

MSE, 미적분학

[연습문제 답안 이용 안내]

- 본 연습문제 답안의 저작권은 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

Chapter 12 연습문제 답안

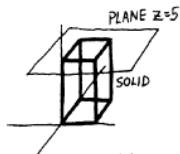
《Section 12.1》

1. 20π

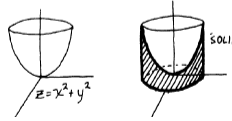
2. (a) $\int_R \sqrt{9-x^2-y^2} dA$

(b) $\int_R 5dA$

3. (a)



(b)



4. (a) 양수

(b) 0

(c) 0

(d) 음수

(e) 양수

5. 아니오

6. 0

7. (a) 참

(b) 거짓

(c) 거짓

(d) 참

(e) 참

8. 36π

9. 아니오

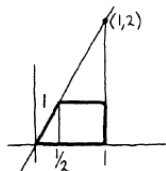
《Section 12.2》

1.
 - (a) $\int_{x=0}^{3/2} \int_{y=2x}^{y=3} x^3 dy = \int_0^3 \frac{1}{64} y^4 dy = \frac{1}{65} \times \frac{1}{5} y^5 \Big|_0^5 = 243/320$
 - (b) $5/4$
 - (c) 0

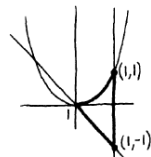
2.
 - (a) $\int_{x=0}^{x=4} \int_{y=x^2/8}^{y=\sqrt{x}} f(x,y) dy dx, \int_{y=0}^{y=2} \int_{x=y^2}^{x=\sqrt{8}y} f(x,y) dx dy$
 - (b) $\int_{x=0}^{x=\ln 2} \int_{y=e^x}^{y=2} f(x,y) dy dx, \int_{y=1}^{y=2} \int_{x=0}^{x=\ln y} f(x,y) dx dy$
 - (c) $\int_{x=0}^{x=2} \int_{y=x/3}^{y=2x} f(x,y) dy dx + \int_{x=2}^{x=3} \int_{y=x/3}^{y=-3x+10} f(x,y) dy dx$
 $\int_{y=1}^{y=4} \int_{x=y/2}^{x=(10-y)/3} f(x,y) dx dy + \int_{y=0}^{y=1} \int_{x=y/2}^{x=3y} f(x,y) dx dy$
 - (d) $\int_{x=2}^{x=7} \int_{y=0}^{y=5} f(x,y) dy dx, \int_{y=0}^{y=5} \int_{x=2}^{x=7} f(x,y) dx dy$
 - (e) $\int_{y=0}^{y=1} \int_{x=-2\sqrt{y}}^{x=2\sqrt{y}} f(x,y) dx dy + \int_{y=1}^{y=4} \int_{x=2y-4}^{x=2\sqrt{y}} f(x,y) dx dy$
 - (f) $\int_{y=0}^{y=1/3} \int_{x=2}^{x=3} f(x,y) dx dy + \int_{y=1/3}^{y=1/2} \int_{x=2}^{x=1/y} f(x,y) dx dy$
 - (g) $\int_{x=-\infty}^{x=\infty} \int_{y=x^2}^{y=\infty} f(x,y) dy dx, \int_{y=0}^{y=\infty} \int_{x=-\sqrt{y}}^{x=\sqrt{y}} f(x,y) dx dy$
 - (h) $\int_{y=-\infty}^{y=0} \int_{x=1/y}^{x=\infty} f(x,y) dx dy + \int_{y=0}^{y=\infty} \int_{x=-\infty}^{x=1/y} f(x,y) dx dy$
 $\int_{x=-\infty}^{x=0} \int_{y=1/x}^{y=\infty} f(x,y) dy dx + \int_{x=0}^{x=\infty} \int_{y=-\infty}^{y=1/x} f(x,y) dy dx$
 - (i) $\int_{x=0}^{x=2} \int_{y=0}^{y=x} f(x,y) dy dx + \int_{x=2}^{x=3} \int_{y=0}^{y=2} f(x,y) dy dx + \int_{x=3}^{x=5} \int_{y=x-3}^{y=2} f(x,y) dy dx$
 - (j) $\int_{x=-\sqrt{2}}^{x=\sqrt{2}} \int_{y=-\sqrt{4-2x^2}}^{y=\sqrt{4-2x^2}} f(x,y) dy dx, \int_{y=-2}^{y=2} \int_{x=-\sqrt{2-y^2/2}}^{x=\sqrt{2-y^2/2}} f(x,y) dx dy$

