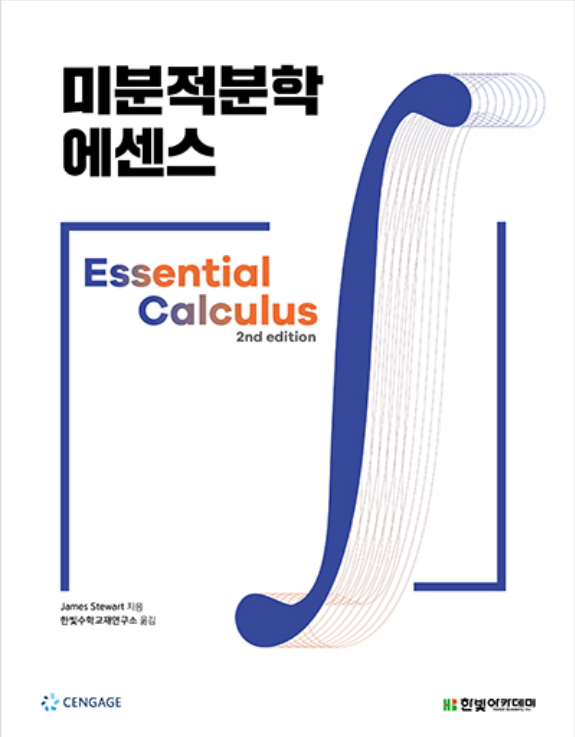


연습문제 해답

게시 일자 : 2018-03-09

1장
함수와 극한

1.1 함수의 정의와 표현 방법

01. 예

02. (a) −2 (b) 2.8 (c) −3.1 (d) −2.5, 0.3 (e) [− 3, 3] , [− 2, 3] (f) [− 1, 3]

03. 그래프가 아니다.

04. 그래프이다. [− 3, 2] , [− 3, 2) ∪ [− 1, 3]

05.

$f(x) = 3x^2 - x + 2.$

$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12.$

$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16.$

$f(a) = 3a^2 - a + 2.$

$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2.$

$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 = 3a^2 + 5a + 4.$

$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4.$

$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2.$

$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2.$

$$[f(a)]^2 = \left[3a^2 - a + 2\right]^2 = \left(3a^2 - a + 2\right)\left(3a^2 - a + 2\right)$$
$$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4.$$

$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2.$

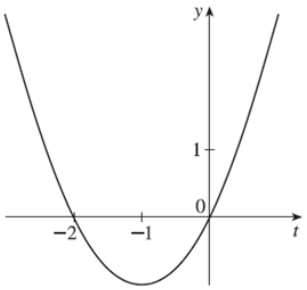
06.

$$\frac{f(3+h) - f(3)}{h} = \frac{(4 - 3h - h^2) - 4}{h} = \frac{h(-3-h)}{h} = -3 - h.$$

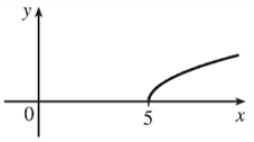
07. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

08. $[0,\ 4]$

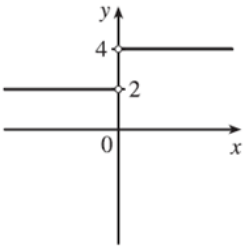
09. $(-\infty, \infty)$



10. $[5, \infty)$



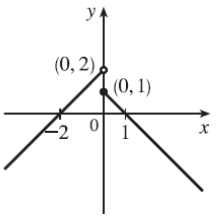
11. $(-\infty, 0) \cup (0, \infty)$



12.

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 1 - x & \text{if } x \geq 0 \end{cases}$$

The domain is $\mathbb{R}$.



