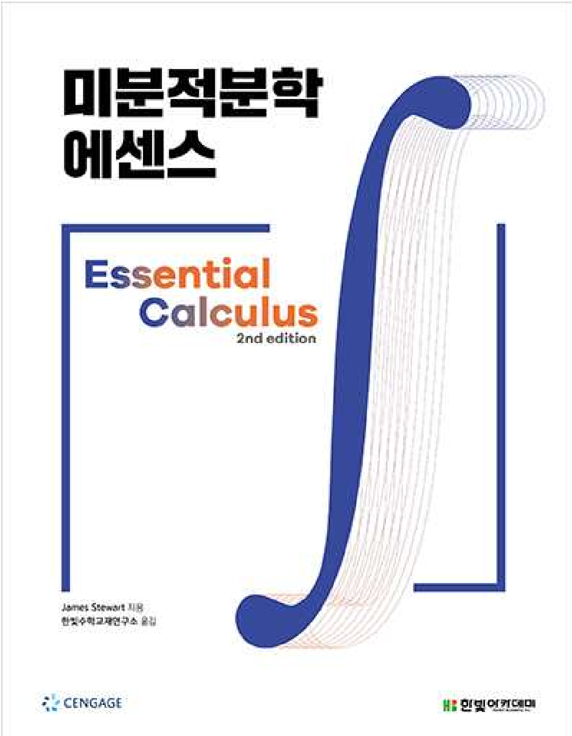


연습문제 해답

게시 일자 : 2018-03-09

2장
역함수
: 지수함수, 로그함수, 역삼각함수

2.1 역함수

01.
(a) 정의 1 참조
(b) 수평선 판정법(Horizontal Line Test)을 통과해야 한다.

02. 일대일 함수가 아니다.

03. 일대일 함수가 아니다.

04. 일대일 함수가 아니다.

05. 일대일 함수가 아니다.

06. 일대일 함수이다.

07. 일대일 함수가 아니다.

08. (a) 6 (b) 3

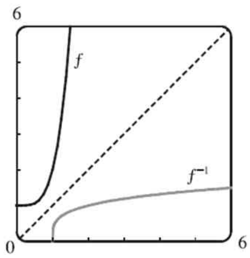
09. 4

10. 생략

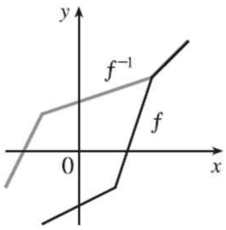
11. $f^{-1}(x) = \dfrac{3-x}{2}$

12. $f^{-1}(x) = \left(\dfrac{1-x}{1+x}\right)^2$ with $-1 < x \le 1$

13. $f^{-1}(x) = \sqrt[4]{x-1}$



14.



15.

(a) $f^{-1}(x) = \sqrt{1-x^2}$, $0 \le x \le 1$; f^{-1}과 f는 같은 함수이다.

(b) 제1사분면에 있는 사분원

16. (a) $g^{-1}(x) = f^{-1}(x) - c$ (b) $h^{-1}(x) = (1/c)f^{-1}(x)$

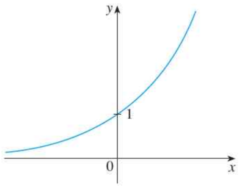
2.2 지수함수

01.

(a) The domain of $f(x) = a^x$ is $\mathbb{R}$.

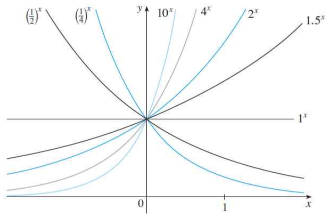
(b) The range of $f(x) = a^x$ $[a \neq 1]$ is $(0, \infty)$.

