ => (한빛아카데미 판) 본문 문제 수정됨

(c) $s^2 Y(s) - s y(0) - y'(0) - 2[s Y(s) - y(0)] + 5 Y(s) = 0$

$(s^2 - 2s + 5) Y(s) = -s + 3, \quad Y(s) = \dfrac{-(s-1)}{(s-1)^2 + 4} + \dfrac{2}{(s-1)^2 + 4}$

$y(t) = -e^t \cos 2t + e^t \sin 2t$

(d) $s^2 Y(s) - s y(0) - y'(0) + 4[s Y(s) - y(0)] + 4 Y(s) = \dfrac{1}{s^2}$

$(s^2 + 4s + 4) Y(s) = \dfrac{1}{s^2}, \quad Y(s) = \dfrac{1}{s^2 (s+2)^2} = -\dfrac{1}{4} \dfrac{1}{s} + \dfrac{1}{4} \dfrac{1}{s^2} + \dfrac{1}{4} \dfrac{1}{s+2} + \dfrac{1}{4} \dfrac{1}{(s+2)^2}$

$y(t) = -\dfrac{1}{4} u(t) + \dfrac{1}{4} t + \dfrac{1}{4} e^{-2t} + \dfrac{1}{4} t e^{-2t}$

(e) $s^2 Y(s) - s y(0) - y'(0) + 2[s Y(s) - y(0)] + 5 Y(s) = \dfrac{1}{s+1}$

$$(s^2 + 2s + 5)\,Y(s) = \frac{1}{s+1} + 1, \quad Y(s) = \frac{s+2}{[(s+1)^2+4](s+1)}$$

$$Y(s) = \frac{1}{4}\frac{1}{s+1} - \frac{1}{4}\frac{s+1}{(s+1)^2+4} + \frac{1}{2}\frac{2}{(s+1)^2+4}$$

$$y(t) = \frac{1}{4}e^{-t} - \frac{1}{4}e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t$$

(f) $s^2 Y(s) - sy(0) - y'(0) + 4[sY(s) - y(0)] + 4Y(s) = \dfrac{1}{s+1}$

$$(s^2 + 4s + 4)\,Y(s) = \frac{1}{s+1}, \quad Y(s) = \frac{1}{(s+1)^2(s+1)} = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$y(t) = e^{-t} - e^{-2t} - te^{-2t}$$

(g) $s^2 Y(s) - sy(0) - y'(0) + Y(s) = \dfrac{1}{(s+1)^2+1}$

$$(s^2 + 1)\,Y(s) = \frac{1}{(s+1)^2+1},$$

$$Y(s) = \frac{1}{(s+1)^2[(s+1)^2+1]} = \frac{1}{5}\frac{1}{s^2+1} - \frac{2}{5}\frac{s}{s^2+1} + \frac{1}{5}\frac{1}{(s+1)^2+1} + \frac{2}{5}\frac{s+1}{(s+1)^2+1}$$

$$y(t) = \frac{1}{5}\sin t - \frac{2}{5}\cos t + \frac{1}{5}e^{-t}\sin t + \frac{2}{5}e^{-t}\cos t$$

5.11 (a) $sY(s) - y(0) - Y(s) = \dfrac{2e^{-2s}}{s}$

$$Y(s) = \frac{2}{s(s-1)}e^{-2s} = \left[-\frac{2}{s} + \frac{2}{s-1}\right]e^{-2s} = -\frac{2}{s}e^{-2s} + \frac{2}{s-1}e^{-2s}$$

$$y(t) = -2u(t-2) + 2e^{t-2}u(t-2)$$

(b) $sY(s) - y(0) + 3Y(s) = \mathcal{L}\,[f(t)]$

여기서 $f(t) = t - (t-2)u(t-2) - 2u(t-2) = \dfrac{1}{s^2} - \dfrac{1}{s^2}e^{-2s} - \dfrac{2}{s}e^{-2s}$

$$Y(s) = \frac{1}{(s+3)s^2} + \frac{-1-2s}{(s+3)s^2}e^{-2s}$$

$$= -\frac{1}{9}\frac{1}{s} + \frac{1}{3}\frac{1}{s^2} + \frac{1}{9}\frac{1}{s+3} + \left[-\frac{1}{3}\frac{1}{s^2} + \frac{1}{9}\frac{1}{s} + \frac{5}{9}\frac{1}{s+3}\right]e^{-2s}$$

$$y(t) = -\frac{1}{9}u(t) + \frac{1}{3}t + \frac{1}{9}e^{-3t} + \frac{1}{9}u(t-2) - \frac{1}{3}(t-2)u(t-2) + \frac{5}{9}e^{-3(t-2)}u(t-2)$$

(c) $f(t) = -u(t) + u(t-2)$

$$s^2 Y(s) - sy(0) - y'(0) + Y(s) = -\frac{1}{s} + \frac{1}{s}e^{-2s}$$

$$Y(s) = -\frac{1}{s(s^2+1)} + \frac{1}{s(s^2+1)}e^{-2s} = -\frac{1}{s} + \frac{s}{s^2+1} + \left(\frac{1}{s} - \frac{s}{s^2+1}\right)e^{-2s}$$

$$y(t) = -u(t) + \cos t + u(t-2) - \cos(t-2)u(t-2)$$

(d) $y'' - y' - 2y = u(t-1)$, $y(0) = 1$, $y'(0) = 1$ => (한빛아카데미 판) 본문 문제 수정됨

$$s^2 Y(s) - sy(0) - y'(0) - [sY(s) - y(0)] - 2Y(s) = \frac{1}{s}e^{-s},$$

$$s^2 Y(s) - s - 1 - [sY(s) - 1] - 2Y(s) = \frac{1}{s}e^{-s}$$

$$(s^2 - s - 2)Y(s) = s + \frac{1}{s}e^{-s}$$

$$Y(s) = \frac{s}{(s+1)(s-2)} + \frac{1}{s(s+1)(s-2)}e^{-s}$$

$$= \frac{1}{3}\frac{1}{s+1} + \frac{2}{3}\frac{1}{s-2} + \left[-\frac{1}{2s} + \frac{1}{3}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s-2}\right]e^{-s}$$

$$y(t) = \frac{1}{3}e^{-t} + \frac{2}{3}e^{2t} - \frac{1}{2}u(t-1) + \frac{1}{3}e^{-(t-1)}u(t-1) + \frac{1}{6}e^{2(t-1)}u(t-1)$$

(e) $y = \dfrac{1}{5}e^{2t} - \dfrac{1}{5}e^{-3t} - \dfrac{1}{6}u(t-1) + \dfrac{1}{10}e^{2(t-1)}u(t-1) + \dfrac{1}{15}e^{-3(t-1)}u(t-1)$

$$+ \frac{1}{6}u(t-2) - \frac{1}{10}e^{2(t-2)}u(t-2) - \frac{1}{15}e^{-3(t-2)}u(t-2)$$

(f) $y = \sin t + \dfrac{1}{2}(t-\pi)\sin(t-\pi)u(t-\pi)$

5.12 (a) $y = -\dfrac{3}{25}\cos t - \dfrac{4}{25}\sin t + \dfrac{2}{5}t\cos t + \dfrac{1}{5}t\sin t + \dfrac{28}{25}e^{-2t}$

(b) $y = -\dfrac{1}{75}e^{-2t} + \dfrac{1}{12}e^t + \dfrac{1}{100}e^{3t}(10t-7)$

5.13 운동방정식 $m\ddot{x} + kx = f(t)$

$m = 1kg$, $k = 2N/m$, $f(t) = 3u(t) - 3u(t-2)$ N, $x(0) = x'(0) = 0$

$$\ddot{x} + 2x = 3u(t) - 3u(t-2)$$

$$s^2 X(s) - sx(0) - x'(0) + 2X(s) = \frac{3}{s} - \frac{3}{s}e^{-2s}$$

$$(s^2 + 2)X(s) = \frac{3}{s} - \frac{3}{s}e^{-2s}$$

$$X(s) = \frac{3}{s(s^2+2)} - \frac{3}{s(s^2+2)}e^{-2s} = \frac{3}{2}\frac{1}{s} - \frac{3}{2}\frac{s}{s^2+2} - \left[\frac{3}{2}\frac{1}{s} - \frac{3}{2}\frac{s}{s^2+2}\right]e^{-2s}$$

$$x(t) = \frac{3}{2}u(t) - \frac{3}{2}\cos\sqrt{2}\,t - \frac{3}{2}u(t-2) + \frac{3}{2}\cos\sqrt{2}\,(t-2)u(t-2)$$

5.14 운동방정식 $Rq' + \dfrac{1}{C} = e(t)$

$2q' + 4q = u(t-1)$

$2[sQ(s) - q(0)] + 4Q(s) = \dfrac{1}{s}e^{-s}$ $\qquad (2s+4)Q(s) = \dfrac{1}{s}e^{-s}$

$Q(s) = \dfrac{1}{2s(s+2)}e^{-s} = \dfrac{1}{2}\left[\dfrac{1}{2}\dfrac{1}{s} - \dfrac{1}{2}\dfrac{1}{s+2}\right]e^{-s}$

$q(t) = \dfrac{1}{4}u(t-1) - \dfrac{1}{4}e^{-2(t-1)}u(t-1)$

5.15 운동방정식 $Lq'' + \dfrac{1}{C}q = e(t)$

$L = 2H, \ C = 0.1F, \ q(0) = 0, \ q'(0) = 0, \ e(t) = 5u(t-1) - 5u(t-2)$

$2q'' + 10q = e(t) \qquad 2[s^2Q(s) - sq(0) - q'(0)] + 10Q(s) = \dfrac{5}{s}e^{-s} - \dfrac{5}{s}e^{-2s}$

$\qquad (2s^2 + 10)Q(s) = \dfrac{5}{s}e^{-s} - \dfrac{5}{s}e^{-2s}$

$Q(s) = \dfrac{5}{2s(s^2+5)}e^{-s} - \dfrac{5}{2s(s^2+5)}e^{-2s}$

$\qquad = \dfrac{5}{2}\left[\dfrac{1}{5}\dfrac{1}{s} - \dfrac{1}{5}\dfrac{s}{s^2+5}\right]e^{-s} - \dfrac{5}{2}\left[\dfrac{1}{5}\dfrac{1}{s} - \dfrac{1}{5}\dfrac{s}{s^2+5}\right]e^{-2s}$

$q(t) = \dfrac{1}{2}u(t-1) - \dfrac{1}{2}\cos\sqrt{5}\,(t-1)u(t-1) - \dfrac{1}{2}u(t-2) + \dfrac{1}{2}\cos\sqrt{5}\,(t-2)u(t-2)$

5.16 (a) $(s^2+1)X(s) = s+1 \quad \rightarrow \quad X(s) = \dfrac{s}{s^2+1} + \dfrac{1}{s^2+1}, \quad x(t) = \cos t + \sin t$

$\qquad (s^2+1)Y(s) = s-1 \quad \rightarrow \quad Y(s) = \dfrac{s}{s^2+1} - \dfrac{1}{s^2+1}, \quad y(t) = \cos t - \sin t$

(b) $[(s^2-1)+4]X(s) = 2(s+1) - 2 \quad \rightarrow \quad X(s) = \dfrac{2s}{s^2+3}, \ x(t) = 2\cos\sqrt{3}\,t$

$\qquad [(s^2-1)+4]Y(s) = s-1+4 \rightarrow Y(s) = \dfrac{s}{s^2+3} + \dfrac{3}{\sqrt{3}}\dfrac{\sqrt{3}}{s^2+3},$

$\qquad y(t) = \cos\sqrt{3}\,t + \sqrt{3}\,\sin\sqrt{3}\,t$

(c) $x(t) = -\dfrac{3}{10}e^{-t} + \dfrac{3}{10}e^{t} - \dfrac{2}{15}\sin 2t - \dfrac{1}{3}\sin t$

$\quad y(t) = \dfrac{3}{10}e^{-t} - \dfrac{3}{10}e^{t} + \dfrac{7}{15}\sin 2t - \dfrac{1}{2}\sin t$

(d) $x(t) = \dfrac{1}{4}e^{t} - \dfrac{2}{7}e^{-2t} + \dfrac{1}{7}e^{2t} - \dfrac{\sqrt{3}}{28}\sin\sqrt{3}\,t - \dfrac{3}{28}\cos\sqrt{3}\,t$

$\quad y(t) = -\dfrac{2}{7}e^{-2t} + \dfrac{1}{7}e^{2t} + \dfrac{\sqrt{3}}{21}\sin\sqrt{3}\,t + \dfrac{1}{7}\cos\sqrt{3}\,t$

Chapter 06 연습문제 답안

6.1 (a) $2\vec{a} - \vec{b}$
$= 2\langle 2, 2, -3 \rangle - \langle 0, 1, -2 \rangle$
$= \langle 4, 4, -6 \rangle - \langle 0, 1, -2 \rangle$
$= \langle 4, 3, -4 \rangle$

(b) $-\vec{a} - \vec{b} + 2\vec{c}$
$= -\langle 2, 2, -3 \rangle - \langle 0, 1, -2 \rangle + 2\langle 3, -1, -4 \rangle$
$= -\langle 2, 2, -3 \rangle - \langle 0, 1, -2 \rangle + \langle 6, -2, -8 \rangle$
$= -\langle 4, -5, -3 \rangle$

(c) $2\vec{a} + \vec{b} - 2\vec{c}$
$= 2\langle 2, 2, -3 \rangle + \langle 0, 1, -2 \rangle - 2\langle 3, -1, -4 \rangle$
$= \langle 4, 4, -6 \rangle + \langle 0, 1, -2 \rangle - \langle 6, -2, -8 \rangle$
$= \langle -2, 7, 0 \rangle$

(d) $\| \vec{a} - 2\vec{b} \|$
$= \| \langle 2, 2, -3 \rangle - 2\langle 0, 1, -2 \rangle \|$
$= \| \langle 2, 2, -3 \rangle - \langle 0, 2, -4 \rangle \|$
$= \| \langle 2, 0, 1 \rangle \|$
$= \sqrt{2^2 + 0^2 + 1^2} = \sqrt{5}$

(e) $\| \vec{a} - 2\vec{b} - 2\vec{c} \|$
$= \| \langle 2, 2, -3 \rangle - \langle 0, 1, -2 \rangle - 2\langle 3, -1, -4 \rangle \|$
$= \| \langle 2, 2, -3 \rangle - \langle 0, 1, -2 \rangle - \langle 6, -2, -8 \rangle \|$
$= \| \langle -4, 3, 7 \rangle \|$
$= \sqrt{(-4)^2 + 3^2 + 7^2} = \sqrt{74}$

(f) $\| -\vec{a} + \vec{b} - 2\vec{c} \|$
$= \| -\langle 2, 2, -3 \rangle + \langle 0, 1, -2 \rangle - 2\langle 3, -1, -4 \rangle \|$
$= \| -\langle 2, 2, -3 \rangle + \langle 0, 1, -2 \rangle - \langle 6, -2, -8 \rangle \|$
$= \| \langle -8, 1, 9 \rangle \|$
$= \sqrt{(-8)^2 + 1^2 + 9^2} = \sqrt{146}$

6.2 (a) $\| \overrightarrow{p_1 p_2} \| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

(b) $\| \overrightarrow{p_1 p_2} \| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20}$

(c) $\| \overrightarrow{p_1p_2} \| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

(d) $\| \overrightarrow{p_1p_2} \| = \sqrt{(-3)^2 + 6^2} = \sqrt{45}$

(e) $\| \overrightarrow{p_1p_2} \| = \sqrt{(-1)^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

(f) $\| \overrightarrow{p_1p_2} \| = \sqrt{(1)^2 + (-3)^2 + (3)^2} = \sqrt{19}$

(g) $\| \overrightarrow{p_1p_2} \| = \sqrt{(-1)^2 + 3^2 + (-1)^2} = \sqrt{11}$

(h) $\| \overrightarrow{p_1p_2} \| = \sqrt{(-5)^2 + 2^2 + 1^2} = \sqrt{30}$

6.3 (a) $\vec{a} \cdot \vec{b} = \langle 1, 4 \rangle \cdot \langle -1, 2 \rangle = 1 \times (-1) + 4 \times 2 = 7$

(b) $\vec{a} \cdot \vec{b} = \langle 2, 2 \rangle \cdot \langle 1, -2 \rangle = 2 \times 1 + 2 \times (-2) = -2$

(c) $\vec{a} \cdot \vec{b} = \langle 1, -4 \rangle \cdot \langle 3, 3 \rangle = 1 \times 3 + (-4) \times 3 = -9$

(d) $\vec{a} \cdot \vec{b} = \langle 2, -2 \rangle \cdot \langle 3, -2 \rangle = 2 \times 3 + (-2) \times (-2) = 10$

(e) $\vec{a} \cdot \vec{b} = \langle 0, -1, 2 \rangle \cdot \langle 1, 1, 2 \rangle = 0 \times 1 + (-1) \times 1 + 2 \times 2 = 3$

(f) $\vec{a} \cdot \vec{b} = \langle 1, -2, 1 \rangle \cdot \langle 3, 3, 1 \rangle = 1 \times 3 + (-2) \times 3 + 1 \times 1 = -2$

(g) $\vec{a} \cdot \vec{b} = \langle 2, -2, 0 \rangle \cdot \langle 4, -1, 2 \rangle = 2 \times 4 + (-2) \times (-1) + 0 \times 2 = 10$

(h) $\vec{a} \cdot \vec{b} = \langle 0, -3, 1 \rangle \cdot \langle 2, -2, -3 \rangle = 0 \times 2 + (-3) \times (-2) + 1 \times (-3) = 3$

6.4 (a) $\theta = \cos^{-1}(\dfrac{1}{\sqrt{5} \cdot \sqrt{13}}) = 82.9°$

(b) $\theta = \cos^{-1}(\dfrac{-2}{\sqrt{8} \cdot \sqrt{5}}) = 108.4°$

(c) $\theta = \cos^{-1}(0) = 90°$

(d) $\theta = \cos^{-1}(0) = 90°$

(e) $\theta = \cos^{-1}(\dfrac{5}{\sqrt{5} \cdot \sqrt{6}}) = 24.1°$

(f) $\theta = \cos^{-1}(\dfrac{2}{3 \cdot \sqrt{22}}) = 81.8°$

(g) $\theta = \cos^{-1}(\dfrac{-1}{\sqrt{10} \cdot \sqrt{21}}) = 94.0°$

(h) $\theta = \cos^{-1}(0) = 90°$

6.5 (a) $c = -2$

(b) $c = -2$

(c) $c = 5$

(d) $c = \dfrac{4}{5}$

6.6 (a) $W = \overrightarrow{F} \bullet \overrightarrow{p_1 p_2} = \langle\, 2,\, -2\,\rangle \bullet \langle\, -2,\, 2\,\rangle$
$= 2\times(-2) + (-2)\times 2 = -8\,(N \bullet m)$