3.
 - (a) $\int_{x=-1}^{x=0} \int_{y=-x-1}^{y=x+1} f dy dx + \int_{x=0}^{x=1} \int_{y=x-1}^{y=-x+1} f dy dx$
 - (b) 아니오

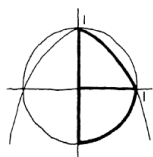
4. (a) $\int_{x=0}^{x=1/2} \int_{y=0}^{y=2x} f(x,y) dy dx + \int_{x=1/2}^{x=1} \int_{y=0}^{y=1} f(x,y) dy dx$



(b) $\int_{y=0}^{y=1} \int_{x=\sqrt{y}}^{x=1} f(x,y) dx dy + \int_{y=-1}^{y=0} \int_{x=-y}^{x=1} f(x,y) dx dy$



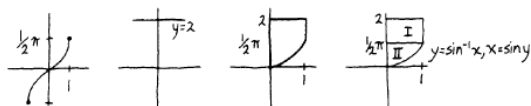
(c) $\int_{y=0}^{y=1} \int_{x=0}^{x=\sqrt{1-y}} f(x,y) dx dy + \int_{y=-1}^{y=0} \int_{x=0}^{x=\sqrt{1-y^2}} f(x,y) dx dy$



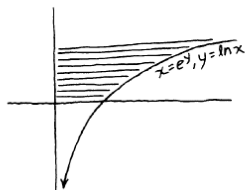
(d) $\int_{y=0}^{y=\infty} \int_{x=0}^{x=y/2} f(x,y) dx dy$



(e) $\int_{y=\pi/2}^{y=2} \int_{x=0}^{x=1} f(x,y) dx dy + \int_{y=-1}^{y=\pi/2} \int_{x=0}^{x=\sin y} f(x,y) dx dy$



(f) $\int_{x=0}^{x=1} \int_{y=0}^{y=\infty} f(x,y) dy dx + \int_{x=1}^{x=\infty} \int_{y=\ln x}^{y=\infty} f(x,y) dy dx$



5.
$$\int_{x=0}^{x=2y} e^{y^2} dx = 2ye^{y^2}$$
$$\int_{y=0}^{y=1} 2ye^{y^2} dy = e - 1$$

《Section 12.3》

1. (a) $\frac{128}{3}$

(b) 32

(c) $\pi \ln 17$

2.
$$\int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} e^{-r^2} r dr d\theta = \frac{1}{4} \pi$$

3.
$$\frac{1}{2} \pi (10 \ln 10 - 9)$$

4. (a)
$$\int_{\theta=0}^{2\pi} \int_{r=3}^4$$

(b)
$$\int_{\theta=0}^{2\pi} \int_{r=2}^{\infty}$$

(c)
$$\int_{\theta=0}^{\pi/2} \int_{r=0}^{2/(\cos\theta + \sin\theta)}$$

(d)
$$\int_{\theta=-\pi/2}^{\pi/2} \int_{r=0}^{4\cos\theta}$$

(e)
$$\int_{\theta=0}^{\pi} \int_{r=5}^{\infty}$$

(f)
$$\int_{\theta=0}^{\tan^{-1}(1/2)} \int_{r=0}^{6\sec\theta} + \int_{\theta=\tan^{-1}(1/2)}^{\pi/2} \int_{r=0}^{3\csc\theta}$$

《Section 12.4》

1. (a) $\int_{-1}^0 (\sqrt{x+1} - -\sqrt{x+1})dx + \int_0^3 (1-x - -\sqrt{x-1})dx = \frac{9}{2}$

(b) $\int_1^4 (5-x-4/x)dx$

(c) $\int_{-2}^2 (4-x^2)dx$

(d) $Inner = \frac{1}{2}\theta^2, outer = \frac{4}{3}\pi^3$

2. (a) $Inner = 12 - \frac{4}{3}\cos\theta - 8\sin\theta, outer = 24\pi$

(b) $\frac{4}{3}\pi R^3$

(c) $Inner = \frac{1}{3}(216 - 27^{3/2}), outer = 4\pi\frac{1}{3}(216 - 27^{3/2})$