13. $f(x) = \frac{5}{2}x - \frac{11}{2},\ 1 \leq x \leq 5$

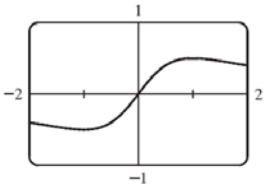
14. $f(x) = 1 - \sqrt{-x}$

15. $A(L) = L(10 - L) = 10L - L^2, \ 0 < L < 10$

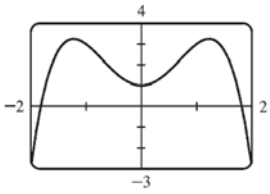
16. $A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$, with domain $x > 0$

17. (a) $(-5, 3)$ (b) $(-5, 3)$

18. 기함수



19. 우함수

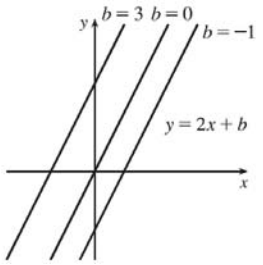


20.
(i) $f + g$는 우함수
(ii) $f + g$는 기함수
(iii) 아무것도 아니다($f = 0$ or $g = 0$이 아닌 경우).

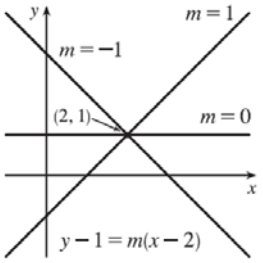
1.2 꼭 필요한 함수 목록

01.

(a) $y = f(x) = 2x + b$, 여기서 b는 y절편이다.

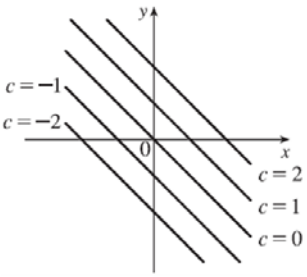


(b) $y = mx + (1-2m)$, 여기서 m은 기울기이다.



(c) $y = 2x - 3$

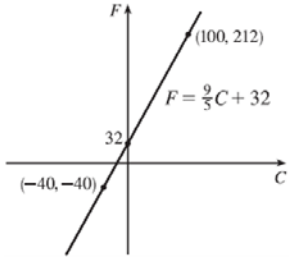
02. 이 그래프들은 기울기 -1을 갖는다.



03. $f(x) = -3x(x+1)(x-2)$

04.

(a)



(b) $\frac{9}{5}$, 1°C당 온도에 대한 °F의 변화; 32, 0°C에 대응하는 화씨온도

05.

(a) $y = f(x) + 3$

(b) $y = f(x) - 3$

(c) $y = f(x - 3)$

(d) $y = f(x + 3)$

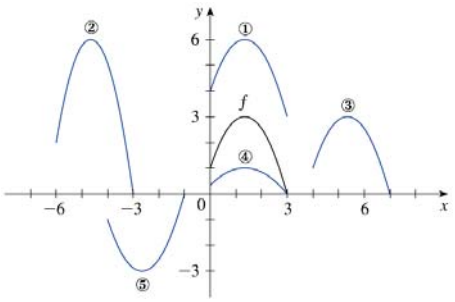
(e) $y = -f(x)$

(f) $y = f(-x)$

(g) $y = 3f(x)$

(h) $y = \frac{1}{3}f(x)$

06.



(a) (graph 3) The graph of f is shifted 4 units to the right and has equation $y = f(x - 4)$.

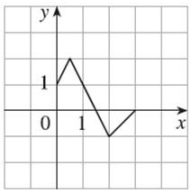
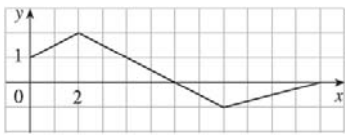
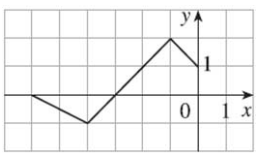
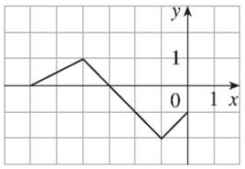
(b) (graph 1) The graph of f is shifted 3 units upward and has equation $y = f(x) + 3$.