(b) $W = \overrightarrow{F} \bullet \overrightarrow{p_1 p_2} = \langle\, 3,\, -1\,\rangle \bullet \langle\, 2,\, -4\,\rangle$
$= 3\times 2 + (-1)\times(-4) = 10\,(N \bullet m)$

(c) $W = \overrightarrow{F} \bullet \overrightarrow{p_1 p_2} = \langle\, 2,\, -1,\, 2\,\rangle \bullet \langle\, -1,\, -3,\, -1\,\rangle$
$= 2\times(-1) + (-1)\times(-3) + 2\times(-1)$
$= -1\,(N \bullet m)$

6.7 (a) 방향코사인 값
$\cos\alpha = \dfrac{3}{\sqrt{14}}\,,\ \cos\beta = \dfrac{-1}{\sqrt{14}}\,,\ \cos\gamma = \dfrac{2}{\sqrt{14}}$

방향각
$\alpha = \cos^{-1}\!\left(\dfrac{3}{\sqrt{14}}\right) = 36.7^\circ$
$\beta = \cos^{-1}\!\left(\dfrac{-1}{\sqrt{14}}\right) = 105.5^\circ$
$\gamma = \cos^{-1}\!\left(\dfrac{2}{\sqrt{14}}\right) = 57.7^\circ$

(b) 방향코사인 값
$\cos\alpha = \dfrac{-2}{3}\,,\ \cos\beta = \dfrac{-2}{3}\,,\ \cos\gamma = \dfrac{1}{3}$

방향각
$\alpha = \cos^{-1}\!\left(\dfrac{-2}{3}\right) = 131.8^\circ$
$\beta = \cos^{-1}\!\left(\dfrac{-2}{3}\right) = 131.8^\circ$
$\gamma = \cos^{-1}\!\left(\dfrac{1}{3}\right) = 70.5^\circ$

(c) 방향코사인 값

$$\cos\alpha = \frac{2}{\sqrt{17}} \,,\, \cos\beta = \frac{-3}{\sqrt{17}} \,,\, \cos\gamma = \frac{-2}{\sqrt{17}}$$

방향각

$$\alpha = \cos^{-1}(\frac{2}{\sqrt{17}}) = 61.0°$$

$$\beta = \cos^{-1}(\frac{-3}{\sqrt{17}}) = 136.7°$$

$$\gamma = \cos^{-1}(\frac{-2}{\sqrt{17}}) = 119.0°$$

6.8 방향코사인값

$$\cos\alpha_S = \frac{8}{\sqrt{180}} \,,\, \cos\beta_E = \frac{10}{\sqrt{180}} \,,\, \cos\gamma_N = \frac{4}{\sqrt{180}}$$

방향각

$$\alpha_S = \cos^{-1}(\frac{8}{\sqrt{180}}) = 53.4°$$

$$\beta_E = \cos^{-1}(\frac{10}{\sqrt{180}}) = 41.8°$$

$$\gamma_N = \cos^{-1}(\frac{4}{\sqrt{180}}) = 72.7°$$

6.9 (a) $\dfrac{-4}{\sqrt{5}}$

(b) $\dfrac{8}{\sqrt{14}}$

6.10 (a) $proj_{\vec{b}}\,\vec{a} = \dfrac{-2}{\sqrt{6}} \cdot \dfrac{\langle 1,1,-2 \rangle}{\sqrt{6}}$

$$= -\frac{1}{3}\langle 1,1-2 \rangle$$

$$= \left\langle -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$$

(b) $proj_{\vec{b}}\,\vec{a} = \dfrac{7}{\sqrt{6}} \cdot \dfrac{\langle 2,-1,1 \rangle}{\sqrt{6}}$

$$= \frac{7}{6}\langle 2,-1,1 \rangle$$

$$= \left\langle \frac{14}{6}, -\frac{7}{6}, \frac{7}{6} \right\rangle$$

6.11 (a) $proj_{\vec{b}}\,\vec{a} = (comp_{\vec{b}}\,\vec{a})\dfrac{\vec{b}}{\|\vec{b}\|} = \left(\dfrac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}\right)\dfrac{\vec{b}}{\|\vec{b}\|}$

$$= \dfrac{\langle 2,2,-1\rangle\cdot\langle 1,2,-2\rangle}{\sqrt{1^2+2^2+(-2)^2}}\cdot\dfrac{\langle 1,2,-2\rangle}{\sqrt{1^2+2^2+(-2)^2}}$$

$$= \dfrac{2\times 1+2\times 2+(-1)\times(-2)}{3}\cdot\dfrac{\langle 1,2,-2\rangle}{3}$$

$$= \dfrac{8}{3}\cdot\dfrac{\langle 1,2,-2\rangle}{3} = \left\langle \dfrac{8}{9},\dfrac{16}{9},\dfrac{-16}{9}\right\rangle$$

(b) $proj_{\vec{b}^{\perp}}\,\vec{a} = \vec{a} - proj_{\vec{b}}\,\vec{a} = \langle 2,2,-1\rangle - \left\langle \dfrac{8}{9},\dfrac{16}{9},\dfrac{-16}{9}\right\rangle$

$$= \left\langle \dfrac{18}{9},\dfrac{18}{9},-\dfrac{9}{9}\right\rangle - \left\langle \dfrac{8}{9},\dfrac{16}{9},\dfrac{-16}{9}\right\rangle$$

$$= \left\langle \left(\dfrac{18}{9}-\dfrac{8}{9}\right),\left(\dfrac{18}{9}-\dfrac{16}{9}\right),\left(\dfrac{-9}{9}+\dfrac{16}{9}\right)\right\rangle$$

$$= \left\langle \dfrac{10}{9},\dfrac{2}{9},\dfrac{7}{9}\right\rangle$$

(c) $proj_{(\vec{a}+\vec{b})}\,\vec{a} = (comp_{(\vec{a}+\vec{b})}\,\vec{a})\dfrac{\vec{a}+\vec{b}}{\|(\vec{a}+\vec{b})\|} = \dfrac{\vec{a}\cdot(\vec{a}+\vec{b})}{\|(\vec{a}+\vec{b})\|}\cdot\dfrac{\vec{a}+\vec{b}}{\|(\vec{a}+\vec{b})\|}$

$$= \dfrac{\langle 2,2,-1\rangle\cdot\langle 3,4,-3\rangle}{\sqrt{3^2+4^2+(-3)^2}}\cdot\dfrac{\langle 3,4,-3\rangle}{\sqrt{3^2+4^2+(-3)^2}}$$

$$= \dfrac{2\times 3+2\times 4+(-1)\times(-3)}{\sqrt{34}}\cdot\dfrac{\langle 3,4,-3\rangle}{\sqrt{34}}$$

$$= \dfrac{17}{34}\langle 3,4,-3\rangle = \dfrac{1}{2}\langle 3,4,-3\rangle = \left\langle \dfrac{3}{2},2,-\dfrac{3}{2}\right\rangle$$

(d) $proj_{(\vec{a}-\vec{b})}\,\vec{a} = (comp_{(\vec{a}-\vec{b})}\,\vec{a})\dfrac{\vec{a}-\vec{b}}{\|(\vec{a}-\vec{b})\|} = \dfrac{\vec{a}\cdot(\vec{a}-\vec{b})}{\|(\vec{a}-\vec{b})\|}\cdot\dfrac{\vec{a}-\vec{b}}{\|(\vec{a}-\vec{b})\|}$

$$= \dfrac{\langle 2,2,-1\rangle\cdot\langle 1,0,1\rangle}{\sqrt{1^2+0^2+1^2}}\cdot\dfrac{\langle 1,0,1\rangle}{\sqrt{1^2+0^2+1^2}}$$

$$= \dfrac{2\times 1+2\times 0+(-1)\times 1}{\sqrt{2}}\cdot\dfrac{\langle 1,0,1\rangle}{\sqrt{2}}$$

$$= \dfrac{1}{2}\langle 1,0,1\rangle = \left\langle \dfrac{1}{2},0,\dfrac{1}{2}\right\rangle$$

6.12 (a) $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 0 \\ 3 & -2 & 0 \end{vmatrix} = \begin{vmatrix} -3 & 0 \\ -2 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & -3 \\ 3 & -2 \end{vmatrix}\vec{k}$

$$= [-2-(-9)]\vec{k} = 7\vec{k}$$

(b) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -1 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix}\vec{k}$

$= (2 \times (-1) - 3 \times (-1))\vec{k} = \vec{k}$

(c) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 4 & 0 \\ 3 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix}\vec{k}$

$= (4 \times 2 - (-1) \times 3)\vec{k} = 11\vec{k}$

(d) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -3 & 0 \\ -1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} -3 & 0 \\ 2 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} -2 & 0 \\ -1 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} -2 & -3 \\ -1 & 2 \end{vmatrix}\vec{k}$

$= ((-2) \times 2 - (-3) \times (-1))\vec{k} = -7\vec{k}$

(e) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & -2 \\ 2 & -4 & 0 \end{vmatrix} = \begin{vmatrix} -3 & -2 \\ -4 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & -2 \\ 2 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & -3 \\ 2 & -4 \end{vmatrix}\vec{k}$

$= -8\vec{i} - 4\vec{j} - 2\vec{k}$

(f) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -2 \\ 1 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 3 & -2 \\ 0 & -2 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & -2 \\ 1 & -2 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix}\vec{k}$

$= -6\vec{i} - 3\vec{k}$

(g) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & -1 \\ 2 & -1 & -2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ -1 & -2 \end{vmatrix}\vec{i} - \begin{vmatrix} -2 & -1 \\ 2 & -2 \end{vmatrix}\vec{j} + \begin{vmatrix} -2 & 4 \\ 2 & -1 \end{vmatrix}\vec{k}$

$= (-8-1)\vec{i} - (4+2)\vec{j} + (2-8)\vec{k}$

$= -9\vec{i} - 6\vec{j} - 6\vec{k}$

(h) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & 2 \\ 1 & -1 & 4 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ -1 & 4 \end{vmatrix}\vec{i} - \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix}\vec{j} + \begin{vmatrix} -1 & -3 \\ 1 & -1 \end{vmatrix}\vec{k}$

$= (-12+2)\vec{i} - (-4-2)\vec{j} + (1+3)\vec{k}$

$= -10\vec{i} + 6\vec{j} + 4\vec{k}$

6.13 (a) $\theta = \sin^{-1}\left(\dfrac{8}{\sqrt{5} \cdot \sqrt{13}}\right) = 82.9°$

(b) $\theta = \sin^{-1}\left(\dfrac{6}{\sqrt{8} \cdot \sqrt{5}}\right) = 71.6°$

(c) $\theta = \sin^{-1}(1) = 90°$

(d) $\theta = \sin^{-1}(\dfrac{9}{\sqrt{18} \cdot \sqrt{5}}) = 71.6°$

(e) $\theta = \sin^{-1}(\dfrac{3}{\sqrt{5} \cdot \sqrt{2}}) = 71.6°$

(f) $\theta = \sin^{-1}(\dfrac{\sqrt{35}}{\sqrt{14} \cdot \sqrt{6}}) = 40.2°$

(g) $\theta = \sin^{-1}(1) = 90°$

(h) $\theta = \sin^{-1}(\dfrac{\sqrt{83}}{\sqrt{14} \cdot \sqrt{6}}) = 83.7°$

6.14 $\dfrac{\| \vec{a} \times \vec{b} \|}{2} = \dfrac{\sqrt{206}}{2} = 7.176$

6.15 $\| \vec{a} \times \vec{b} \| = \sqrt{(-12)^2 + (-15)^2 + (18)^2} = 26.325$

6.16 $M = \vec{F} \times \vec{r} = \| \vec{F} \| \cdot \| \vec{r} \| \cdot \sin\theta$
$= 500 \cdot 2 \cdot 1 = 1000(N \cdot m)$

방향은 오른속 법칙에 의해 지면을 뚫고 나오는 방향이다.

6.17   (a) $|\vec{c} \cdot (\vec{a} \times \vec{b})| = |3| = 3$
      (b) $|\vec{c} \cdot (\vec{a} \times \vec{b})| = |6| = 6$

6.18   (a) 주어진 세 벡터는 동일 평면상에 있지 않다.
      (b) 주어진 세 벡터는 동일 평면상에 있다.

6.19   (a) 일차종속
      (b) 일차독립

6.20 (a) $\vec{a} \cdot \vec{b} = \langle\, 2\,,\, -1\,,\, -2\,,\, -3\,\rangle \cdot \langle\, -2\,,\, -1\,,\, 3\,,\, 2\,\rangle$
$= 2\times(-2) + (-1)\times(-1) + (-2)\times3 + (-3)\times2$
$= -4 + 1 - 6 - 6$
$= -15$

(b) $\vec{a} \cdot \vec{b} = \langle\, -2\,,\, -1\,,\, -1\,,\, 3\,\rangle \cdot \langle\, 2\,,\, -2\,,\, -1\,,\, 1\,\rangle$
$= (-2)\times2 + (-1)\times(-2) + (-1)\times(-1) + 3\times1$
$= -4 + 2 + 1 + 3$
$= 2$

6.21 (a) 두 벡터 $\vec{a}, \vec{b}$는 서로 직교한다.

(b) 두 벡터 $\vec{a}, \vec{b}$는 서로 직교한다.

Chapter 07 연습문제 답안

7.1 (a) $x = \cos t,\ y = t$ 이므로 $x = \cos y$ 그래프는 코사인 그래프

(b) $x = t,\ y = 2t^2$ 이므로 $y = 2x^2$ 그래프는 포물선

(c) $x = 2\cos t,\ y = 2\sin t$ 이므로 $x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2 = 2^2$
그래프는 중심이 원점이고, 반경이 2인 원

(d) $y^2 = 4\sin^2 t = 4(1 - x^2)$, $4x^2 + y^2 = 4$, $x^2 + \dfrac{y^2}{2^2} = 1$
그래프는 중심이 원점이고, 장축이 2, 단축이 1인 타원

(e) $\vec{r}(t) = 3\sin t\,\vec{i} + 3\cos t\,\vec{j} + 2\vec{k}$
$x = 3\sin t,\ y = 3\cos t,\ z = 2$
$x^2 = 9\sin^2 t,\ y^2 = 9\cos^2 t$
$x^2 + y^2 = 9,\ z = 2$
그래프는 $z = 2$인 xy평면상에서 중심이 원점이고, 반경이 3인 원

(f) $\vec{r}(t) = 2\vec{i} + \cos t\,\vec{j} + \sin t\,\vec{k}$
$x = 2,\ y = \cos t,\ z = \sin t$
$x = 2,\ y^2 + z^2 = 1$
그래프는 $z = 2$인 xy평면상에서 중심이 원점이고, 반경이 3인 원

7.2 (a) $\vec{i}$

(b) $-\vec{j}$

(c) $\vec{i} + 2\vec{j}$

(d) $-\vec{j} + \vec{k}$

7.3 (a) $2\vec{j}$

(b) $2\vec{i}$

7.4 (a) $-2\vec{i}-3\vec{j}+7\vec{k}$

(b) $3(4-1)=9$

7.5 (a) $\vec{r}'(t)=(2t+2)\vec{i}-\dfrac{2}{(t-1)^2}\vec{j}-4e^{-2t}\vec{k}$

$\vec{r}'(0)=2\vec{i}-2\vec{j}-4\vec{k}$

(b) $\vec{r}'(t)=(e^{-t}\cos t-e^{-t}\sin t)\vec{i}+(2t+2)\vec{j}+(-2te^{-2t}+e^{-2t})\vec{k}$

$\vec{r}'(0)=\vec{i}+2\vec{j}+\vec{k}$

7.6 (a) $\dfrac{d}{dt}\big[\vec{r}'(t)\times t\vec{r}(t)\big]=\vec{r}'(t)\times\dfrac{d}{dt}\big[t\vec{r}(t)\big]+\dfrac{d}{dt}\big[\vec{r}'(t)\big]\times t\vec{r}(t)$

$=\vec{r}'(t)\times\Big[t\vec{r}'(t)+\dfrac{d}{dt}\vec{r}(t)\Big]+\vec{r}''(t)\times t\vec{r}(t)$

$=t\vec{r}'(t)\times\vec{r}'(t)+\vec{r}'(t)\times\vec{r}'(t)+\vec{r}''(t)\times t\vec{r}(t)$

$=t\vec{r}''(t)\times\vec{r}(t)$

(b) $\dfrac{d}{dt}\big[\vec{r}'(t)\big]\times e^{t}\vec{r}(t)=\vec{r}'(t)\times\dfrac{d}{dt}\big[e^{t}\vec{r}(t)\big]$

$=\vec{r}'(t)\times[e^{t}\vec{r}'(t)\times e^{t}\vec{r}(t)]$

$=e^{t}\vec{r}'(t)\times\vec{r}(t)$

(c) $\dfrac{d}{dt}\big[\vec{r}(t)\;\bullet\;\{\vec{r}(t)\times\vec{r}'(t)\}\big]=\vec{r}(t)\;\bullet\;\dfrac{d}{dt}\{\vec{r}(t)\times\vec{r}'(t)\}+\dfrac{d}{dt}\vec{r}(t)\;\bullet\;\{\vec{r}(t)\times\vec{r}'(t)\}$

$=\vec{r}(t)\;\bullet\;[\vec{r}(t)\times\vec{r}''(t)+\vec{r}'(t)\times\vec{r}'(t)]+\vec{r}'(t)\;\bullet\;\{\vec{r}(t)\times\vec{r}'(t)\}$

$=0$

(d) $\dfrac{d}{dt}\big[\vec{r}'(t^2)+t\vec{r}(t^2)\big]=\vec{r}''(t^2)(2t)+t\big[2t\vec{r}'(t)+\vec{r}(t^2)\big]=2t\vec{r}''(t^2)+2t^2\vec{r}'(t)+t\vec{r}(t^2)$

7.7 (a) $\vec{r}(t) = (t^2 e^t - 2)\vec{i} + (t\ln t)\vec{j} + (e^{-t}\sin 2t)\vec{k}$

$\vec{r}'(t) = (t^2 e^t + 2te^t)\vec{i} + (t \times \frac{1}{t} + \ln t)\vec{j} + (e^{-t}2\cos 2t - e^{-t}\sin 2t)\vec{k}$

$\vec{r}''(t) = (t^2 e^t + 2te^t + 2te^t + 2e^t)\vec{i} + \frac{1}{t}\vec{j} + (-4e^{-t}\sin 2t - 2e^{-t}\cos 2t - 2e^{-t}\cos 2t + e^{-t}\sin 2t)\vec{k}$

$\qquad = (t^2 e^t + 4te^t + 2e^t)\vec{i} + \frac{1}{t}\vec{j} + (-3e^{-t}\sin 2t - 4e^{-t}\cos 2t)\vec{k}$

$\vec{r}''(t)$을 한번 더 미분하면 $\vec{r}'''(t)$가 구해진다.