(d) $\frac{1}{3}\pi R^2 h$

(e) $\pi R^2 h$

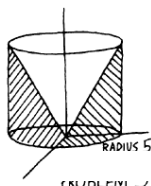
3. (a) $\int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{6}} (12-2r^2)rdrd\theta$

(b) $\int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{2}} (8-4r^2)rdrd\theta$

(c) $\int_{\theta=0}^{2\pi} \int_{r=0}^2 (\sqrt{25-r^2} - \sqrt{21})rdrd\theta$

4. 144

5. 부피 계산하는 식



《Section 12.5》

1. (a) $5/3$
(b) $625/6$

2. $500\ln 3$

3. $6^5\pi/5$

4. (a) $\int_0^{2\pi} \int_{r=0}^2 r^2 dr d\theta$
(b) $\int_1^5 \int_{y=0}^{-3(x-5)/4} \sqrt{x^2+y^2} dy dx$

5. (a) 단위면적당 인구 수
(b) 인구밀도. (2,3)에서의 인구밀도가 8이다.

6. $\int_{y=0}^4 \int_{x=y/2}^{x=8-3y/2} \frac{f(x,y)}{\sqrt{(x-8)^2+y^2}} dx dy$

7. $\int_0^5 \frac{5-x}{8-x} dx$

8. $\int_{x=0}^6 \int_{y=0}^{(6-x)/2} \frac{x+2}{7-y} dy dx$

9. (a) 6π
(b) $\int_{\theta=0}^{2\pi} \int_{r=0}^3 \frac{1}{\sqrt{25+r^2}} r dr d\theta \int_0^5 \frac{5-x}{8-x} dx$

10. $\frac{200}{3}(25^3 - 1000)$

《Section 12.6》

1. (a) $1/6$ (b) 50π

2. (a) $\int_{y=0}^3 \int_{z=0}^{(12-4y)/3} \int_{x=0}^{(12-4y-3z)/6} f(x, y, z) dx dz dy$

(b) $\int_{\theta=0}^{2\pi} \int_{r=0}^R \int_{z=-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$

(c) $\int_{z=0}^h \int_{y=0}^R \int_{x=-\sqrt{R^2z^2/h^2-y^2}}^{\sqrt{R^2z^2/h^2-y^2}} f(x, y, z) dx dy dz$

(d) $\int_{z=0}^h \int_{y=0}^R \int_{x=0}^{\sqrt{R^2-y^2}} f(x, y, z) dx dy dz$

(e) $\int_{\theta=0}^{\pi/2} \int_{r=0}^3 \int_{x=0}^{\sqrt{9-r^2 \sin^2 \theta}} f(x, r \cos \theta, r \sin \theta) r dx dr d\theta$

(f) $\int_{y=0}^k \int_{x=-R}^R \int_{z=\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} f(x, y, z) dz dx dy$

(g) $\int_{y=-\sqrt{2}}^{\sqrt{2}} \int_{z=y^2}^{4-y^2} \int_{x=-\sqrt{(z-y^2)/2}}^{\sqrt{(z-y^2)/2}} f(x, y, z) dx dz dy$

(h) $\int_{x=-\sqrt{5}}^{\sqrt{5}} \int_{z=x^2}^5 \int_{y=-2}^3 f(x, y, z) dy dz dx$

(i) $\int_{y=\sqrt{6}}^{\sqrt{6}} \int_{x=-\sqrt{12-2y^2}}^{\sqrt{12-2y^2}} \int_{z=-\sqrt{(12-x^2-2y^2)/3}}^{\sqrt{(12-x^2-2y^2)/3}} f(x, y, z) dz dx dy$

(j) $\int_{y=0}^2 \int_{x=0}^1 \int_{z=0}^x f(x, y, z) dz dx dy$

(k) $\int_{y=-1}^1 \int_{z=0}^{5-y} \int_{x=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y, z) dx dz dy$

(l) $\int_{\theta=0}^{\pi/2} \int_{r=0}^1 \int_{z=0}^{r \cos \theta} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$

(m) $\int_{z=H-h}^H \int_{y=-R_1z/H}^{R_1z/H} \int_{x=-\sqrt{R_1^2z^2/H^2-y^2}}^{\sqrt{R_1^2z^2/H^2-y^2}} f(x, y, z) dx dy dz$