(c) (graph 4) The graph of f is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.

(d) (graph 5) The graph of f is shifted 4 units to the left and reflected about the x-axis. Its equation is $y = -f(x + 4)$.

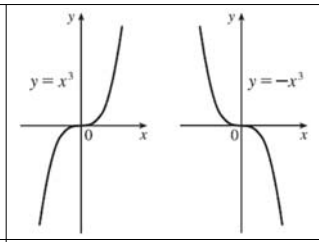
(e) (graph 2) The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x + 6)$.

07.

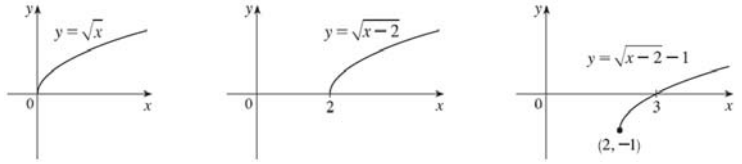
(a)		(b)	
(c)		(d)	

08. ~ 11.

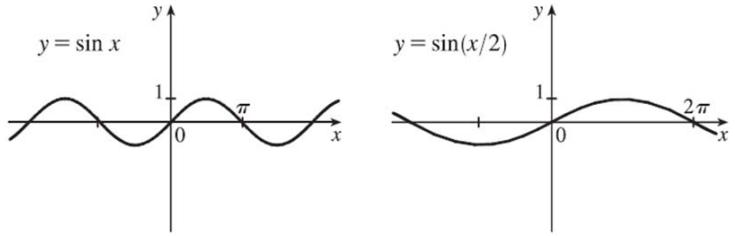
08.



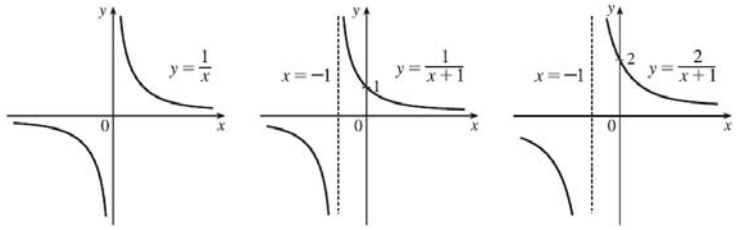
09.



10.



11.



12.

(a) $(f+g)(x) = \sqrt{1+x} + \sqrt{1-x}$, $[-1, 1]$

(b) $(f-g)(x) = \sqrt{1+x} - \sqrt{1-x}$, $[-1, 1]$

(c) $(fg)(x) = \sqrt{1+x} \cdot \sqrt{1-x} = \sqrt{1-x^2}$, $[-1, 1]$

(d) $(f/g)(x) = \dfrac{\sqrt{1+x}}{\sqrt{1-x}}$, $[-1, 1)$

13.

(a) $(f \circ g)(x) = 4x^2 + 4x$, $(-\infty, \infty)$

(b) $(g \circ f)(x) = 2x^2 - 1$, $(-\infty, \infty)$

(c) $(f \circ f)(x) = x^4 - 2x^2$, $(-\infty, \infty)$

(d) $(g \circ g)(x) = 4x + 3$, $(-\infty, \infty)$

14.

$(f \circ g \circ h)(x) = \sqrt{x^6 + 4x^3 + 1}$

15.

(a) $g(2) = 5$, because the point $(2, 5)$ is on the graph of g. Thus, $f(g(2)) = f(5) = 4$, because the point $(5, 4)$ is on the graph of f.

(b) $g(f(0)) = g(0) = 3$

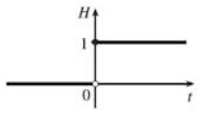
(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$. This value is not defined, because there is no point on the graph of g that has x-coordinate 6.

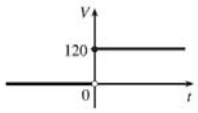
(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

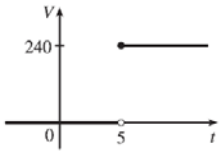
16.