(b) $\vec{r}(t) = (e^{-2t}\cos t)\vec{i} + (t^2 + 2t)\vec{j} + (e^{-2t} - 2\sin t)\vec{k}$

$\vec{r}'(t) = (-e^{-2t}\sin t - 2e^{-2t}\cos t)\vec{i} + (2t+2)\vec{j} + (-2e^{-2t} - 2\cos t)\vec{k}$

$\vec{r}''(t) = (3e^{-2t}\cos t + 4e^{-2t}\sin t)\vec{i} + 2\vec{j} + (4e^{-2t} + 2\sin t)\vec{k}$

$\vec{r}''(t)$을 한번 더 미분하면 $\vec{r}'''(t)$가 구해진다.

7.8 (a) $\vec{r}(t) = (t^2 - 2t)\vec{i} + (te^{-2t})\vec{j}$

속도벡터함수 : $\vec{r}'(t) = (2t-2)\vec{i} + (-2te^{-2t} + e^{-2t})\vec{j}$

가속도 벡터함수 : $\vec{r}''(t) = 2\vec{i} + (4te^{-2t} - 4e^{-2t})\vec{j}$

(b) $\vec{r}(t) = (t^2\cos t)\vec{i} + (t-3)\vec{j}$

$\vec{r}'(t) = (-t^2\sin t + 2t\cos t)\vec{i} + \vec{j}$

$\vec{r}''(t) = (-t^2\cos t - 2t\sin t - 2t\sin t + 2\cos t)\vec{i}$

$\qquad = (-t^2\cos t - 4t\sin t + 2\cos t)\vec{i}$

(c) $\vec{r}(t) = (2t^2 - 2)\vec{i} + \left(\frac{2}{t-1}\right)\vec{j}$

$\vec{r}'(t) = 4t\vec{i} - \frac{2}{(t-1)^2}\vec{j}$

$\vec{r}'(t) = 4\vec{i} + \frac{4}{(t-1)^3}\vec{j}$

(d) $\vec{r}(t) = (t^2 e^{-t})\vec{i} + (2t\sin t - e^t)\vec{j} + (e^{-2t}\cos t)\vec{k}$

$\vec{r}'(t) = (2te^{-t} - t^2 e^{-t})\vec{i} + (2\sin t + 2t\cos t - e^t)\vec{j} + (-2e^{-2t}\cos t - e^{-2t}\sin t)\vec{k}$

$\vec{r}''(t) = (2e^{-t} - 4te^{-t} + t^2 e^{-t})\vec{i} + (4\cos t - 2t\sin t - e^t)\vec{j} + (3e^{-2t}\cos t + 4e^{-2t}\sin t)\vec{k}$

7.9 (a) $z_x = -2x + 4xy$

$z_y = 2x^2 - 3y^2$

(b) $z_x = 6x^2y + 2xy^2 + 2y$

$\quad\; z_y = 2x^3 + 2x^2y + 2x - 3y^2$

(c) $z_x = \dfrac{2xy + 2}{y^2 + 1}$

$\quad\; z_y = \dfrac{2x(y^2+1) - 2y(x^2y + 2x)}{(y^2+1)^2} = \dfrac{2xy^2 + 2x - 2x^2y^2 - 4xy}{(y^2+1)^2}$

(d) $z_x = \dfrac{\sqrt{\dfrac{y}{x}}\,(x^2+y^2) - 2x(2\sqrt{xy} + y)}{(x^2+y^2)^2}$

$\quad\; z_y = \dfrac{\left(\sqrt{\dfrac{x}{y}} + 1\right)(x^2+y^2) - 2y(2\sqrt{xy} + y)}{(x^2+y^2)^2}$

(e) $z_x = 9\cos^2 2x\,(-2\sin 2x) - 2x\sin^3 2y$

$\quad\; z_y = -3x^2\sin^2 2y\,(2\cos 2x)$

(f) $z_x = 2(x^2 - 2xy + 2y^3)(2x - 2y)$

$\quad\; z_y = 2(x^2 - 2xy + 2y^3)(-2x + 6y^2)$

7.10 (a) $z_{xx} = 4\sin y - 4y^3$
$\qquad z_{yy} = -2x^2\sin y - 12x^2y$
$\qquad z_{xy} = 4x\cos y - 12xy^2 - 2$

(b) $z_{xx} = -12x^2y^2$
$\qquad z_{yy} = -2x^4 - 6xy$
$\qquad z_{xy} = 2 - 8x^3y - 1 - 3y^2 = 1 - 8x^3y - 3y^2$

(c) $z_{xx} = 8y^2\cos 4x$
$\qquad z_{yy} = 16x\cos 4y - 2\cos^2 2x$
$\qquad z_{xy} = 4\sin 2y(2\cos 2y) + 4y\sin 4x = 4\sin 4y + 4y\sin 4x = 4\sin 4y(y+1)$

(d) $z_{xx} = 4(x^2 - xy + 2y^2) + 2(2x-y)^2 + 2$

$z_{yy} = 8(x^2 - xy + 2y^2) + 2(4y-x)^2 + 2$

$z_{xy} = -2(x^2 - xy + 2y^2) + 2(4y-x)(2x-y)$

7.11 (a) $\dfrac{\partial z}{\partial u} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial u} = (2x - 2xy^2)(2uv - e^{-v}) + (-2x^2y + 2y)(4uv - 2u)$

$\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v} = (2x - 2xy^2)(u^2 + ue^{-v}) + (-2x^2y + 2y)(2u^2)$

(b) $\dfrac{\partial x}{\partial u} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial u} = (4xy - y^2)(-2u + 4uv) + (2x^2 - 2xy)(6u^2v + 2uv^2 + 2v)$

$\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v} = (4xy - y^2)(2u^2 - 3v^2) + (2x^2 - 2xy)(2u^3 + 2u^2v + 2u)$

7.12 (a) $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$

$= (2x + 2y - 2xy^2)(4\cos 2t - 4\sin 2t) + (2x - 2x^2y)(-2\sin t - \cos t)$

(b) $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$

$= (-ye^{-x} - e^{-y})(-e^{-2t}\sin t - 2e^{-2t}\cos t - e^{-2t}\cos t + 2e^{-2t}\sin t) + (e^{-x} + xe^{-y})(4t - 1)$

$= (-ye^{-x} - e^{-y})(e^{-2t}\sin t - 3e^{-2t}\cos t) + (e^{-x} + xe^{-y})(4t - 1)$

7.13 (a) $\nabla f(x,y) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} = (2y - y^3)\vec{i} + (2x - 3xy^2 + 2)\vec{j}$

$\nabla f(1,2) = [2(2) - 2^3]\vec{i} + [2(1) - 3(1)2^2 + 2]\vec{j} = -4\vec{i} - 8\vec{j}$

(b) $\nabla f(x,y) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} = (2\sin^2 2y + 2y^2\sin 2x)\vec{i} + [(4x\sin 2y)(2\cos 2y) - 2y\cos 2x]\vec{j}$

$\nabla f\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) = \left[2\sin^2 \pi + 2\left(\dfrac{\pi}{2}\right)^2\sin \pi\right]\vec{i} + \left[4\left(\dfrac{\pi}{2}\right)\sin \pi \times 2\cos \pi - 2\left(\dfrac{\pi}{2}\cos \pi\right)\right]\vec{j} = \pi\vec{j}$

(c) $\nabla f(x,y,z) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} + \dfrac{\partial f}{\partial z}\vec{k} = (2x - yz + 2y^2)\vec{i} + (-xz + 4xy)\vec{j} + (-xy + 2z)\vec{k}$

$\nabla f(1,2,1) = (2\times 1 - 2\times 1 + 2\times 2^2)\vec{i} + (-1 + 4\times 1\times 2)\vec{j} + (-1\times 2 + 2\times 1)\vec{k} = 8\vec{i} + 7\vec{j}$

7.14 (a) $\nabla f(x,y) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} = (6x-2y)\vec{i} + (-2x-2y)\vec{j}$

$\nabla f(1,1) = 4\vec{i} - 4\vec{j}$

$D_{\vec{a}}f = \nabla f(1,1) \cdot \dfrac{\vec{a}}{\|\vec{a}\|} = (4\vec{i}-4\vec{j}) \cdot \dfrac{\vec{i}-\vec{j}}{\sqrt{2}} = \dfrac{4}{\sqrt{2}} + \dfrac{4}{\sqrt{2}} = 4\sqrt{2}$

(b) $\nabla f(x,y) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} = \left[2x^2(2xy-y)2y\right]\vec{i} + \left[2x^2(2xy-y)(2x-1)\right]\vec{j}$

$\nabla f(-1,1) = \left[2(1)(2(-1)(1)-1)2(1)\right]\vec{i} + \left[2(1)2(-1)(1)-1)(2(-1)-1\right]\vec{j}$

$\quad = -12\vec{i} - 12\vec{j}$

$D_{\vec{a}}f = \nabla f(-1,1) \cdot \dfrac{\vec{a}}{\|\vec{a}\|} = (-12\vec{i}-12\vec{j}) \cdot \dfrac{2\vec{i}+\vec{j}}{\sqrt{5}}$

$= \dfrac{-24}{\sqrt{5}} + \dfrac{-12}{\sqrt{5}} = \dfrac{-36}{\sqrt{5}}$

(c) $\nabla f(x,y,z) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} + \dfrac{\partial f}{\partial z}\vec{k}$

$= 2y(z+2)^2\vec{i} + \left[2x(z+2)^2+z\right]\vec{j} + \left[4xy(z+2)+y\right]\vec{k}$

$\nabla f(1,-1,2) = \left[2(-1)(2+2)^2\right]\vec{i} + \left[2(1)(2+2)^2+2\right]\vec{j} + \left[4(1)(-1)(2+2)-1\right]\vec{k}$

$= -32\vec{i} + 34\vec{j} - 17\vec{k}$

$D_{\vec{a}}f = \nabla(1,-1,2) \cdot \dfrac{\vec{a}}{\|\vec{a}\|} = (-32\vec{i}+34\vec{j}-17\vec{k}) \cdot \dfrac{1}{\sqrt{3}}(\vec{i}+\vec{j}-\vec{k})$

$= -\dfrac{32}{\sqrt{3}} + \dfrac{34}{\sqrt{3}} + \dfrac{17}{\sqrt{3}} = \dfrac{19}{\sqrt{3}}$

(d) $\nabla f(x,y,z) = \left[yz + \dfrac{2xy}{2\sqrt{x^2y-y^2z}}\right]\vec{i} + \left[xz + \dfrac{x^2-2yz}{2\sqrt{x^2y-y^2z}}\right]\vec{j} + \left[xy + \dfrac{-y^2}{2\sqrt{x^2y-y^2z}}\right]\vec{k}$

$\nabla f(1,1,1) = \left[(1)(1) + \dfrac{2(1)(1)}{2\sqrt{1^2(1)-1^2(1)}}\right]\vec{i} + \left[(1)(1) + \dfrac{1^2-2(1)(1)}{2\sqrt{1^2(1)-1^2(1)}}\right]\vec{j}$

$\quad + \left[(1)(1) + \dfrac{-1^2}{2\sqrt{1^2(1)-1^2(1)}}\right]\vec{k}$

그래디언트가 존재하지 않음.

따라서 점 $(1,1,1)$에서의 방향도함수는 구할 수 없음.

7.15 (한빛아카데미 판) 본문 문제 수정됨 [연습문제 7.14] -> [연습문제 7.13]

(a) 방향도함수의 최댓값 : $\|\nabla f\|$, $\nabla f(1,1) = 4\vec{i} - 4\vec{j}$이므로

$\|\nabla f\| = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}$

방향도함수의 최솟값 : $-\|\nabla f\| = -4\sqrt{2}$

(b) 방향도함수의 최댓값 : $\parallel \nabla f \parallel$, $\nabla f(-1, 1) = -12\vec{i} - 12\vec{j}$ 이므로

$\parallel \nabla f \parallel = \sqrt{(-12)^2 + (-12)^2} = 12\sqrt{2}$

방향도함수의 최솟값 : $-\parallel \nabla f \parallel = -12\sqrt{2}$

(c) $\nabla f(1,-1,2) = -32\vec{i} + 34\vec{j} - 17\vec{k}$

방향도함수의 최댓값

$\parallel \nabla f \parallel = \sqrt{(-32)^2 + (34)^2 + (-17)^2} = \sqrt{2469}$

최솟값 $= -\sqrt{2469}$

(d) 구할 수 없음

7.16 $\nabla H(5,1) = -20\vec{i} - 6\vec{j}$

7.17 가장 급격히 증가하는 방향 : $\nabla T(1,1,1) = 4\vec{i} - 2\vec{k}$

크기 : $2\sqrt{5}$

가장 급격히 감소하는 방향 : $-\nabla T(1,1,1) = -4\vec{i} + 2\vec{k}$

크기 : $-2\sqrt{5}$

7.18 (a) $\mathrm{div}\,\vec{F}(x,y,z) = \nabla \cdot \vec{F}$

$= \dfrac{\partial}{\partial x}(x) + \dfrac{\partial}{\partial y}(y) + \dfrac{\partial}{\partial z}(z) = 3$

(b) $\mathrm{div}\,\vec{F}(x,y,z) = \dfrac{\partial}{\partial x}(xy) + \dfrac{\partial}{\partial y}(yz) + \dfrac{\partial}{\partial z}(xz)$

$= y + z + x$

(c) $\mathrm{div}\,\vec{F}(x,y,z) = \dfrac{\partial}{\partial x}(y^2 x) + \dfrac{\partial}{\partial y}(x^2 z) + \dfrac{\partial}{\partial z}(xy)$

$= 0$

(d) $\mathrm{div}\,\vec{F}(x,y,z) = \dfrac{\partial}{\partial x}(2x^2 y) + \dfrac{\partial}{\partial y}(x^2 + yz) + \dfrac{\partial}{\partial z}(2x^3)$

$= 4xy + z$

(e) $\operatorname{div} \overrightarrow{F}(x,y,z) = \frac{\partial}{\partial x}(xe^y) + \frac{\partial}{\partial y}(e^{-x}z) + \frac{\partial}{\partial z}(xyz)$

$= e^y + xy$

(f) $\operatorname{div} \overrightarrow{F}(x,y,z) = \frac{\partial}{\partial x}(xy^2z^2) + \frac{\partial}{\partial y}(yz - xz) + \frac{\partial}{\partial z}(x^2y^2)$

$= y^2z^2 + z$

(g) $\operatorname{div} \overrightarrow{F}(x,y,z) = \frac{\partial}{\partial x}(xe^{-z}) + \frac{\partial}{\partial y}(ye^{-x}) + \frac{\partial}{\partial z}(ze^{-y})$

$= e^{-z} + e^{-x} + e^{-y}$

(h) $\operatorname{div} \overrightarrow{F}(x,y,z) = \frac{\partial}{\partial x}(x^2\cos yz) + \frac{\partial}{\partial y}(y^2\sin xz) + \frac{\partial}{\partial z}(z^2)$

$= 2x\cos yz + 2y\sin xz + 2z$

7.19   => (한빛아카데미 판) 본문 문제 수정됨

$\operatorname{div} \overrightarrow{F} = \frac{\partial \overrightarrow{F}}{\partial x} + \frac{\partial \overrightarrow{F}}{\partial y} + \frac{\partial \overrightarrow{F}}{\partial z} = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(2xy^2z) - \frac{\partial}{\partial z}(2xyz + 2xyz^2)$

$= 2xy + 4xyz - 2xy - 4xyz$

$= 0$

7.20   (a) $\operatorname{curl} \overrightarrow{F} = \nabla \times \overrightarrow{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$

$= (\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z})\vec{i} + (\frac{\partial y}{\partial x} - \frac{\partial y}{\partial z})\vec{j} + (\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y})\vec{k}$

$= \vec{0}$

(b) $\nabla \times \overrightarrow{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix}$

$= (\frac{\partial(xz)}{\partial y} - \frac{\partial(yz)}{\partial z})\vec{i} + (\frac{\partial(xy)}{\partial z} - \frac{\partial(xz)}{\partial x})\vec{j} + (\frac{\partial(yz)}{\partial x} - \frac{\partial(xy)}{\partial y})\vec{k}$

$= -y\vec{i} - z\vec{j} - x\vec{k}$

(c) $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z & x^2z & xy \end{vmatrix}$

$= (\frac{\partial(xy)}{\partial y} - \frac{\partial(x^2z)}{\partial z})\vec{i} + (\frac{\partial(y^2z)}{\partial z} - \frac{\partial(xy)}{\partial x})\vec{j} + (\frac{\partial(x^2z)}{\partial x} - \frac{\partial(y^2z)}{\partial y})\vec{k}$

$= (x - x^2)\vec{i} + (y^2 - y)\vec{j} + (2xz - 2yz)\vec{k}$

(d) $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2y & (x^2+yz) & 2x^3 \end{vmatrix}$