3. (a) (i) $\frac{2}{3}\pi R^3 h$ (ii) $\frac{1}{2}\pi h^2 R^2$

(b) $\frac{1}{10}\pi h R^4 \delta$

(c) 7500π

(d) $\int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{6}} \int_{z=2r^2}^{12} r dz dr d\theta$

4. $\int_{z=0}^2 \int_{y=-\sqrt{(6+3z^2)/2}}^{\sqrt{(6+3z^2)/2}} \int_{x=-\sqrt{6+3z^2-2y^2}}^{\sqrt{6+3z^2-2y^2}} dx dy dz$

5. 아니오

《Section 12.7》

1. $\frac{4}{3}\pi R^3$

2. $\frac{1}{3}\pi R^2 h$

3. $\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/2} \int_{\rho=0}^R (1/\rho^2) \rho^2 \sin\phi d\rho d\phi d\theta = 4\pi R$

4. (a) ∞

(b) $\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/2} \int_{\rho=0}^{\infty}$

(c) $\int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} \int_{\rho=0}^{\infty}$

5. $18\pi(1 - \frac{\sqrt{5}}{3})$

6. $\delta \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\phi_0} \int_{\rho=0}^{h \sec\phi} (\rho \sin\phi)^2 \rho^2 \sin\phi d\rho d\phi d\theta$

7. $\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\phi_0} \int_{\rho=3/\cos\phi}^{6/\cos\phi} (1/\rho) \rho^2 \sin\phi d\rho d\phi d\theta$

《Section 12.8》

1. $\frac{3}{8}R$

2. (a) $\frac{3}{4}h$

(b) $\frac{4R}{3\pi}$

3. (a) $\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\phi_0} \int_{\rho=4\sqrt{2}\sec\phi}^6 \rho^2 \sin\phi d\rho d\phi d\theta$

(b) $\frac{1}{6} \int_{x=0}^3 \int_{y=0}^{4-4x/3} y dy dx$

4. (a)
$$\frac{\int_{\theta=0}^{\pi} \int_{r=0}^2 (r \sin\theta)(r^2) r dr d\theta}{\int_{\theta=0}^{\pi} \int_{r=0}^2 r^2 r dr d\theta}$$

(b) (i) $h - z$

(ii) r

《복습문제》

1. (a) $\int_{x=-\sqrt{2}}^{-1} \int_{y=-\sqrt{2-x^2}}^{\sqrt{2-x^2}} dy dx + \int_{x=-1}^1 \int_{y=-\sqrt{2-x^2}}^1 dy dx + \int_{x=1}^{\sqrt{2}} \int_{y=-\sqrt{2-x^2}}^{\sqrt{2-x^2}} dy dx$
 (b) $\int_{\theta=\pi/4}^{3\pi/4} \int_{r=0}^{\csc\theta} (r\cos\theta)^3 r \sin\theta r dr d\theta + \int_{\theta=3\pi/4}^{9\pi/4} \int_{r=0}^{\sqrt{2}} (r\cos\theta)^3 r \sin\theta r dr d\theta$
 (c) 0
2. 아니오
3. $\int_{x=-1}^2 \int_{y=x^2}^{y=x+2} dy dx$
4. π/e^9
5. $\int_{\theta=0}^{\pi/2} \int_{r=0}^2 (r\cos\theta)(r\sin\theta) r dr d\theta$
6. $\int_{x=0}^3 \int_{y=-4}^4 \int_{z=0}^{16-y^2} dz dy dx$
 $\int_{y=-4}^4 \int_{z=0}^{16-y^2} \int_{x=0}^3 dx dz dy$
 $\int_{x=0}^3 \int_{z=0}^{16} \int_{y=-\sqrt{16-z}}^{\sqrt{16-z}} dy dz dx$
7. $\int_{y=0}^2 \int_{z=4-2y}^{4-y^2} \int_{x=-\sqrt{4-y^2-z}}^{\sqrt{4-y^2-z}} dx dz dy$
8. 243π
9. (a) $\int_{\theta=0}^{2\pi} \int_{r=0}^2 (10-3r^2) r dr d\theta$
 (b) $\int_{\theta=0}^{2\pi} \int_{r=0}^2 \int_{z=0}^{10-r^2} r dz dr d\theta$
10. $\frac{8}{15}\pi R^5 \delta$