(a) 

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

(b) 

$$V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 120 & \text{if } t \geq 0 \end{cases}$$ so $V(t) = 120H(t)$.

(c) 

$$V(t) = 240H(t-5)$$

17. 그렇다.

1.3 함수의 극한

01.

(a)

(i) $[1.5, 2]$: $h = 0.5$, $v_{\text{ave}} = -7.15$ m/s

(ii) $[1.5, 1.6]$: $h = 0.1$, $v_{\text{ave}} = -5.19$ m/s

(iii) $[1.5, 1.55]$: $h = 0.05$, $v_{\text{ave}} = -4.945$ m/s

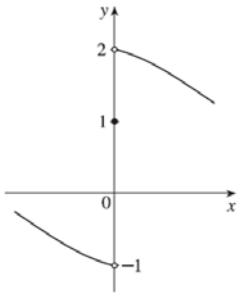
(iv) $[1.5, 1.51]$: $h = 0.01$, $v_{\text{ave}} = -4.749$ m/s

(b) -4.7 m/s

02.

(a) 2 (b) 1 (c) 4 (d) 존재하지 않는다. (e) 3

03.



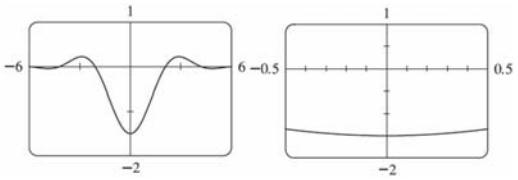
04. $\frac{2}{3}$

05. $\frac{1}{4}$

06. $\frac{3}{5}$

07.

(a) -1.5



(b)

x	$f(x)$
± 0.1	-1.493759
± 0.01	-1.499938
± 0.001	-1.499999
± 0.0001	-1.500000

08.

(a) 0

x	$f(x)$
1	0.998000
0.8	0.638259
0.6	0.358484
0.4	0.158680
0.2	0.038851
0.1	0.008928
0.05	0.001465

(b) −0.001

x	$f(x)$
0.04	0.000572
0.02	−0.000614
0.01	−0.000907
0.005	−0.000978
0.003	−0.000993
0.001	−0.001000

1.4 극한 계산

01.
(a) −6 (b) −8 (c) 2 (d) −6 (e) 존재하지 않는다. (f) 0

02. $\frac{7}{8}$

03. $\frac{\pi}{2}$

04. 4

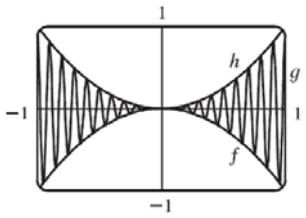
05. $\frac{6}{5}$

06. −10

07. $3x^2$

08. (a) ≈ $\frac{2}{3}$ (b) $\frac{2}{3}$ (c) 생략

09. 0



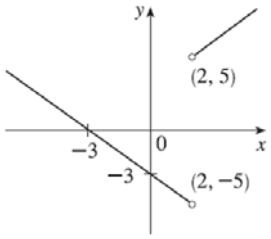
10. 생략

11. 6

12. −4

13.
(a) (i) 5 (ii) −5
(b) 존재하지 않는다.

(c)



14.

(a)

(i) -2 (ii) 존재하지 않는다. (iii) -3

(b)

(i) $n-1$ (ii) n

(c) a는 정수가 아닐 때 존재한다.

15. 생략

16. $\frac{2}{3}$

17. 9

18. 15

19. 0

20. ~ 23. 생략

1.5 연속성

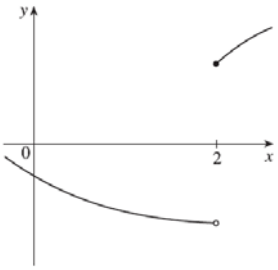
01. $\lim_{x \to 4} f(x) = f(4)$

02.