$= (\frac{\partial(2x^3)}{\partial y} - \frac{\partial(x^2+yz)}{\partial z})\vec{i} + (\frac{\partial(2x^2y)}{\partial z} - \frac{\partial(2x^3)}{\partial x})\vec{j} + (\frac{\partial(x^2+yz)}{\partial x} - \frac{\partial(2x^2y)}{\partial y})\vec{k}$

$= -y\vec{i} - 6x^2\vec{j} + (2x - 2x^2)\vec{k}$

(e) $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xe^y & e^{-x}z & xyz \end{vmatrix}$

$= (\frac{\partial(xyz)}{\partial y} - \frac{\partial(e^{-x}z)}{\partial z})\vec{i} + (\frac{\partial(xe^y)}{\partial z} - \frac{\partial(xyz)}{\partial x})\vec{j} + (\frac{\partial(e^{-x}z)}{\partial x} - \frac{\partial(xe^y)}{\partial y})\vec{k}$

$= (xz - e^{-x})\vec{i} - yz\vec{j} + (e^{-x}z + xe^y)\vec{k}$

(f) $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2z^2 & (yz-xz) & x^2y^2 \end{vmatrix}$

$= \left(\frac{\partial(x^2y^2)}{\partial y} - \frac{\partial(yz-xz)}{\partial z}\right)\vec{i} + \left(\frac{\partial(xy^2z^2)}{\partial z} - \frac{\partial(x^2y^2)}{\partial x}\right)\vec{j} + \left(\frac{\partial(yz-xz)}{\partial x} - \frac{\partial(xy^2z^2)}{\partial y}\right)\vec{k}$

$= (2x^2y + x - y)\vec{i} + (2xy^2z - 2xy^2)\vec{j} - (z + 2xyz^2)\vec{k}$

(g) $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xe^{-z} & ye^{-x} & ze^{-y} \end{vmatrix} = \left(\frac{\partial(ze^{-y})}{\partial y} - \frac{\partial(ye^{-y})}{\partial z}\right)\vec{i}$

$+ \left(\frac{\partial(xe^{-z})}{\partial z} - \frac{\partial(ze^{-y})}{\partial x}\right)\vec{j} + \left(\frac{\partial(ye^{-x})}{\partial x} - \frac{\partial(xe^{-z})}{\partial y}\right)\vec{k}$

$= -ze^{-y}\vec{i} - xe^{-z}\vec{j} - ye^{-x}\vec{k}$

(h) $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2\cos yz & y^2\sin xz & z^2 \end{vmatrix}$

$$= \left(\frac{\partial(z^2)}{\partial y} - \frac{\partial(y^2\sin xz)}{\partial z}\right)\vec{i} + \left(\frac{\partial(x^2\cos yz)}{\partial z} - \frac{\partial(z^2)}{\partial x}\right)\vec{j} + \left(\frac{\partial(y^2\sin xz)}{\partial x} - \frac{\partial(x^2\cos yz)}{\partial y}\right)\vec{k}$$

$$= -xy^2\cos xz\,\vec{i} - x^2y\sin yz\,\vec{j} + (y^2z\cos xz + x^2z\sin yz)\vec{k}$$

7.21 $\vec{v}(x,\,y) = \dfrac{2x}{(x^2+y^2)^2}\vec{i} + \dfrac{2y}{(x^2+y^2)^2}\vec{j},\quad (x \neq 0,\, y \neq 0)$

$curl\,\vec{v} = \nabla \times \vec{v} = \left(\dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y}\right)\vec{k}$

$\dfrac{\partial v_y}{\partial x} = \dfrac{\partial}{\partial x}\left(\dfrac{2y}{(x^2+y^2)^2}\right) = \dfrac{-4y(x^2+y^2)(2x)}{(x^2+y^2)^4} = \dfrac{-8xy(x^2+y^2)}{(x^2+y^2)^4} = \dfrac{-8xy}{(x^2+y^2)^3}$

$\dfrac{\partial v_x}{\partial y} = \dfrac{\partial}{\partial y}\left(\dfrac{2x}{(x^2+y^2)^2}\right) = \dfrac{-4x(x^2+y^2)(2y)}{(x^2+y^2)^4} = \dfrac{-8xy}{(x^2+y^2)^3}$

$\therefore curl\,v = \left(\dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y}\right)\vec{k} = \vec{0},$ 비회전적

7.22 (a) => (한빛아카데미 판) 본문 문제 수정

$\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$: 라플라시안

$\mathrm{div}(\nabla f) = \nabla \cdot (\nabla f) = \left(\dfrac{\partial}{\partial x}\vec{i} + \dfrac{\partial}{\partial y}\vec{j} + \dfrac{\partial}{\partial z}\vec{k}\right) \cdot \left(\dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} + \dfrac{\partial f}{\partial z}\vec{k}\right) = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

$\therefore \nabla^2 f = \mathrm{div}(\nabla f)$

(b) $\mathrm{div}(curl\,\vec{f}) = \nabla \cdot \left[(R_y - Q_z)\vec{i} - (R_x - P_z)\vec{j} + (Q_x - P_y)\vec{k}\right]$

$$= (R_{yx} - Q_{zx}) - (R_{xy} - P_{zy}) + (Q_{xz} - P_{yz}) = 0$$

(c) => (한빛아카데미 판) 본문 문제 수정됨

$$curl(\nabla f) = \nabla \times \nabla f = \nabla \times \left(\frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{y}\vec{j} + \frac{\partial f}{z}\vec{k} \right)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial y} & \dfrac{\partial f}{\partial z} \end{vmatrix}$$

$$= \left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial y \partial z} \right)\vec{i} + \left(\frac{\partial^2 f}{\partial x \partial z} - \frac{\partial^2 f}{\partial x \partial z} \right)\vec{i} + \left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial x \partial y} \right)\vec{k} = \vec{0}$$

Chapter 08 연습문제 답안

8.1 (a) $\displaystyle\int_c (xy-x)dx = \int_0^2 [t(2t+1)-t]\frac{dx}{dt}dt = \int_0^2 2t^2(1)dt = \frac{2}{3}t^3 \Big|_0^2 = \frac{16}{3}$

$\displaystyle\int_c (xy-x)dy = \int_0^2 [t(2t+1)-t]\frac{dy}{dt}dt = \int_0^2 2(t^2(2)dt = \frac{4}{3}t^3 \Big|_0^2 = \frac{32}{3}$

$\displaystyle\int_c (xy-x)ds = \int_0^2 [t(2t+1)-t]\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt = \int_0^2 2t^2\sqrt{1^2+2^2}dt = \frac{2\sqrt{5}}{3}t^3\Big|_0^2 = \frac{16\sqrt{5}}{3}$

(b) => (한빛아카데미 판) 본문 문제 수정됨

$\displaystyle\int_c (x^2+xy+y^2)dx = \int_0^1 [(2t)^2+(2t)(-t+1)+(-t+1)^2]\frac{dx}{dt}dt$

$\displaystyle = \int_0^1 (4t^2-2t^2+2t+t^2-2t+1)(2)dt = \int_0^1 (6t^2+2)dt = 2t^3+2t\Big|_0^1 = 4$

$\displaystyle\int_c (x^2+xy+y^2)dy = \int_0^1 (3t^2+1)\frac{dy}{dt}dt = \int_0^1 (dt^2+1)(-1)dt = -t^3-t\Big|_0^1 = -2$

$\displaystyle\int_c (x^2+xy+y^2)ds = \int_0^1 (3t^2+1)\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt$

$\displaystyle = \int_0^1 (3t^2+1)\sqrt{(2)^2+(-1)^2}dt = \int_0^1 (3\sqrt{5}t^2+\sqrt{5})dt = \sqrt{5}t^3+\sqrt{5}t\Big|_0^1 = 2\sqrt{5}$

8.2 (a) $\displaystyle\int_c 2xydx-3xy^2dy, \ c: y=2x+3, \ 0 \leq x \leq 2$

$\displaystyle\int_c (2xydx-3xy^2dy)dy = \int_0^2 2x(2x+3)dx-3x(2x+3)^2\frac{dy}{dx}dx$

$\displaystyle = \int_0^2 (4x^2+6x)dx-3x(4x^2+12x+9)(2)dx$

$\displaystyle = \int_0^2 (-24x^3-68x^2-48x)dx = -6x^4-\frac{68}{3}x^3-24x^2\Big|_0^2$

$\displaystyle = -6(2)^4-\frac{68}{3}(2)^3-24(2)^2 = -\frac{1120}{3}$

(b) $\displaystyle\int_c 3x^2ydx-2xy^2dy, \ c: x=y^2+1, \ 0 \leq y \leq 1$

$$\int_c 3x^2 y\, dx - 2xy^2\, dy = \int_0^1 3(y^2+1)^2 y \frac{dx}{dy}\, dy - 2(y^2+1)y^2\, dy$$

$$= \int_0^1 (3y^5 + 6y^3 + 3y)(2y)\, dy - (2y^4 + 2y^2)\, dy = \int_0^1 (6y^6 + 10y^4 + 4y^2)\, dy = \frac{6}{7}y^7 + 2y^5 + \frac{4}{3}y^3 \Big|_0^1 = \frac{88}{21}$$

8.3 (a) $\displaystyle\int_c 2x^2 y\, dx - xy\, dy = \int_{-1}^2 2x^2(-1)\, dx - x(-1)(0) + \int_{-1}^3 2(2)^2 y(0) - (2)y\, dy = -\frac{2}{3}x^3 \Big|_{-1}^2 - y^2 \Big|_{-1}^3 = -\frac{44}{3}$

(b) $\displaystyle\int_c 2x^2 y\, dx - xy\, dy = \int_{-1}^3 2(-1)^2 y(0) - (-1)y\, dy + \int_{-1}^2 2x^2(3)\, dx - x(3)(0) = \frac{1}{2}y^2 \Big|_{-1}^3 + 2x^3 \Big|_{-1}^2 = 22$

(c) $\displaystyle\int_c 2x^2 y\, dx - xy\, dy, \quad y = 2x, \; dy = 2dx$

$$= \int_0^2 2x^2(2x)\, dx - x(2x)(2dx) = x^4 \Big|_0^2 - \frac{4}{3}x^3 \Big|_0^2 = \frac{16}{3}$$

(d) $\displaystyle\int_c 2x^2 y\, dx - xy\, dy, \; y = x^2, \quad dy = 2xdx$

$$= \int_0^2 2x^2(x^2)\, dx - x(x^2)(2xdx) = 0$$

8.4 (a) $= \displaystyle\int_0^1 x(0)^2 dx + [2x(0)+1](0) + \int_0^1 (1)y^2(0) + [2(1)y+1]\, dy$

$$= \int_1^0 x(x)^2 dx + [2x(x)+1]\, dx$$

$$= 0 + \big[y^2 + y\big]_0^1 + \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 + x\right]_1^0 = \frac{35}{12}$$

(b) $= \displaystyle\int_0^1 x(x^2)\, dx + [2x(x)+1]\, dx + \int_1^0 (1)y^2(0) + [2(1)y+1]\, dy + \int_1^0 x(0)^2 dx + 2x(0)+1](0)$

$$= \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 + x\right]_0^1 + \big[y^2 + y\big]_1^0 + 0 = -\frac{1}{12}$$

(c) $= \displaystyle\int_0^1 x(x)^2 dx + [2x(x)+1]\, dx + \int_1^0 (1)y^2(0) + [2(1)y+1]\, dy + \int_1^0 (0)y^2(0) + [2(0)y+1]\, dy$

$$= \left[\frac{1}{4}x^4 + \frac{2}{3}x^2 + x \right]_0^1 + \left[y^2 + y \right]_1^0 + y \Big|_1^0 = -\frac{13}{12}$$

(d) $= \int_0^1 (0)y^2(0) + [2(0)y+1]dy + \int_0^1 x(1)^2 dx + [2x(1)+1](0) + \int_1^0 x(x)^2 dx + [2x(x)+1]dx$

$$= y \Big|_0^1 + \frac{1}{3}x^3 \Big|_0^1 + \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 + x \right]_1^0 = -\frac{7}{12}$$

8.5 $\vec{r}(t) = \sin t \vec{i} + \cos t \vec{j} + t \vec{k}, \ (0 \leq t \leq 2\pi)$

$x = \sin t, \quad y = \cos t, \quad z = t$

$\dfrac{dx}{dt} = \cos t, \quad \dfrac{dy}{dt} = -\sin t, \quad \dfrac{dz}{dt} = 1$

$\displaystyle\int_c x dx + y dy + z dz = \int_0^{2\pi} [\sin t \cos t + \cos t(-\sin t) + t] dt = \frac{1}{2} t^2 \Big|_0^{2\pi} = 2\pi^2$

8.6 $W = \displaystyle\int_c P dx + Q dy = \int_0^\pi [(-\sin t)\cos t + \cos t(-\sin t)] dt$

$$= \int_0^\pi (-2\sin t \cos t) dt = -\int_0^\pi \sin 2t \, dt = \frac{1}{2} \cos 2t \Big|_0^\pi = 0$$

8.7 $W = \displaystyle\int_c P dx + Q dy = \int_c (-2xy) dx + 3y dy = \int_{-1}^2 [-2x(0)dx + 3(0)(0)] + \int_0^3 [-2(2)y(0) + 3y dy]$

$$= \int_0^3 eydy = \frac{3}{2}y^2 \Big|_0^3 = \frac{27}{2}$$

8.8 $W = \displaystyle\int_c P dx + Q dy = \int_c (x-2y)dx + (2xy-y)dy$

$$= \int_{-2}^2 [x - 2(-2)]dx + [2x(-2) - (-2)](0) + \int_{-2}^2 (2-2y)(0) + [2(2)y-y]dy$$

$$+ \int_2^{-2} [x - 2(2)]dx + [2x(2) - (2)](0) + \int_2^{-2} [(-2)-2y](0) + [2(-2)y-y]dy$$

$$= \int_{-2}^2 (x+4)dx + \int_{-2}^2 3y dy + \int_2^{-2} (x-4)dx + \int_2^{-2} (-5y)dy$$

$$= \frac{1}{2}x^2 + 4x \Big|_{-2}^2 + \frac{3}{2}y^2 \Big|_{-2}^2 + \frac{1}{2}x^2 - 4x \Big|_2^{-2} - \frac{5}{2}y^2 \Big|_2^{-2}$$

$$= 2 + 8 - 2 + 8 + 6 - 6 + 2 + 8 - 2 + 8 - \frac{5}{2}(4-4) = 32$$

8.9　$W = \int_c P dx + \theta dy$

$$= \int_c (x-2y)dx + (2xy-y)dy$$

$$= \int_0^{2\pi}]2\cos t - 2(2\sin t)](-2\sin t)dt + [2(2\cos t)(2\sin t) - (2\sin t)](2\cos t dt)$$

$$= \int_0^{2\pi} [-2\sin 2t + 8\sin^2 t + 8\cos^2 t \sin t - 2\sin 2t]dt$$

$$= \int_0^{2\pi} [-4\sin 2t + 8\sin^2 t + 8\cos^2 t \sin t]dt$$

$$= \left[\frac{1}{8}\cos 2t + 7t - 2\sin 2t - \frac{8}{3}cos^3 t \right]_0^{2\pi} = 8\pi$$

8.10　$W = \int_c P dx + Q dy = \int_0^{2\pi} 2(-2\sin t dt) + 3(-2\cos t)dt = 4\cos t \Big|_0^{2\pi} - 6\sin t \Big|_0^{2\pi} = 0$

8.11　$W = \int_c P dx + Q dy + R dz = \int_1^2 (3t)(t)(3dt) + (t)(-2t)dt + (3t)(-2t)(-2)dt$

$$= \int_1^2 (9t^2 - 2t^2 + 12t^2)dt = \int_1^2 19t^2 dt = \frac{19}{3}t^3 \Big|_1^2 = \frac{133}{3}$$

8.12　(a)　$= \int_0^2 [x(x^2)-2]dx + x^2(2x)dx = \int_0^2 (3x^3 - 2x)dx = \left[\frac{3}{4}x^4 - x^2 \right]_0^2 = 8$

(b)　$= \int_0^4 [(0)y-2](0) + (0)^2 dy + \int_0^4 [x(4)-2]dx + x^2(0) = [2x^2 - 2x]_0^2 = 4$

선적분 값은 경로에 따라 다르다.