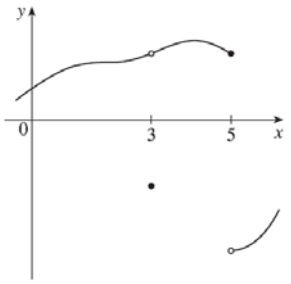
(a) f is discontinuous at -4 since $f(-4)$ is not defined and at -2, 2, and 4 since the limit does not exist (the left and right limits are not the same).

(b) f is continuous from the left at -2 since $\lim_{x \to -2^-} f(x) = f(-2)$. f is continuous from the right at 2 and 4 since $\lim_{x \to 2^+} f(x) = f(2)$ and $\lim_{x \to 4^+} f(x) = f(4)$. It is continuous from neither side at -4 since $f(-4)$ is undefined.

03.

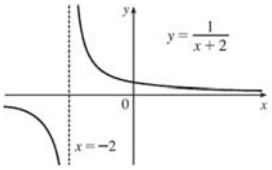


04.

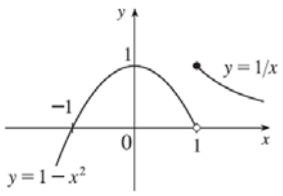


05. 4

06. $f(-2)$가 정의되지 않았다.

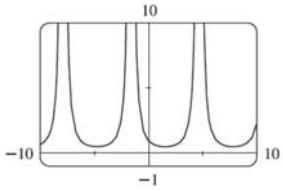


07. $\lim_{x \to 1} f(x)$가 존재하지 않는다.



08. $(-\infty, \infty)$

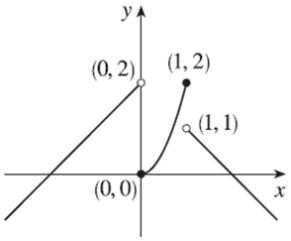
09. $(-\infty, -1] \cup (0, \infty)$

10. 

$x = -\frac{\pi}{2} + 2\pi n$, n은 정수

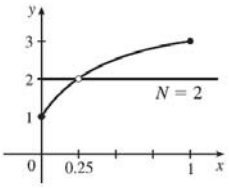
11. $\frac{7}{3}$

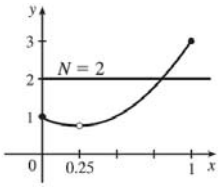
12. 0, 오른쪽으로부터 연속
1, 왼쪽으로부터 연속



13. $\frac{2}{3}$

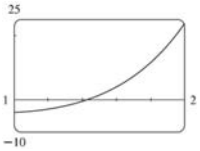
14.

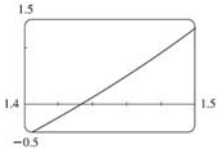




15. ~ 16. 생략

17. (a) 생략 (b) $x \approx 1.434$





18. 생략

1.6 연속성

01.

(a) $\lim_{x\to\infty} f(x) = -2$ (b) $\lim_{x\to-\infty} f(x) = 2$ (c) $\lim_{x\to 1} f(x) = \infty$

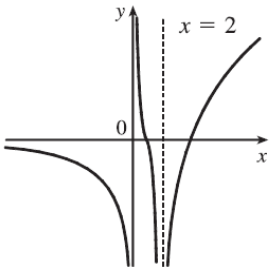
(d) $\lim_{x\to 3} f(x) = -\infty$ (e) Vertical: $x = 1$, $x = 3$; horizontal: $y = -2$, $y = 2$

02.

$\lim_{x\to 2} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$,

$\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to 0^+} f(x) = \infty$,

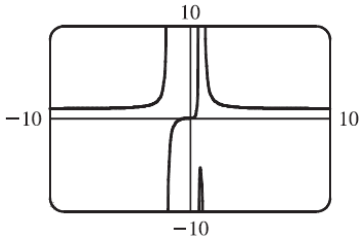
$\lim_{x\to 0^-} f(x) = -\infty$



03.

Vertical: $x \approx -1.62$, $x \approx 0.62$, $x = 1$;

Horizontal: $y = 1$



04. $-\infty$

05. ∞

06. $-\infty$

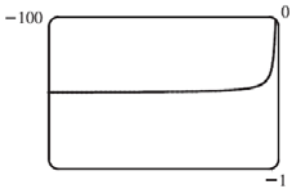
07. $\frac{1}{2}$

08. 4

09. ∞

10.

(a) −0.5



(b)

x	$f(x)$
−10,000	−0.4999625
−100,000	−0.4999962
−1,000,000	−0.4999996

(c) 생략

11. (a) $\frac{5}{4}$ (b) 5

12. (a) 0 (b) $\pm\infty$

13. 4

14.

(a) 생략

(b) 탱크 안으로 들어온 소금물의 농도로 접근한다.

15. 생략

1장 복습문제

연습문제

01.

(a) $f(2) \approx 2.7$

(b) $x \approx 2.3, 5.6$

(c) $[-6, 6]$

(d) $[-4, 4]$

(e) $[-4, 4]$

(f) 기함수, 이 그래프는 원점에 대해 대칭이다.

02. $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$, $(-\infty, 0) \cup (0, \infty)$

03.

(a) 그래프를 위쪽으로 8단위만큼 이동

(b) 그래프를 왼쪽으로 8단위만큼 이동

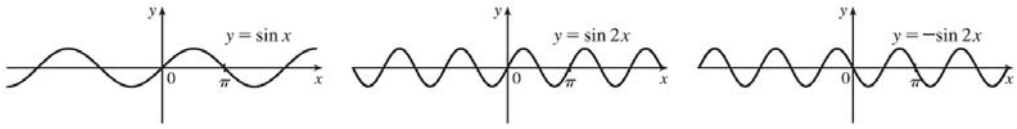
(c) 그래프를 수직으로 2배로 늘리고 위쪽으로 1단위만큼 이동

(d) 그래프를 오른쪽으로 2단위만큼 이동하고 아래쪽으로 2단위만큼 이동

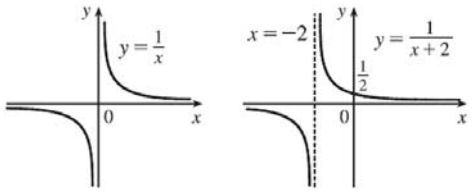
(e) 그래프를 x축에 대해 대칭이동

(f) 그래프를 x축에 대해 대칭이동하고 위쪽으로 3단위만큼 이동

04.



05.



06.

(a) 아무것도 아니다.

(b) 기함수

(c) 우함수

(d) 아무것도 아니다.

07.

(a) $(f \circ g)(x) = \sqrt{\sin x}$, $\{x \mid x \in [2n\pi, \pi + 2n\pi],\ n$은 정수$\}$

(b) $(g \circ f)(x) = \sin \sqrt{x}$, $[0, \infty)$

(c) $(f \circ f)(x) = \sqrt[4]{x}$, $[0, \infty)$

(d) $(g \circ g)(x) = \sin(\sin x)$, $\mathbb{R}$

08.

(a) (i) 3 (ii) 0 (iii) 존재하지 않는다. (iv) 2 (v) ∞ (vi) $-\infty$
(vii) 4 (viii) -1

(b) $y = 4,\ y = -1$ (c) $x = 0,\ x = 2$ (d) -3, 0, 2, 4

09. 1

10. 0

11. $-\frac{1}{8}$

12. 2

13. 1

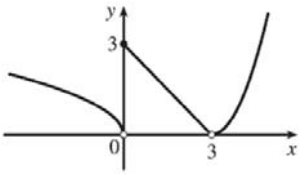
14. ~ 15. 생략

16.

(a) (i) 3 (ii) 0 (iii) 존재하지 않는다. (iv) 0 (v) 0 (vi) 0

(b) 0과 3에서

(c)



17. 생략