8.13　(a)　$= \int_{-1}^1 [2x(-1)+1]dx + (x^2-1)(0) + \int_{-1}^1 [2(1)y+1](0) + [(1)^2-1]dy = \left[-x^2 + x\right]_{-1}^1 = 2$

(b)　$= \int_{-1}^1 [2x(x)+1]dx + [x^2-1]dx = \left[\frac{2}{3}x^3 + x + \frac{1}{3}x^3 - x \right]_{-1}^1 = [x^3]_{-1}^1 = 2$

(c) $= \int_{-1}^{1} [2(-1)y+1](0) + [(-1)^2-1]dy + \int_{-1}^{1} [2x(1)+1]dx + (x^2-1)(0)] = [x^2+x]_{-1}^{1} = 2$

8.14 $\int_{c} (2xy+1)dx + (x^2-1)dy = \int_{a}^{b} d\phi = [x^2y+x-y+C]_{(2,1)}^{(-2,-1)}$

$= (-2)^2(-1)+(-1)-(-2)+C - [2^2(1)+2-1+C] = -8$

8.15 $\int_{c} (y-z)dz + (x+y)dy + (-x+z)dz = \int_{a}^{b} d\phi = \left[xy - xz + \frac{1}{2}y^2 + \frac{1}{2}z^2 + c \right]_{(-1,0,-1)}^{(2,1,1)}$

$= \left[2(1) - 2(1) + \frac{1}{2}(1)^2 \frac{1}{2}(1)^2 + c - \left\{ (-1)(0) - (-1)(-1) + \frac{1}{2}(0)^2 + \frac{1}{2}(-1)^2 + C \right\} \right]$

$= \frac{3}{2}$

8.16 (a) $\vec{F} = (x+y)\vec{i} + (x-y)\vec{j}$

$curl\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (x+y) & (x-y) & 0 \end{vmatrix} = \dfrac{\partial}{\partial x}(x-y)\vec{k} - \dfrac{\partial}{\partial y}(x+y)\vec{k} = \vec{0}$ ∴ 퍼텐셜함수 존재

$\dfrac{\partial \phi}{\partial x} = x+y, \ \dfrac{\partial \phi}{\partial y} = x-y$

$\phi = \dfrac{1}{2}x^2 + yx + c(y)$

$\dfrac{\partial \phi}{\partial y} = x + \dfrac{\partial c}{\partial y} = x-y, \ \dfrac{\partial c(y)}{\partial y} = -y, \ c(y) = -\dfrac{1}{2}y^2 + c$

$\phi = \dfrac{1}{2}x^2 + yx - \dfrac{1}{2}y^2 + c$

(b) $\vec{F} = (x^2 + 2xy^2 + 1)\vec{i} + (2x^2y - 2y - 3)\vec{j}$ 퍼텐셜함수 존재

$\dfrac{\partial \phi}{\partial x} = x^2 + 2xy + 1, \ \dfrac{\partial \phi}{\partial y} = 2x^2y - 2y - 3$

$\therefore \phi(x,y) = \dfrac{1}{3}x^2 + x^2y^2 + x - y^2 - 3y + c$

(c) $\vec{F} = (2xy\cos y + x - y)\vec{i} + (-x^2\sin y - x - 1)\vec{j}$ 퍼텐셜함수 존재

$\dfrac{\partial \phi}{\partial x} = 2x\cos y + x - y, \dfrac{\partial \phi}{\partial y} = -x^2\sin y - x - 1$

$$\phi = x^2\cos y + \frac{1}{2}x^2 - yx + c(y)$$

$$\frac{\partial \phi}{\partial y} = -x^2\sin y - x + c'(y) = -x^2\sin y - x - 1, \ c'(y) = -1, \ c(y) = -y + c$$

$$\therefore \phi = x^2\cos y + \frac{1}{2}x^2 - yx - y + c$$

(d) $\overrightarrow{F} = (2xy+z)\vec{i} + (x^2+z)\vec{j} + (x+y)\vec{k}$

$$curl\overrightarrow{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (2xy+z) & (x^2+z) & (x+y) \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(x+y) - \frac{\partial}{\partial z}(x^2+z)\right]\vec{i} + \left[\frac{\partial}{\partial z}(2xy+z) - \frac{\partial}{\partial x}(x+y)\right]\vec{j} + \left[\frac{\partial}{\partial x}(x^2+z) - \frac{\partial}{\partial y}(2xy+z)\right]\vec{k}$$

$$= \vec{0} \quad \therefore \text{퍼텐셜함수 존재}$$

$$\frac{\partial \phi}{\partial x} = 2xy + z, \ \frac{\partial \phi}{\partial y} = x^2 + z, \ \frac{\partial \phi}{\partial z} = x + y$$

$$\phi = x^2y + zx + c(y,z), \ \frac{\partial \phi}{\partial y} = x^2 + c'(y,z) = x^2 + z$$

$$c'(y,z) = z, \ c(y,z) = \frac{1}{2}z^2 + c(z)$$

$$\phi = x^2y + xz + \frac{1}{2}z^2 + c(z)$$

$$\frac{\partial \phi}{\partial z} = x + z + c'(z) = x + y, \ c'(z) = y - z, \ c(z) = yz - \frac{1}{2}z^2 + c$$

$$\therefore \phi = x^2y + xz + \frac{1}{2}z^2 + yz - \frac{1}{2}z^2 + c = x^2y + xz + yz + c$$

8.17 $\displaystyle W = \int_c Pdx + Qdy = \int_a^b d\phi = \left[-\frac{1}{2}x^2 + \frac{1}{2}xe^{-y} + x + \frac{1}{2}y^2 - y + c\right]_{(-1,-1)}^{(1,1)}$

$$= \left[-\frac{1}{2}(1)^2 + \frac{1}{2}(1)e^{-1} + 1 + \frac{1}{2}(1)^2 - 1 + c - \left\{-\frac{1}{2}(-1)^2 + \frac{1}{2}(-1)e^1 + (-1) + \frac{1}{2}(-1)^2 - (-1) + c\right\}\right]$$

$$= \frac{1}{2}(e + e^{-1})$$

8.18 $\displaystyle \iint_R f(x,y)dA = \int_b^a\left[\int_{g(x)}^{h(x)} f(x,y)dy\right]dx = \int_0^1\left[\int_{x^2}^x (2x-y)dy\right]dx = \int_0^1\left(\left[2xy - \frac{1}{2}y^2\right]_{x^2}^x\right)dx$

$$= \int_0^1\left(2x^2 - \frac{1}{2}x^2 - 2x^3 + \frac{1}{2}x^4\right)dx = \left[\frac{3}{2}x^3 - \frac{1}{2}x^4 + \frac{1}{10}x^5\right]_0^1 = \frac{11}{10}$$

8.19 $\displaystyle\iint_R f(x,y)dA = \int_a^b\left[\int_{p(y)}^{q(y)}f(x,y)ds\right]dy = \int_0^1\left[\int_y^{\sqrt{y}}(2x-y)dx\right]dy$

$\displaystyle= \int_0^1\left(\left[x^2-xy\right]_y^{\sqrt{y}}\right)dy = \int_0^1(y-y\sqrt{y}-y^2+y^2)dy$

$\displaystyle= \left[\frac{1}{2}y^2+\frac{2}{5}y^{\frac{5}{2}}\right]_0^1 = \frac{9}{10}$

8.20 $\displaystyle\iint_R (x+y^2)dxdy = \int_0^\pi\int_0^1[\{r\cos\theta+(r\sin\theta^2)r\}dr]d\theta$

$\displaystyle= \int_0^\pi\left(\left[\frac{1}{3}r^3\cos\theta+\frac{1}{4}r^4\sin^2\theta\right]_0^1\right)d\theta = \int_0^\pi\left(\frac{1}{3}\cos\theta+\frac{1}{4}\sin^2\theta\right)d\theta$

$\displaystyle= \int_0^\pi\left(\frac{1}{3}\cos\theta+\frac{1}{4}\sin^2\theta\right)d\theta = \int_0^\pi\left(\frac{1}{3}\cos\theta+\frac{1}{4}\frac{1-\cos2\theta}{2}\right)d\theta$

$\displaystyle= \left[\frac{1}{3}\sin\theta+\frac{1}{8}\theta-\frac{1}{16}\sin2\theta\right]_0^\pi = \frac{\pi}{8}$

8.21 $\displaystyle\iint_R dA = \pi r^2 = \pi$

8.22 $\displaystyle\oint_R (2x^2-4y)dx+(x+e^y)dy = 10\pi$

8.23 $\displaystyle\iint_s 2xds = \int_0^2\int_0^{\sqrt{2}}2x\sqrt{1+4x^2}\,dxdy = \frac{1}{6}\int_0^2\left[(1+4x^2)^{\frac{3}{2}}\right]_0^{\sqrt{2}}dy = \frac{26}{6}\int_0^2 dy = \frac{26}{3}$

8.24 $\displaystyle\oint_c \vec{F}\cdot d\vec{r} = \iint_S curl\vec{F}\cdot\vec{n}dS = \iint_S(-\frac{1}{\sqrt{3}})dS$

$\displaystyle= \iint_R(-\frac{1}{\sqrt{3}})\sqrt{1+(-1)^2+(-1)^2}\,dA = -\iint_R dA = -\frac{1}{2}$

여기서 $\displaystyle\iint_R dA$은 아래 그림의 빗금친 부분의 면적
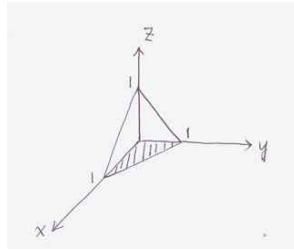
8.25 $\displaystyle\iint_S \vec{F} \cdot \vec{n} \, dS = \iiint_T 3 \, dv = 3\left(\frac{4}{3}\pi(2)^3\right) = 32\pi$

Chapter 09 연습문제 답안

9.1 (a) $\left.\begin{array}{l} -1 = a+2 \\ 3 = b-1 \end{array}\right\} \rightarrow a=3,\ b=4$

(b) $\left.\begin{array}{l} 4 = 2a \\ a-b = b \end{array}\right\} \rightarrow a=2,\ b=1$

9.2 (a) $2\vec{A}-4\vec{B}=2\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} -4\begin{bmatrix} 0 & 4 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 4-0 & -2-16 & 0-8 \\ -4+4 & 6-12 & 4+8 \\ 2-8 & 4+8 & -2-12 \end{bmatrix} = \begin{bmatrix} 4 & -18 & -8 \\ 0 & -6 & 12 \\ -6 & 12 & -14 \end{bmatrix}$

(b) $-2\vec{A}-\vec{B}=-2\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} -\begin{bmatrix} 0 & 4 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -2-0 & 2-4 & 0-2 \\ 4+1 & -6-3 & -4+2 \\ -2-2 & -4+2 & 2-3 \end{bmatrix} = \begin{bmatrix} -2 & -2 & -2 \\ 5 & -9 & -2 \\ -4 & -2 & -1 \end{bmatrix}$

(c) $2(\vec{A}+2\vec{B}) = 2\vec{A}+4\vec{B}$

$= 2\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} +4\begin{bmatrix} 0 & 4 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 14 & 8 \\ -8 & 18 & -4 \\ 10 & -4 & 10 \end{bmatrix}$

(d) $2\vec{A}+\vec{B}-\vec{C}= 2\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} +\begin{bmatrix} 0 & 4 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} -\begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & -2 \\ 1 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ -4 & 7 & 4 \\ 3 & 5 & 1 \end{bmatrix}$

(e) $-2\vec{A}-(\vec{B}+2\vec{C})=-2\vec{A}-\vec{B}-2\vec{C}$

$=-2\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} -\begin{bmatrix} 0 & 4 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} -2\begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & -2 \\ 1 & -3 & 0 \end{bmatrix} = \begin{bmatrix} -10 & 0 & -6 \\ 7 & -13 & 2 \\ -6 & 4 & -1 \end{bmatrix}$

(f) $-2(\vec{A}-\vec{B})-\vec{C}=-2\vec{A}+\vec{B}-\vec{C}$

$=-2\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} +2\begin{bmatrix} 0 & 4 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} -\begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & -2 \\ 1 & -3 & 0 \end{bmatrix} = \begin{bmatrix} -7 & 11 & 2 \\ 3 & -2 & -6 \\ 1 & 5 & 8 \end{bmatrix}$

(g) $-\vec{A}+\vec{B}^T+\vec{C}=-\begin{bmatrix} 2 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} +\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} +\begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & -2 \\ 1 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 4 \\ 7 & 2 & -6 \\ 2 & -7 & 4 \end{bmatrix}$

(h) $(2\vec{A}-\vec{B})^T-\vec{C}= 2\vec{A}^T-\vec{B}^T-\vec{C}$

$$= 2\begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & 2 \\ 0 & 2 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & -2 \\ 1 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -2 \\ -5 & 1 & 8 \\ -3 & 9 & -5 \end{bmatrix}$$

(i) $(\vec{A} + \vec{B} + \vec{C})^T = \vec{A}^T + \vec{B}^T + \vec{C}^T$

$$= \begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & 2 \\ 0 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 1 \\ -1 & 2 & -3 \\ 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -4 & 4 \\ 2 & 8 & -3 \\ 4 & -2 & 2 \end{bmatrix}$$

9.3 (a) $\vec{A}\vec{B} = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 11 \\ -3 & -17 \end{bmatrix}$

(b) $\vec{B}\vec{A} = \begin{bmatrix} 0 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -8 & 12 \\ 4 & -8 \end{bmatrix}$

(c) $\vec{C}(\vec{A}\vec{B}) = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 11 \\ -3 & -17 \end{bmatrix} = \begin{bmatrix} 6 & 50 \\ -7 & -45 \end{bmatrix}$

(d) $(\vec{A}\vec{B})\vec{C} = \begin{bmatrix} 1 & 11 \\ -3 & -17 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -8 & 21 \\ 8 & -31 \end{bmatrix}$

(e) $\vec{A}(\vec{B}\vec{C})$

$\vec{B}\vec{C} = \begin{bmatrix} 0 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ 0 & -5 \end{bmatrix}$

$\vec{A}(\vec{B}\vec{C}) = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -4 & 8 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} -8 & 21 \\ 8 & -31 \end{bmatrix}$

(f) $(\vec{C}\vec{A})\vec{B}$

$\vec{C}\vec{A} = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ -6 & 7 \end{bmatrix}$

$(\vec{C}\vec{A})\vec{B} = \begin{bmatrix} 8 & -6 \\ -6 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 50 \\ -7 & -45 \end{bmatrix}$

(g) $\vec{A}^T\vec{B} = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ -3 & -13 \end{bmatrix}$

(h) $\vec{A}^T\vec{B}^T\vec{C}^T = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ 12 & -8 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -28 & 16 \\ 44 & -28 \end{bmatrix}$

9.4 (a) 행렬 계수는 3이다.
   (b) 행렬계수는 2이다
   (c) 행렬계수는 1이다.

9.5 (a) $\vec{A} = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$, $\det\vec{A} = \begin{vmatrix} 1 & -3 \\ -2 & 2 \end{vmatrix} = 4$

(b) $\vec{B} = \begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 3 \\ -1 & -4 & -2 \end{bmatrix}$, $\det\vec{B} = \begin{vmatrix} 1 & 4 & 2 \\ 1 & 2 & 3 \\ -1 & -4 & -2 \end{vmatrix} = -4-8-12+4+8+12 = 0$

(c) $\vec{C} = \begin{bmatrix} 1 & -1 & 2 & 0 \\ -2 & 1 & -2 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$,

행렬식을 구하기 위해 3행에 대해 여인수 전개를 하면 편리하다.

$\det\vec{C} = a_{31}c_{31} + 1_{32}c_{32} + a_{33}c_{33} + a_{34}c_{34}$

$= (1)(-1)^{3+1} \begin{vmatrix} -1 & 2 & 0 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} + (1)(-1)^{3+3} \begin{vmatrix} 1 & -1 & 0 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 1-2 = -1$

9.6 $\vec{A} = \begin{vmatrix} 1 & 2 & -3 \\ -4 & 0 & 3 \\ 5 & -2 & 4 \end{vmatrix}$, $\vec{B} = \begin{bmatrix} 5 & -2 & 4 \\ -4 & 0 & 3 \\ 1 & 2 & -3 \end{bmatrix}$

$\det\vec{A} = 44$, $\det\vec{B} = -44$ $\therefore det\vec{A} = -\det\vec{B}$

9.7 => (한빛아카데미 판) 본문 문제 수정됨

$\vec{A} = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$, $\vec{B} = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 0 & 3 \\ 2 & -2 & 4 \end{bmatrix}$

$\det\vec{A} = 12$, $\det\vec{B} = 24$

$\therefore det\vec{B} = 2\det\vec{A}$

9.8 $\vec{A} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$, $\vec{B} = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 0 & 3 \\ 2 & -2 & 4 \end{bmatrix}$

$\det(\vec{A}\vec{B}) = -144$, $(\det\vec{A})(\det\vec{B}) = -144$

$\therefore det(\vec{A}\vec{B}) = \det(\vec{A})\det(\vec{B})$

9.9 $\vec{A} = \begin{bmatrix} 1 & -1 & -2 \\ 3 & 2 & 4 \\ -2 & -3 & -1 \end{bmatrix}$, $\vec{A}^T = \begin{bmatrix} -1 & 3 & -2 \\ -1 & 2 & -3 \\ -2 & 4 & -1 \end{bmatrix}$

$$\det \overrightarrow{A}^T = \det \overrightarrow{A} = 25$$

9.10 $\overrightarrow{A} = \begin{bmatrix} 3 & 0 & 0 \\ -1 & -2 & 0 \\ 2 & 3 & -1 \end{bmatrix}, \quad \det \overrightarrow{A} = (3)(-2)(-1) = 6$

9.11 $\overrightarrow{A} = \begin{bmatrix} -2 & 5 \\ -1 & -4 \end{bmatrix}, \quad \overrightarrow{B} = \begin{bmatrix} -1 & 2 & -1 \\ 2 & 0 & 3 \\ 1 & -3 & -2 \end{bmatrix}$

$\overrightarrow{A}^{-1} = \dfrac{1}{13}\begin{bmatrix} -4 & -5 \\ 1 & -2 \end{bmatrix}, \quad \overrightarrow{B}^{-1} = \dfrac{1}{11}\begin{bmatrix} 9 & 7 & 6 \\ 7 & 3 & 1 \\ -6 & -1 & -4 \end{bmatrix}$

9.12 $\det \overrightarrow{A} = 0, \quad \therefore$ 행렬 $\overrightarrow{A}$는 특이행렬

Chapter 10 연습문제 답안

10.1 (a) $\begin{bmatrix} -1 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$

10.2 $-x + y - 2z = 1$
$-2y + 4z = 2$
$-x - y + 2z = 3$

10.3 (a) $\begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2.2 \\ 1.3 \end{bmatrix}$, $x_1 = 2.2 , x_2 = 1.3$

(b) $\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$, $x_1 = 5, x_2 = 7$

(c) $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$,

그러나 행렬 $\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$ 은 역행렬이 존재하지 않으므로 해가 존재하지 않음.

(d) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & -3 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & -3 \\ 2 & -2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{4}{15} & \frac{7}{15} \\ \frac{1}{3} & -\frac{1}{15} & -\frac{2}{15} \\ 0 & -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{7}{15} \\ -\frac{2}{15} \\ -\frac{4}{5} \end{bmatrix}$,

$x_1 = \frac{7}{15}, x_2 = -\frac{2}{15}, x_3 = -\frac{4}{5}$

(e) $\begin{bmatrix} 3 & 1 & -1 \\ -1 & 3 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ -1 & 3 & -1 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \\ \frac{1}{6} & -\frac{1}{3} & -\frac{5}{6} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 2 \end{bmatrix}, x_1 = \frac{1}{2}, x_2 = \frac{1}{2}, x_3 = 2$$

(f) $\begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ -3 & -4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$, $\det \begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ -3 & -4 & 2 \end{bmatrix} = 0$ 이므로 $\begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ -3 & -4 & 2 \end{bmatrix}$ 은 역행렬이

존재하지 않음 ∴ 해가 존재하지 않는다.

10.4 (a) $x_1 = \dfrac{\det \begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix}}{\det \begin{bmatrix} 1 & -2 \\ 2 & -2 \end{bmatrix}} = -2$, $x_2 = \dfrac{\det \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}}{\det \begin{bmatrix} 1 & -2 \\ 2 & -2 \end{bmatrix}} = -\dfrac{5}{2}$

(b) $x_1 = \dfrac{\det \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}}{\det \begin{bmatrix} -2 & 1 \\ 2 & 2 \end{bmatrix}} = -\dfrac{1}{6}$, $x_2 = \dfrac{\det \begin{bmatrix} -2 & 2 \\ 2 & 3 \end{bmatrix}}{\det \begin{bmatrix} -2 & 1 \\ 2 & 2 \end{bmatrix}} = \dfrac{5}{3}$

(c) $x_1 = \dfrac{\det \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix}}{\det \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}} = -\dfrac{14}{3}$, $x_2 = \dfrac{\det \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix}}{\det \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}} = -\dfrac{5}{3}$

(d) $x_1 = \dfrac{\det \begin{bmatrix} 2 & 2 & -2 \\ 4 & -2 & -1 \\ 2 & -2 & -3 \end{bmatrix}}{\det \begin{bmatrix} -1 & 2 & -2 \\ 2 & -2 & -1 \\ 1 & -2 & -3 \end{bmatrix}} = \dfrac{18}{5}$, $x_2 = \dfrac{\det \begin{bmatrix} -1 & 2 & -2 \\ 2 & 4 & -1 \\ 1 & 2 & -3 \end{bmatrix}}{\det \begin{bmatrix} -1 & 2 & -2 \\ 2 & -2 & -1 \\ 1 & -2 & -3 \end{bmatrix}} = 2$, $x_3 = \dfrac{\det \begin{bmatrix} -1 & 2 & 2 \\ 2 & -2 & 4 \\ 1 & -2 & 2 \end{bmatrix}}{\det \begin{bmatrix} -1 & 2 & -2 \\ 2 & -2 & -1 \\ 1 & -2 & -3 \end{bmatrix}} = -\dfrac{4}{5}$

(e) $x_1 = \dfrac{\det \begin{bmatrix} 0 & 2 & -1 \\ 2 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}}{\det \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}} = -\dfrac{2}{3}$, $x_2 = \dfrac{\det \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{bmatrix}}{\det \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}} = \dfrac{3}{4}$, $x_3 = \dfrac{\det \begin{bmatrix} 2 & 2 & 0 \\ -1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}}{\det \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}} = \dfrac{4}{3}$

(f) $x_1 = \dfrac{\det \begin{bmatrix} 2 & -2 & 1 \\ 4 & -2 & 1 \\ 3 & -4 & 3 \end{bmatrix}}{\det \begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ -1 & -4 & 3 \end{bmatrix}} = -2$, $x_2 = \dfrac{\det \begin{bmatrix} -1 & 2 & 1 \\ -2 & 4 & 1 \\ -1 & 3 & 3 \end{bmatrix}}{\det \begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ -1 & -4 & 3 \end{bmatrix}} = \dfrac{1}{2}$, $x_3 = \dfrac{\det \begin{bmatrix} -1 & -2 & 2 \\ -2 & -2 & 4 \\ -1 & -4 & 3 \end{bmatrix}}{\det \begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 1 \\ -1 & -4 & 3 \end{bmatrix}} = 1$

10.5 (a) $x_2 = c$(상수)로 놓으면 $x_1 = 4c$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} c$

(b) $x_1 = x_2 = 0$ (자명한 해)

(c) $x_1 = x_2 = 0$ (자명한 해)

(d) $\begin{matrix} x_1 - \dfrac{7}{3}x_3 = 0 \\ x_2 - \dfrac{5}{3}x_3 = 0 \end{matrix}$, $x_3 = 3c$로 놓으면 $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix} c$

(e) $x_2 = 0$이면, $x_1 = -x_3$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} c$

(f) $x_1 = x_2 = x_3 = 0$ (자명한 해)

10.6 (a) $x_1 = -\dfrac{1}{11}$, $x_2 = -\dfrac{3}{11}$

(b) $x_1 = \dfrac{1}{4}$, $x_2 = -\dfrac{1}{2}$

(c) 해가 존재하지 않음

(d) $x_1 = \dfrac{3}{4}$, $x_2 = -1$, $x_3 = \dfrac{1}{4}$

(e) $x_1 = -\dfrac{7}{5} - \dfrac{4}{5}c$, $x_2 = -\dfrac{4}{5} - \dfrac{3}{5}c$

(f) 해가 존재하지 않음

10.7 (a) $\left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ -1 & 1 & -2 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ -1 & 1 & -2 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1 + R_3]{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 & 0 \\ 0 & 4 & -2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{2R_2}$

$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -1 & 1 & 2 & 0 \\ 0 & 4 & -2 & -1 & 0 & 1 \end{array}\right] \xrightarrow[-4R_2 + R_3]{\frac{1}{2}R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -5 & -8 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 2 & 0 \\ 0 & 0 & 1 & -\frac{5}{2} & -4 & \frac{1}{2} \end{array}\right] \xrightarrow[R_3 + R_2]{-R_3 + R_1}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{array}{ccc} \dfrac{7}{2} & 5 & -\dfrac{1}{2} \\[2mm] -\dfrac{3}{2} & -2 & \dfrac{1}{2} \\[2mm] -\dfrac{5}{2} & -4 & \dfrac{1}{2} \end{array} \right.$$

(b) $\begin{bmatrix} 2 & -1 & 3 & 0 \\ -1 & 1 & -2 & 1 \\ 2 & 0 & 1 & -1 \\ 1 & -1 & 0 & -2 \end{bmatrix} \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right.$, 풀이 (a)와 같이 기본행연사을 수행하여 위 확대행렬의

좌측부분을 단위행렬로 바꾸면,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left| \begin{array}{cccc} \dfrac{3}{5} & 1 & \dfrac{1}{5} & \dfrac{2}{5} \\[2mm] -1 & -1 & 1 & -1 \\[2mm] -\dfrac{2}{5} & -1 & \dfrac{1}{5} & -\dfrac{3}{5} \\[2mm] \dfrac{4}{5} & 1 & -\dfrac{2}{5} & \dfrac{1}{5} \end{array} \right.$$

10.8 (a) $R_1 i_1 + R_2 i_2 = 5 - ①$, $R_1 i_1 + R_2 i_2 = 5$, $(2R_1 + R_2)i_1 = 10$, $i_1 = \dfrac{10}{2R_1 + R_2}$,
$R_2 i_2 - R_2 i_2 = 0 - ②$ $R_2 i_1 - 2R_2 i_2 = 0$
$i_1 = i_2 + i_3 \quad - ③$
$i_2 = \dfrac{5}{2R_1 + R_2}$, $i_3 = \dfrac{5}{2R_1 + R_2}$

(b) $R_2 i_2 - (R_2 + R_3)i_3 = 0$, $3i_2 - 5i_3 = 0$, $R_1 = 5\Omega$,
$R_1 i_1 + R_2 i_2 = 10$ $\quad\quad$ $5i_1 + 3i_2 = 10$
$i_1 = i_2 + i_3$ $\quad\quad$ $i_1 - i_2 - i_3 = 0$

$$\begin{bmatrix} 0 & 3 & -5 \\ 5 & 3 & 0 \\ 1 & -1 & -1 \end{bmatrix} \left| \begin{array}{c} 0 \\ 10 \\ 0 \end{array} \right. \xrightarrow{R_{13}} \begin{bmatrix} 1 & -1 & -1 \\ 5 & 3 & 0 \\ 0 & 3 & -5 \end{bmatrix} \left| \begin{array}{c} 0 \\ 10 \\ 0 \end{array} \right. \xrightarrow{-5R_1 + R_2} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 8 & 5 \\ 0 & 3 & -5 \end{bmatrix} \left| \begin{array}{c} 0 \\ 10 \\ 0 \end{array} \right.$$

$$\xrightarrow{\frac{1}{8}R_2} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 5/8 \\ 0 & 3 & -5 \end{bmatrix} \left| \begin{array}{c} 0 \\ 5/4 \\ 0 \end{array} \right. \xrightarrow[-3R_2 + R_3]{R_2 + R_1} \begin{bmatrix} 1 & -1 & -3/8 \\ 0 & 1 & 5/8 \\ 0 & 0 & -55/8 \end{bmatrix} \left| \begin{array}{c} 5/4 \\ 5/4 \\ -15/4 \end{array} \right.,$$

$$\xrightarrow{-8/55R_3} \begin{bmatrix} 1 & 0 & -3/8 \\ 0 & 1 & 5/8 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{array}{c} 5/4 \\ 5/4 \\ 6/11 \end{array} \right. \xrightarrow[-5/8R_3 + R_2]{3/8R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{array}{c} 16/11 \\ 10/11 \\ 6/11 \end{array} \right.$$

$\therefore i_1 = 16/11\,A, i_2 = 10/11\,A, i_3 = 6/11 A$

10.9 (a) $\lambda_1 = -1$인경우, $\overrightarrow{v_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} c$

$$\lambda_2 = 1 \text{인 경우}, \quad \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} c$$

(b) $\lambda_1 = -1 \text{인 경우}, \quad \overrightarrow{v_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} c$

$\lambda_2 = 5 \text{인 경우}, \quad \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} c$

(c) $\lambda_1 = 0 \text{인 경우}, \quad \overrightarrow{v_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} c$

$\lambda_2 = -5 \text{인 경우}, \quad \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} c$

(d) $\lambda_1 = \lambda_2 = 4 \text{인 경우}, \quad \overrightarrow{v_1} = \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} c$

(e) $\lambda_1 = \lambda_2 = 3 \text{인 경우}, \quad \overrightarrow{v_1} = \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} c$

(f) $\lambda_1 = \lambda_2 = -1 \text{인 경우}, \quad \begin{bmatrix} 5 & 5 \\ -5 & -5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \overrightarrow{v_1} = \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} c$

(g) $\lambda_1 = -1 + 2i \text{인 경우}, \quad \overrightarrow{v_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} i \\ 2 \end{bmatrix} c$

$\lambda_2 = -1 - 2i \text{인 경우}, \quad \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_i = \begin{bmatrix} -i \\ 2 \end{bmatrix} c$

(h) $\lambda_1 = 4 + i \text{인 경우}, \quad \begin{bmatrix} 2-4-i & -1 \\ 5 & 6-4+i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \overrightarrow{v_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -2-i \end{bmatrix} c$

$\lambda_2 = 4 - i \text{인 경우}, \quad \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_i = \begin{bmatrix} 1 \\ -2+i \end{bmatrix} c$

(i) $\lambda_1 = 2i \text{인 경우}, \quad \begin{bmatrix} 2-2i & 2 \\ -4 & -2-2i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \overrightarrow{v_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -1+i \end{bmatrix} c$

$\lambda_2 = -2i \text{인 경우}, \quad \overrightarrow{v_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_i = \begin{bmatrix} 1 \\ -1-i \end{bmatrix} c$

(j) $\lambda_1 = 1 \text{인 경우}, \quad \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} c$

$$\lambda_2 = \sqrt{2}\,\text{인경우.}\ \begin{bmatrix} 1-\sqrt{2} & 1 & -1 \\ 0 & 1-\sqrt{2} & 1 \\ 0 & 1 & -1-\sqrt{2} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} \sqrt{2} \\ 1 \\ \sqrt{2}-1 \end{bmatrix}c$$

$$\lambda_3 = -\sqrt{2}\,\text{인경우,}\ \begin{bmatrix} 1+\sqrt{2} & 1 & -1 \\ 0 & 1+\sqrt{2} & 1 \\ 0 & 1 & -1+\sqrt{2} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} -\sqrt{2} \\ 1 \\ -\sqrt{2}-1 \end{bmatrix}c$$

(k) $\lambda_1 = 1\,$인경우, $\begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix}c$

$\lambda_2 = -1\,$인경우, $\begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}c$

$\lambda_3 = 3\,$인경우, $\begin{bmatrix} -2 & 1 & 4 \\ 0 & -2 & 0 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}c$

(l) $\lambda_1 = 1\,$인경우, $\begin{bmatrix} -2 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}c$

$\lambda_2 = 2\,$인경우, $\begin{bmatrix} -3 & 1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}c$

$\lambda_3 = -2\,$인경우, $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}c$

(m) $\lambda_1 = 1\,$인경우, $\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}c$

$\lambda_2 = \lambda_3 = 2\,$인경우, $\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

$$\overrightarrow{V_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\ ,\ \overrightarrow{V_3} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(n) $\lambda_1 = 7\,$인경우, $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}c$

$\lambda_2 = \lambda_3 = 4\,$인경우, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

$$\overrightarrow{V_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} c \ , \ \overrightarrow{V_3} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} c$$

(o) $\lambda_1 = 1$인경우, $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 2 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} c$

$\lambda_2 = \lambda_3 = 2$인경우, $\begin{bmatrix} -1 & 1 & -1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} c \ , \ \overrightarrow{V_3} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} c$

(p) $\lambda_1 = \lambda_2 = \lambda_3 = 1$인경우, $\begin{bmatrix} 2 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \overrightarrow{V_2} = \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} c$

(q) $\lambda_1 = \lambda_2 = \lambda_3 = -2$인경우, $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \overrightarrow{V_2} = \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} c$

(r) $\lambda_1 = \lambda_2 = \lambda_3 = 2$인경우, $\begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \overrightarrow{V_2} = \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} c$

(s) $\lambda_1 = -3$인경우, $\begin{bmatrix} 3 & 3 & 6 \\ 1 & 5 & 2 \\ 0 & -4 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} c$

$\lambda_2 = 1 + 2i$인경우, $\begin{bmatrix} -1-2i & 3 & 6 \\ 1 & 2-1-2i & 2 \\ 0 & -4 & -3-1-2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} -3i \\ -2-i \\ 2 \end{bmatrix} c$$

$\lambda_3 = 1 - 2i$인경우, $\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 3i \\ -2+i \\ 2 \end{bmatrix} c$

(t) $\lambda_1 = 1$인경우, $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} c$

$\lambda_2 = 1 + i$인경우, $\begin{bmatrix} 1-1-i & -1 & 0 \\ -1 & 1-1-i & 2 \\ 0 & -1 & 1-1-i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ -i \\ 1 \end{bmatrix} c$

$\lambda_3 = 1-i$인경우, $\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 1 \\ i \\ 1 \end{bmatrix} c$

(u) $\lambda_1 = -2$인경우, $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} c$

$\lambda_2 = 2i$인경우, $\begin{bmatrix} -2-2i & 0 & -1 \\ 0 & -2-2i & -1 \\ 4 & 4 & 2-2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ 1 \\ -2-2i \end{bmatrix} c$

$\lambda_3 = -2i$인경우, $\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 1 \\ 1 \\ -2+2i \end{bmatrix} c$

10.10 (a) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\overrightarrow{D} = \begin{bmatrix} 4+i & 0 \\ 0 & 4-i \end{bmatrix}$

(c) 대각화 불가능

(d) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(e) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \begin{bmatrix} 1+i & 0 & 0 \\ 0 & 1-i & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(f) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(g) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(h) 대각화 불가능

(i) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \begin{bmatrix} 1+2i & 0 & 0 \\ 0 & 1-2i & 0 \\ 0 & 0 & -3 \end{bmatrix}$

10.11 (a) $\overrightarrow{D} = \overrightarrow{p^{-1}} \overrightarrow{A} \overrightarrow{p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(b) $\vec{D} = \vec{p}^{-1}\vec{A}\vec{p} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $\vec{D} = \vec{p}^{-1}\vec{A}\vec{p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

10.12 (a) $\vec{A}^2 - 4\vec{A} - 5\vec{I} = \vec{0}$

$\vec{A}^2 = 4\vec{A} + 5\vec{I} = 4\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 16 & 9 \end{bmatrix}$

$\vec{A}^3 = 4\vec{A}^2 + 5\vec{A} = 4\begin{bmatrix} 17 & 8 \\ 16 & 9 \end{bmatrix} + 5\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 83 & 42 \\ 84 & 41 \end{bmatrix}$

$\vec{A}^4 = 4\vec{A}^3 + 5\vec{A}^2 = 4\begin{bmatrix} 83 & 42 \\ 84 & 41 \end{bmatrix} + 5\begin{bmatrix} 17 & 8 \\ 16 & 9 \end{bmatrix} = \begin{bmatrix} 417 & 208 \\ 416 & 209 \end{bmatrix}$

(b) $\vec{A}^3 - 3\vec{A}^2 - \vec{A} + 3\vec{I} = \vec{0}$

$\vec{A}^3 = 3\vec{A}^2 - \vec{A} - 3\vec{I} = 3\begin{bmatrix} 5 & 6 & 8 \\ 0 & 1 & 0 \\ 2 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 19 & 28 \\ 0 & 1 & 0 \\ 9 & 10 & 13 \end{bmatrix}$

$\vec{A}^4 = 3\vec{A}^3 - \vec{A}^2 + 5\vec{I} = 3\begin{bmatrix} 13 & 19 & 28 \\ 0 & 1 & 0 \\ 7 & 10 & 13 \end{bmatrix} + \begin{bmatrix} 5 & 6 & 8 \\ 0 & 1 & 0 \\ 2 & 3 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 41 & 60 & 80 \\ 0 & 1 & 0 \\ 20 & 30 & 41 \end{bmatrix}$

11.1 (a) $\dfrac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(b) $\dfrac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(c) $\dfrac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(d) $\dfrac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(e) $\dfrac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ -1 & -1 & -1 \\ -2 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(f) $\dfrac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & -1 \\ -3 & -2 & -1 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

11.2 (a) $\dfrac{dx}{dt} = -x + 2y, \quad \dfrac{dy}{dt} = -2x - y$

(b) $\dfrac{dx}{dt} = x - 2y, \quad \dfrac{dy}{dt} = 3x - 2y$

(c) $\dfrac{dx}{dt} = -x + 2y + z, \quad \dfrac{dy}{dt} = x - 2y + 2z, \quad \dfrac{dz}{dt} = 3y - z$

(d) $\dfrac{dx}{dt} = -x + 2y, \quad \dfrac{dy}{dt} = 2x - y - 2z, \quad \dfrac{dz}{dt} = -x + 3y + 3z$

11.3 (a) $\dfrac{d\overrightarrow{X}}{dt} = \dfrac{d}{dx}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, 즉, $\overrightarrow{X}' = \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}\overrightarrow{X}$

$\overrightarrow{X} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{6t}$를 위 식에 대입

좌변 : $\overrightarrow{X}' = \begin{bmatrix} 6 \\ 12 \end{bmatrix}e^{6t}$

우변 : $\begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{6t} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}e^{6t}$

(b) $\dfrac{d\vec{X}}{dt} = \dfrac{d}{dx}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, 즉, $\vec{X}' = \begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}\vec{X}$

$\vec{X} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{3t}$를 위 식에 대입

좌변 : $\vec{X}' = 3\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{3t}$

우변 : $\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{3t} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}e^{3t}$, 좌변 = 우변 $\therefore \begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{3t}$가 해 벡터임

(c) $\dfrac{d\vec{X}}{dt} = \dfrac{d}{dx}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, 즉, $\vec{X}' = \begin{bmatrix} 5 & -2 \\ 0 & 5 \end{bmatrix}\vec{X}$

$\vec{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{5t}$를 위 식에 대입

좌변 : $\vec{X}' = \begin{bmatrix} 5 \\ 0 \end{bmatrix}e^{5t}$

우변 : $\begin{bmatrix} 5 & -2 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{5t} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}e^{5t}$, 좌변 = 우변 $\therefore \begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{5t}$가 해 벡터임

(d) $\vec{X}' = \begin{bmatrix} -2 & -1 \\ 2 & 0 \end{bmatrix}\vec{X}$

$\vec{X} = \begin{bmatrix} \frac{1}{2}\cos t - \frac{1}{2}\sin t \\ \sin t \end{bmatrix}e^{-t}$를 위 식에 대입 => (한빛아카데미 판) 본문 문제 수정됨

좌변 : $\vec{X}' = \begin{bmatrix} -\frac{1}{2}\cos t + \frac{1}{2}\sin t \\ -\sin t \end{bmatrix}e^{-t} + \begin{bmatrix} -\frac{1}{2}\sin t - \frac{1}{2}\cos t \\ \cos t \end{bmatrix}e^{-t} = \begin{bmatrix} -\cos t \\ \cos t - \sin t \end{bmatrix}e^{-t}$

우변 : $\begin{bmatrix} -2 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -\frac{1}{2}\cos t - \frac{1}{2}\sin t \\ \sin t \end{bmatrix}e^{-t} = \begin{bmatrix} -\cos t + \sin t - \sin t \\ \cos t - \sin t \end{bmatrix}e^{-t} = \begin{bmatrix} -\cos t \\ \cos t - \sin t \end{bmatrix}e^{-t}$,

좌변 = 우변 : $\begin{bmatrix} \frac{1}{2}\cos t - \frac{1}{2}\sin t \\ \sin t \end{bmatrix}e^{-t}$ 가 해 벡터임

11.4 (a) $x = c_1 e^t - c_2 e^{-t}, y = -3c_1 e^t + c_2 e^{-t}$

(b) $x = -2c_1 + c_2 e^{-5t}, y = c_1 + 3c_2 e^{-5t}$

(c) $\vec{V_2} = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}, \therefore \vec{X_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}te^{4t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}e^{4t}$

$\vec{X} = c_1\vec{X_1} + c_2\vec{X_2} = c_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{4t} + c_2\begin{bmatrix} 1 \\ 0 \end{bmatrix}te^{4t} + c_2\begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}e^{4t}$

$x = c_1 e^{4t} + c_2 te^{4t}$

$y = -\dfrac{1}{2}c_2 e^{4t}$

11.5 (a) $\dfrac{d}{dt}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix},\ \ \ \vec{W} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} c_1 e^t \\ c_2 e^{-t} \\ c_3 e^{3t} \end{bmatrix}$

$\vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{P}\vec{W} = \begin{bmatrix} 4 & -2 & 2 \\ -4 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} c_1 e^t \\ c_2 e^{-t} \\ c_e e^{3t} \end{bmatrix} = \begin{bmatrix} 4c_1 e^t - 2c_2 e^{-t} + 2c_3 e^{3t} \\ -4c_1 e^t \\ c_1 e^t + c_2 e^{-t} + c_3 e^{3t} \end{bmatrix}$

(b) $\dfrac{d}{dt}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix},\ \ \ \vec{W} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} c_1 e^{5t} \\ c_2 e^{5t} \\ c_3 e^{8t} \end{bmatrix}$

$\vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{P}\vec{W} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} c_1 e^{5t} \\ c_2 e^{5t} \\ c_e e^{8t} \end{bmatrix} = \begin{bmatrix} -c_1 e^{5t} - c_2 e^{5t} + c_3 e^{8t} \\ c_1 e^{5t} + c_3 e^{8t} \\ c_2 e^{5t} + c_3 e^{8t} \end{bmatrix}$

11.6 (a) $\vec{X} = \begin{bmatrix} x \\ y \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{6t} + \begin{bmatrix} -\dfrac{13}{18} \\ \dfrac{8}{9} \end{bmatrix}$

(b) $\vec{X} = \vec{X_c} + \vec{X_P} = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t} + \begin{bmatrix} \dfrac{2}{5} - \dfrac{2}{9} e^{2t} \\ -\dfrac{8}{5} + \dfrac{1}{9} e^{2t} \end{bmatrix}$

(c) $\vec{X} = \vec{X_c} + \vec{X_P} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^t + \begin{bmatrix} 2t-3 \\ -3t+4 \end{bmatrix}$

(d) $\vec{X} = \vec{X_c} + \vec{X_P} = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} -\dfrac{1}{26}\cos t + \dfrac{31}{26}\sin t + \dfrac{4}{5}t + \dfrac{4}{5} \\ \dfrac{3}{26}\cos t - \dfrac{15}{26}\sin t - \dfrac{2}{5}t \end{bmatrix}$

(e) $\vec{X} = \vec{X_c} + \vec{X_P} = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} t e^{5t} + \begin{bmatrix} 0 \\ -\dfrac{1}{2} \end{bmatrix} e^{5t} \right\} + \begin{bmatrix} -\dfrac{3}{5} + \dfrac{1}{18} e^{-t} \\ \dfrac{1}{6} e^{-t} \end{bmatrix}$

(f) $\vec{X} = \vec{X_c} + \vec{X_P} = c_1 \begin{bmatrix} \cos t \\ \sin t - 2\cos t \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} \sin t \\ -2\sin t - \cos t \end{bmatrix} e^{4t} + \begin{bmatrix} -\dfrac{16}{17} \\ \dfrac{19}{17} \end{bmatrix}$

11.7 (a) $\vec{\Phi}(t) = \begin{bmatrix} -e^{3t} & e^{6t} \\ e^{3t} & 2e^{6t} \end{bmatrix}$

$\vec{\Phi}^{-1}(t) = -\dfrac{1}{3e^{9t}}\begin{bmatrix} 2e^{6t} & -e^{6t} \\ -e^{3t} & -e^{3t} \end{bmatrix},\ \ \vec{U}(t) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$$\overrightarrow{X_P} = \overrightarrow{\Phi}(t) \int \overrightarrow{\Phi}^{-1}(t) \overrightarrow{U}(t)\,dt$$

$$= \frac{1}{3}\begin{bmatrix} -e^{3t} & e^{6t} \\ e^{3t} & 2e^{6t} \end{bmatrix} \int \begin{bmatrix} -2e^{-3t} & e^{-3t} \\ e^{-6t} & e^{-6t} \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix}dt$$

$$= \frac{1}{3}\begin{bmatrix} -e^{3t} & e^{6t} \\ e^{3t} & 2e^{6t} \end{bmatrix} \int \begin{bmatrix} -7e^{-3t} \\ -e^{-6t} \end{bmatrix}dt = \frac{1}{3}\begin{bmatrix} -e^{3t} & e^{6t} \\ e^{3t} & 2e^{6t} \end{bmatrix}\begin{bmatrix} \frac{7}{3}e^{-3t} \\ \frac{1}{6}e^{-6t} \end{bmatrix} = \begin{bmatrix} -\frac{13}{18} \\ \frac{8}{9} \end{bmatrix},$$

문제 11.6의 (a)결과와 동일

(b) $\overrightarrow{\Phi}(t) = \begin{bmatrix} e^{-t} & e^{5t} \\ -2e^{-t} & e^{5t} \end{bmatrix}$

$\overrightarrow{\Phi}^{-1}(t) = \frac{1}{3}e^{-4t}\begin{bmatrix} e^{5t} & -e^{5t} \\ 2e^{-t} & e^{-t} \end{bmatrix}, \overrightarrow{U}(t) = \begin{bmatrix} 2 \\ e^{2t} \end{bmatrix}$

$\overrightarrow{X_P} = \overrightarrow{\Phi}(t) \int \overrightarrow{\Phi}^{-1}(t) \overrightarrow{U}(t)\,dt$

$$= \frac{1}{3}\begin{bmatrix} e^{-t} & e^{5t} \\ -2e^{-t} & e^{5t} \end{bmatrix} \int \begin{bmatrix} e^{t} & -e^{t} \\ 2e^{-5t} & e^{-5t} \end{bmatrix}\begin{bmatrix} 2 \\ e^{2t} \end{bmatrix}dt$$

$$= \frac{1}{3}\begin{bmatrix} e^{-t} & e^{5t} \\ -2e^{-t} & e^{5t} \end{bmatrix} \int \begin{bmatrix} 2e^{t}-e^{3t} \\ 4e^{-5t}+e^{-3t} \end{bmatrix}dt = \frac{1}{3}\begin{bmatrix} e^{-t} & e^{5t} \\ -2e^{-t} & e^{5t} \end{bmatrix}\begin{bmatrix} 2e^{t}-\frac{1}{3}e^{3t} \\ -\frac{4}{5}e^{-5t}-\frac{1}{3}e^{-3t} \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2-\frac{1}{3}e^{2t}-\frac{4}{5}-\frac{1}{3}e^{2t} \\ -4+\frac{2}{3}e^{2t}-\frac{4}{5}-\frac{1}{3}e^{2t} \end{bmatrix} = \begin{bmatrix} \frac{2}{5}-\frac{2}{9}e^{2t} \\ -\frac{8}{5}+\frac{1}{9}e^{2t} \end{bmatrix}, 문제 11.6의 (b)의 결과와 동일$$

(c) ~ (f) 이 문제들도 위와 같은 방법으로 풀면 문제 11.6에서 구한 것과 같은 결과들을 얻을 수 있으며, 그 풀이과정은 유사하므로 생략한다.

11.8 (a) $\overrightarrow{X}' = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\overrightarrow{X} + \begin{bmatrix} 2 \\ 1 \\ t \end{bmatrix}$

$\overrightarrow{X_c} = c_1\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}e^{3t} + c_2\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}e^{-t} + c_3\begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix}e^{t}$

$\overrightarrow{X_P}$는 문제 11.6의 미정계수법 또는 11.7의 매개변수 변환법에 의해 구하면

$$\overrightarrow{X_P} = \begin{bmatrix} \frac{23}{9}-\frac{4}{3}t \\ 0 \\ -\frac{11}{9}+\frac{1}{3}t \end{bmatrix}, \therefore \overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P}$$

(b) $\quad \overrightarrow{X}' = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 6 & 1 \\ 1 & 1 & 6 \end{bmatrix} \overrightarrow{X} + \begin{bmatrix} e^{3t} \\ 2 \\ -e^{3t} \end{bmatrix}$

$\quad \overrightarrow{X_c} = c_1 \begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ \dfrac{3}{2} \\ -\dfrac{5}{2} \end{bmatrix} e^{5t} + c_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} e^{7t}$

$\quad \overrightarrow{X_P} = \begin{bmatrix} \dfrac{2}{5} + \dfrac{1}{4}e^{3t} \\ -\dfrac{2}{5} - \dfrac{3}{16}e^{3t} \\ \dfrac{5}{16}e^{3t} \end{bmatrix}, \; \therefore \overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P}$

11.9 (a) $x = c_1 e^{3t} + c_2 e^{-t} - \dfrac{2}{3} + \dfrac{2}{5}e^{4t}$

$\quad y = c_1 e^{3t} - c_2 e^{-t} + \dfrac{4}{3} + \dfrac{3}{5}e^{4t}$

$\quad z = c_3 e^{2t} - \dfrac{1}{4} - \dfrac{1}{2}t$

(b) $x = -c_1 e^{-2t} - 2c_2 e^{-2t} + c_3 e^t + \dfrac{3}{10}e^t + 2$

$\quad y = c_2 e^{-2t} + c_3 e^t + \dfrac{1}{10}e^{3t}$

$\quad z = c_1 e^{-2t} + c_2 e^{-2t} + c_3 e^t + \dfrac{1}{10}e^{3t} + 1$

11.10 (a) 키르히호프 전압법칙과 전류법칙에 의해

$\quad \begin{cases} L_1 \dfrac{di_1}{dt} + R_1 i_2 = e_1(t) \\ L_2 \dfrac{di_3}{dt} + R_2 i_3 - R_1 i_2 = 0 \\ i_1 = i_2 + i_3 \end{cases}$

값들을 대입하고 i_2를 소거하면

$\quad \begin{matrix} \dfrac{di_1}{dt} = 10 \\ \dfrac{di_3}{dt} = -i_3 \end{matrix} \Bigg\} \begin{matrix} i_1 = 10t + c_1 \\ i_3 = c_2 e^{-t} \\ i_2 = i_1 - i_3 \text{에 의해 구함} \end{matrix}$

위 미분방정식을 풀면 된다.

=> (한빛아카데미 판) 본문 문제 수정됨

(b) $\begin{cases} L_1\dfrac{di_1}{dt} + R_2 i_3 + R_1 i_1 = e_2(t) \\ L_2\dfrac{di_2}{dt} - R_2 i_3 = 0 \\ i_1 = i_2 + i_3 \end{cases}$

$\left.\begin{array}{l} \dfrac{di_1}{dt} = -i_1 + i_2 + 10 \\ \dfrac{di_2}{dt} = i_1 - i_2 \end{array}\right)$

위 미분방정식을 풀면 된다.

$i_1 = -\dfrac{1}{2}c_1 e^{-2t} + c_2 + 5t$

$i_2 = \dfrac{1}{2}c_1 e^{-2t} + c_2 + 5t - 5$

$i_3 = i_1 - i_2$

Chapter 12 연습문제 답안

12.1 $\langle f,g\rangle = \displaystyle\int_0^{\frac{\pi}{2}} \cos x \cos 3x\, dx$

$\displaystyle = \frac{1}{2}\int_0^{\frac{\pi}{2}}(\cos 2x\, 4\cos 4x)dx$

$\displaystyle = \frac{1}{2}\left[\frac{\sin 2x}{2}+\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$

$= 0$

$\therefore \cos x$ 와 $\cos 3x$ 는 구간 $\left[0,\dfrac{\pi}{2}\right]$ 에서 직교한다.

12.2 $f_0(x)=1, f_1(x)=\sin x, f_2(x)=\sin 2x$ 는 구간 $\left[0,\dfrac{\pi}{2}\right]$ 에서 서로 직교한다.

각 함수들의 크기는

$\displaystyle \|f_0(x)\| = \sqrt{\int_0^{\frac{\pi}{2}}f_0^2(x)dx} = \sqrt{\int_0^{\frac{\pi}{2}}(1)dx} = \sqrt{\frac{\pi}{2}}$

$\displaystyle \|f_1(x)\| = \sqrt{\int_0^{\frac{\pi}{2}}f_1^2(x)dx} = \sqrt{\int_0^{\frac{\pi}{2}}\sin^2 x\, dx} = \sqrt{\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}dx}$

$\displaystyle = \sqrt{\frac{x-\frac{1}{2}\sin 2x}{2}}\Big|_0^{\frac{\pi}{2}} = \sqrt{\frac{\pi}{4}} = \frac{\sqrt{\pi}}{2}$

$\displaystyle \|f_2(x)\| = \sqrt{\int_0^{\frac{\pi}{2}}f_2^2(x)dx} = \sqrt{\int_0^{\frac{\pi}{2}}\sin^2 2x\, dx} = \sqrt{\int_0^{\frac{\pi}{2}}\frac{1-\cos 4x}{2}dx} = \sqrt{\frac{\pi}{4}} = \frac{\sqrt{\pi}}{2}$

$\displaystyle \therefore \frac{f_0(x)}{\sqrt{\frac{\pi}{2}}} = \frac{1}{\sqrt{\frac{\pi}{2}}} = \frac{\sqrt{2}}{\sqrt{\pi}}, \quad \frac{f_1(x)}{\frac{\sqrt{\pi}}{2}} = \frac{2\sin x}{\sqrt{\pi}}, \quad \frac{f_2(x)}{\frac{\sqrt{\pi}}{2}} = \frac{2\sin 2x}{\sqrt{\pi}}$ 는

서로 정규 직교 함수이다.

12.3 $a_0 = \dfrac{1}{T}\displaystyle\int_{-T}^{T}f(x)dx = \dfrac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx = \dfrac{1}{\pi}\int_{-\pi}^{0}(-2)dx + \dfrac{1}{\pi}\int_{0}^{\pi}(2)dx = 0$

$$a_n = \frac{1}{T} \int_{-T}^{T} f(x)\cos\frac{n\pi}{T}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \int_{-\pi}^{0}(-2)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{-2\sin nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{2\sin nx}{n}\right]_{0}^{\pi} = 0$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\frac{n\pi}{T}x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-2)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{2\cos nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{-2\cos nx}{n}\right]_{0}^{\pi} = \frac{2-2\cos n\pi + 2\cos n\pi + 2}{n\pi} = \frac{4(1-\cos n\pi)}{n\pi}$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{4(1-\cos n\pi)}{n\pi}\sin nx$$

12.4
$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)\,dx = \frac{1}{2}\int_{-2}^{2} x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_{-2}^{2} x\,dx = 0$$

$$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\frac{n\pi}{T}x\,dx = \frac{1}{2}\left[\frac{2x}{n\pi}\sin\frac{n\pi}{2}x + \frac{2^2}{(n\pi)^2}\cos\frac{n\pi}{2}x\right]_{-2}^{2} = 0$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\frac{n\pi}{T}x\,dx = \frac{1}{2}\int_{-2}^{2} x\sin\frac{n\pi}{2}x\,dx$$

$$= \frac{1}{2}\left[\frac{-2x}{n\pi}\cos\frac{n\pi}{2}x + \frac{2^2}{(n\pi)^2}\sin\frac{n\pi}{2}x\right]_{-2}^{2} = -\frac{2}{n\pi}\cos n\pi$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{-2}{n\pi}\cos n\pi \sin\frac{n\pi}{2}x$$

12.5
$$a_0 = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx = \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}}\cos x\,dx = \frac{2}{\pi}$$

$$a_n = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\cos 2nx\,dx = \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}}\cos x\cos 2nx\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\frac{\pi}{2}}\left[\cos(2n-1)x + \cos(2n+1)\right]dx$$

$$= \frac{2(-1)^{n+1}}{\pi(4n^2-1)}$$

$$b_n = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)\sin 2nx\,dx = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\cos x\,\sin 2nx\,dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\sin(2n-1)x + \sin(2n+1)x\right]dx$$

$$= \frac{4n}{\pi(4n^2-1)}$$

$$\therefore f(x) = \frac{1}{\pi} + \sum_{n=1}^{\infty}\left[\frac{2(-1)^{n+1}}{\pi(4n^2-1)}\cos 2nx + \frac{4n}{\pi(4n^2-1)}\sin 2nx\right]$$

12.6 $$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)dx + \frac{1}{\pi}\int_0^{\pi}(-2x+\pi)dx$$

$$= \frac{1}{\pi}\left[\frac{1}{2}x^2 + \pi x\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-x^2 + \pi x\right]_0^{\pi}$$

$$= -\frac{1}{\pi}\left(\frac{1}{2}\pi^2 - \pi^2\right) + \frac{1}{\pi}(-\pi^2 + \pi^2) = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\cos nx\,dx + \frac{1}{\pi}\int_0^{\pi}(-2x+\pi)\cos nx\,dx$$

$$= \frac{3(1-\cos n\pi)}{\pi n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\sin nx\,dx + \frac{1}{\pi}\int_0^{\pi}(-2x+\pi)\sin nx\,dx$$

$$= \frac{1}{n}\cos n\pi$$

$$\therefore f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{3(1-\cos n\pi)}{\pi n^2}\cos nx + \frac{1}{n}\cos n\pi\sin nx\right]$$

12.7 (a) $$a_0 = \frac{2}{\pi}\int_0^{\pi}(-x+\pi)dx = \frac{2}{\pi}\left[-\frac{1}{2}x^2 + \pi x\right]_0^{\pi} = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}(-x+\pi)\cos nx\,dx = \frac{2}{\pi n^2}(1-\cos nx)$$

$$\therefore f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{\pi n^2}(1-\cos n\pi)\cos nx$$

(b) $$a_0 = \frac{2}{2}\int_0^{2}f(x)dx = \int_0^{1}(1)dx + \int_1^{2}(-1)dx = 0$$

$$a_n = \frac{2}{2}\int_0^{2}f(x)\cos\frac{n\pi}{2}x\,dx$$

$$= \int_0^{1}\cos\frac{n\pi}{2}x\,dx + \int_1^{2}(-1)\cos\frac{n\pi}{2}x\,dx = \frac{2}{n\pi}\sin\frac{n\pi}{2}x\Big|_0^{1} - \frac{2}{n\pi}\sin\frac{n\pi}{2}\Big|_1^{2} = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{4}{n\pi} sin\frac{n\pi}{2} cos\frac{n\pi}{2}x$$

(c) $a_0 = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} x dx = \pi$

$a_n = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} x cosnx dx = \dfrac{2}{\pi n^2}\left[(-1)^n - 1\right]$

$\therefore f(x) = \dfrac{\pi}{2} + \displaystyle\sum_{n=1}^{\infty} \dfrac{2}{\pi n^2}\left[(-1)^n - 1\right] cosnx$

12.8 (a) $b_n = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} x sinnx dx$ 부분함수법에 의해 적분하면

$b_n = \dfrac{2}{\pi}\left(-\dfrac{\pi}{n}cosn\pi\right) = -\dfrac{2}{n}cosn\pi = -\dfrac{2}{n}(-1)^n$

$\therefore f(x) = \displaystyle\sum_{n=1}^{\infty}\left[-\dfrac{2}{n}(-1)^n\right]sinnx = \displaystyle\sum_{n=1}^{\infty}\dfrac{2}{n}(-1)^{n+1}sin\,nx$

(b) $b_n = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} (-x)sinnx dx$

$= -\dfrac{2}{\pi}\left(-\dfrac{\pi}{n}cosn\pi\right) = \dfrac{2}{n}(-1)^n$

$\therefore f(x) = \displaystyle\sum_{n=1}^{\infty}\left[\dfrac{2}{n}(-1)^n\right]sinnx$

(c) $b_n = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} x^3 sinnx dx = \dfrac{2}{\pi}\left[-\dfrac{x^3}{n}cosnx\Big|_0^{\pi} + \dfrac{3}{n}\displaystyle\int_0^{\pi} x^2 cosnx dx\right]$

$= \dfrac{2\pi^2}{n}(-1)^{n+1} - \dfrac{12}{n^2\pi}\displaystyle\int_0^{\pi} x sinnx dx$

$= \dfrac{2\pi^2}{n}(-1)^{n+1} - \dfrac{12}{n^2\pi}\left[-\dfrac{x}{n}cosnx\Big|_0^{\pi} + \dfrac{1}{n}\displaystyle\int_0^{\pi} cosnx dx\right]$

$= \dfrac{2n^2}{n}(-1)^{n+1} + \dfrac{12}{n^3}(-1)^n$

$\therefore f(x) = \displaystyle\sum_{n=1}^{\infty}\left[\dfrac{2\pi^2}{n}(-1)^{n+1} + \dfrac{12}{n^3}(-1)^n\right]sinnx$

12.9 (a) $a_0 = 4\displaystyle\int_0^1 (1)dx = 4$

$$a_n = 4\int_0^1 (1)\cos\frac{n\pi}{2}xdx = \frac{8}{n\pi}sin\frac{n\pi}{2}$$

$$\therefore f(x) = 2 + \sum_{n=1}^{\infty}\frac{8}{n\pi}sin\frac{n\pi}{2}cos\frac{n\pi}{2}x$$

(b) $a_0 = 4\int_0^1 (1)dx = 4$

$$a_n = 4\int_1^2 (1)\cos\frac{n\pi}{2}xdx = -\frac{8}{n\pi}sin\frac{n\pi}{2}$$

$$\therefore f(x) = 2 + \sum_{n=1}^{\infty}\left(-\frac{8}{n\pi}sin\frac{n\pi}{2}\right)\cos\frac{n\pi}{2}x$$

12.10 (a) $b_n = 4\int_0^1 (1)\sin\frac{n\pi}{2}xdx = \frac{8}{n\pi}\left(1-\cos\frac{n\pi}{2}\right)$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{8}{n\pi}(1-\cos\frac{n\pi}{2})\sin\frac{n\pi}{2}x$$

(b) $b_n = 4\int_1^2 (1)\sin\frac{n\pi}{2}xdx = \frac{8}{n\pi}\left(\cos\frac{n\pi}{2}-\cos n\pi\right)$

$$\therefore f(x) = \sum_{n=1}^{\infty}\frac{8}{n\pi}\left(\cos\frac{n\pi}{2}-\cos n\pi\right)\sin\frac{n\pi}{2}x$$

12.11 (a) $c_n = \frac{1}{2T}\int_{-T}^{T}f(x)e^{-i\frac{n\pi x}{T}}dx = \frac{1}{2}\int_{-1}^{1}f(x)e^{-in\pi x}dx$

$$= \frac{1}{2}\int_{-1}^{0}(1)e^{-in\pi x}dx + \frac{1}{2}\int_{0}^{1}(-1)e^{-in\pi x}dx$$

$$= -\frac{1}{2in\pi}e^{-in\pi x}\Big|_{-1}^{0} + \frac{1}{2in\pi}e^{-in\pi x}\Big|_{0}^{1} = \frac{1}{2in\pi}\left[e^{in\pi}-1+e^{-in\pi}-1\right]$$

$$= \frac{i}{n\pi}(i-\cos n\pi)$$

$$\therefore f(x) = \sum_{n=-\infty}^{\infty}\frac{i}{n\pi}(1-\cos n\pi)e^{in\pi x}$$

(b) $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)e^{-inx}dx = \frac{1}{2\pi}\int_{-\pi}^{0}(0)e^{-inx}dx + \frac{1}{2\pi}\int_{0}^{\pi}(1)e^{-inx}dx$

$$= -\frac{1}{i2\pi n}e^{-inx}\Big|_0^{\pi} = \frac{i}{2\pi n}\left(e^{-in\pi}-1\right)$$

$$= \frac{i}{2\pi n}(\cos n\pi - i\sin n\pi - 1) = \frac{i}{2\pi n}(\cos n\pi - 1)$$

$$\therefore f(x) = \sum_{n=-\infty}^{\infty} \frac{i}{2\pi n}(\cos n\pi - 1)e^{inx}$$

12.12 $f(x) = \int_0^\infty \left[-\frac{\sin w}{\pi w}\cos wx + \frac{\cos w - 1}{\pi w}sinwx \right] dw$

12.13 (a) $f(x) = \int_0^\infty \frac{2\sin 2w}{\pi w}\cos wx\,dw$

(b) $f(x) = \int_0^\infty A(w)\cos wx\,dw$

12.14 (a) $f(x) = \int_0^\infty \frac{2}{\pi w}(1 - \cos 2w)\sin wx\,dw$

(b) $f(x) = \int_0^\infty B(w)\sin wx\,dw$

12.15 (a) $\hat{f}(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx}dx = \int_0^2 (c)e^{-iwx}dx = \frac{c}{-iw}e^{-iwx}\bigg|_0^2$

$$= \frac{c}{iw}(1 - e^{-2wi})$$

(b) $\hat{f}(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx}dx = \int_{-\infty}^0 e^{-x}e^{-iwx}dx = \int_{-\infty}^0 e^{-(1+iw)x}dx$

$$= \frac{1}{-(1+iw)}e^{-(1+iw)x}\bigg|_{-\infty}^0 = \infty$$

Chapter 13 연습문제 답안

13.1 (a) $\sqrt{3}+i$

(b) $\dfrac{\sqrt{2}}{2}+i\dfrac{\sqrt{6}}{2}$

(c) $2\sqrt{2}+i2\sqrt{2}$

(d) -3

13.2 (a) $(z)=\sqrt{5},\ \theta=\tan^{-1}2,\ z=|z|\angle\theta$

(b) $|z|=2,\theta=\dfrac{\pi}{2},z=2\angle\dfrac{\pi}{2}$

(c) $\dfrac{1-i}{1+i}=\dfrac{(1-i)^2}{(1+i)(1-i)}=-i,|z|=1,\theta=\dfrac{3\pi}{2}z=1\angle\dfrac{3}{2}\pi$

(d) $\dfrac{i}{2+2i}=\dfrac{i(2-2i)}{(2+2i)(2-2i)}=\dfrac{1+2i}{4}=\dfrac{1}{4}+\dfrac{1}{2}i$

$|z|=\dfrac{\sqrt{5}}{4},\theta=\tan^{-1}2,z=|z|\angle\theta$

13.3 (a) $z_1+3z_2=2+i+3(1-i)=5-2i$

(b) $(z_1+z_2)^2=(2+i+1-i)^2=9$

(c) $2z_1-3z_2=2(2+i)-3(1-i)=1+5i$

(d) $z_1z_2=(2+i)(1-i)=3-i$

(e) $\dfrac{z_1}{z_1+z_2}=\dfrac{2+i}{2+i+1-i}=\dfrac{2}{3}+\dfrac{1}{3}i$

(f) $\dfrac{z_1}{z_2^2}=\dfrac{2+i}{(1-i)^2}=\dfrac{2+i}{-2i}=\dfrac{-1+2i}{2}=-\dfrac{1}{2}+i$

13.4 (a) $z=\sqrt{3}+i=2(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6})$

$z^4=2^4(\cos4\cdot\dfrac{\pi}{6}+i\sin4\cdot\dfrac{\pi}{6})=16(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i)=-8+8\sqrt{3}i$

$$f'(z) = \frac{(4z+3)(z^2+1)-(sz^2+3z-1)2z}{(z^2+1)^2}$$

13.8 (a) 해석적

(b) 해석적

(c) 해석적이 아니다

(d) 해석적이 아니다

(e) $z=1$을 제외한 모든점에서 해석적

(f) (e)와 같은 방법으로 구하면 된다.

13.9 (a) $\ln z = \ln 2 + i(\pi \pm 2\pi n), n = 0,1,2,\ldots$

$\mathrm{Ln}\, z = \ln 2 + i\pi$

(b) $\ln z = 2\sqrt{2} + i(\frac{3}{4}\pi \pm 2\pi n), n = 0,1,2,\ldots$

$\mathrm{Ln}\, z = \ln 2\sqrt{2} + i\frac{3}{4}\pi$

(c) $\ln z = \ln 2 + i(\frac{2}{3}\pi \pm 2n\pi), n = 0,1,2,\ldots$

$\mathrm{Ln}\, z = \ln 2 + i\frac{2}{3}\pi$

(d) $\ln z = \ln\sqrt{2} + i(-\frac{\pi}{4} \pm 2n\pi), n = 0,1,2,\ldots$

$\mathrm{Ln}\, z = \ln\sqrt{2} - i\frac{\pi}{4}$

(e) $\ln z = \ln\sqrt{2} + i(\frac{\pi}{4} \pm 2n\pi), n = 0,1,2,\ldots$

$\mathrm{Ln}\, z = \ln\sqrt{2} + i\frac{\pi}{4}$

(f) $\ln z = \ln 2 + i(-\frac{\pi}{2} \pm 2n\pi), n = 0,1,2,\ldots$

$\mathrm{Ln}\, z = \ln 2 - i\frac{\pi}{2}$

13.10 (a) $z_1 + z_2 = 4 - 4i$

$$\ln(z_1 z_2) = \ln 4\sqrt{2} + i(-\frac{\pi}{4} \pm 2n\pi), n = 1, 2, 3, \dots$$

$$\mathrm{Ln}(z_1 z_2) = \ln 4\sqrt{2} - i\frac{\pi}{4}$$

(b) $\ln(z_1 z_2) = \ln 2\sqrt{2} + i(-\frac{3}{4}\pi \pm 2n\pi), n = 0, 1, 2, \dots$

$$\mathrm{Ln}(z_1 z_2) = \ln 2\sqrt{2} - i\frac{3}{4}\pi$$

13.11 (a) $\displaystyle\int_C (z^2 - 2z)dz, \quad C: z(t) = t + 2ti \; (0 \le t \le 2)$

$$\int_C (z^2 - 2z)dz = \int_0^2 [(t + 2ti)^2 - 2(t + 2ti)](1 + 2i)dt$$

$$= \int_0^2 (-3t^2 + 4t^2 i - 2t - 4ti)(1 + 2i)dt$$

$$= -\frac{52}{3} - \frac{64}{3}i$$

(b) $\displaystyle\int_C (2z^2)dz, \quad C: z(t) = t + t^2 i \; (0 \le t \le 1)$

$$\int_C (2z^2)dz = \int_0^1 [2(t + t^2 i)^2 (1 + 2ti)]dt$$

$$= \int_0^1 (2t^2 + 4t^3 i - 2t^4)(1 + 2ti)dt$$

$$= -\frac{4}{3} + \frac{4}{3}i$$

(c) $\displaystyle\int_C (2z^2 - 2z + 1)dz, \quad C: z(t) = t + ti \; (0 \le t \le 1)$

$$\int_C (2z^2 - 2z + 1)dz = \int_0^1 [2(t + ti)^2 - 2(t + ti) + 1](1 + i)dt$$

$$= \int_0^1 (2t^2 + 4t^2 i - 2t^2 - 2t - 2ti + 1)(1 + i)dt$$

$$= -\frac{1}{3} + \frac{1}{3}i$$

13.12 (a) $\displaystyle\int_C (z^2)dz = 0$

(b) $\displaystyle\int_C (2z^2 - 1)dz = 0$

13.13 (a) $f(z) = 3z - 1$, $F(z) = \dfrac{3}{2}z^2 - z$ (원시함수), 적분경로에 무관

$$\int_C (3z-1)dz = F(z_2) - F(z_1)$$

$$= \left[\frac{3}{2}z^2 - z\right]_0^{1+2i} = \frac{3}{2}(1+2i)^2 - (1+2i) = -\frac{11}{2} + 4i$$

(b) $f(z) = e^z$, $F(z) = e^z$ (원시함수), 적분경로에 무관

$$\int_C e^z dz = e^z \Big|_0^{1+2i} = e^{1+2i} - 1$$

13.14 $\displaystyle\int_C \frac{e^z}{z^2 - 2z - 8}dz = \int_C \frac{e^z}{(z+2)(z-4)}dz$

$f(z) = \dfrac{e^z}{z-4}$, $a = -2$

코시적분공식에 의해

$$\int_C \frac{e^z/(z-4)}{z+2}dz = \frac{e^{-2}}{-6}(2\pi i) = -e^{-2}\frac{\pi i}{3}$$

13.15 $\displaystyle\int_C \frac{\sin z}{z^3 - 3z^2 + 4}dz = 2\pi i\left(-\frac{1}{9}\sin 1 - \frac{1}{9}\sin 2 + \frac{1}{3}\cos 2\right)$

13.16 $\displaystyle e^{-1/z^2} = \sum_{n=0}^{\infty} \frac{\left(-\dfrac{1}{z^2}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\frac{1}{z^{2n}} = 1 - \frac{1}{z^2} + \frac{1}{2!}\frac{1}{z^4} - + \ldots$

$$= Res\left(e^{-\frac{1}{z^2}}, 0\right) = 0$$

13.17 $f(z) = \dfrac{3}{z-2} + \dfrac{1}{6} - \dfrac{1}{6^2}(z-2) + \dfrac{1}{6^3}(z-2)^2 - + \ldots,\ \ 0 < |z-2| < 6$

13.18 $\displaystyle\int_C \frac{\cos z}{z^3 + z^2}\, dz = 2\pi i(-1 + \cos 1)$