(c) (i)

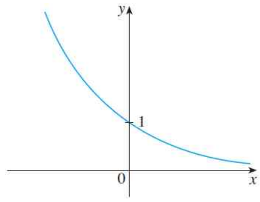


$\lim_{x\to-\infty} a^x = 0$, $\lim_{x\to\infty} a^x = \infty$

(ii)



(iii)



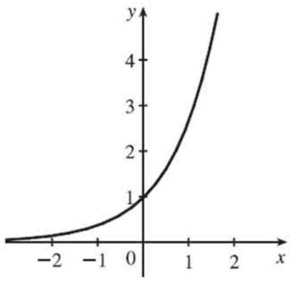
$\lim_{x\to-\infty} a^x = \infty$, $\lim_{x\to\infty} a^x = 0$

02.

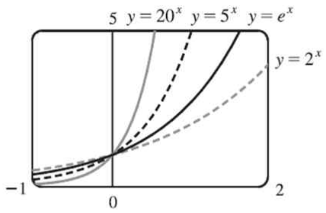
(a) e는 $\ln e = 1$을 만족하는 수이다.

(b) $e \approx 2.71828$

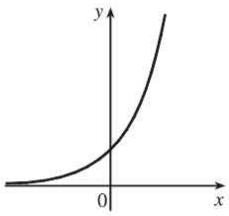
(c) $f(x) = e^x$이면 $f'(0) = 1$



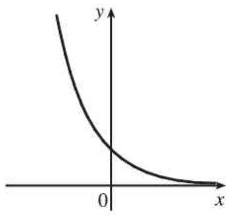
03.



04.

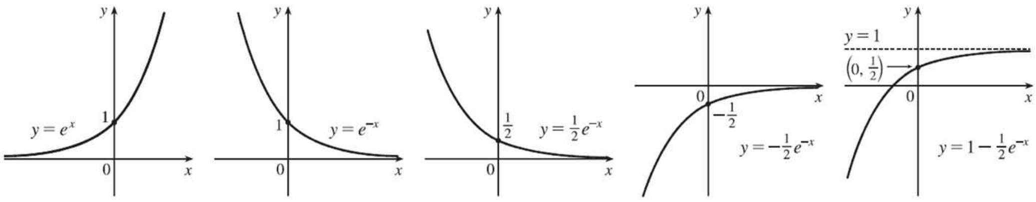


$y = e^x$



$y = e^{-x}$

05.



06. 1

07. 0

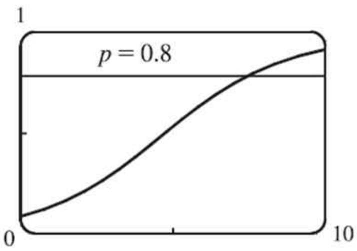
08. $f(x)=3 \cdot 2^x$

09. 생략

10.

(a) 1

(b) $\dfrac{kae^{-kt}}{(1+ae^{-kt})^2}$



$t \approx 7.4$

2.3 로그함수

01.

(a) $\log_a x$ is the number y such that $a^y = x$.

(b) The domain of $f(x) = \log_a x$ is $(0, \infty)$.

(c) The range of $f(x) = \log_a x$ is $\mathbb{R}$.

(d) See Figure 7.

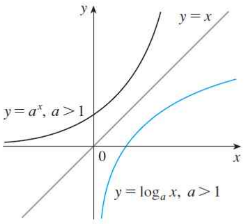


FIGURE 7

02.

(a) 3

(b) $\log_3 \dfrac{1}{27} = -3$ (책에 실린 문제를 변경함)

03. (a) 8

(b) $\dfrac{1}{9}$

04. (a) $\dfrac{1}{25}$

(b) 10

05.

$\ln \sqrt{ab} = \ln(ab)^{1/2} = \frac{1}{2}\ln(ab) = \frac{1}{2}(\ln a + \ln b) = \frac{1}{2}\ln a + \frac{1}{2}\ln b$ [assuming that the variables are positive]

06.

$\ln \dfrac{x^2}{y^3 z^4} = \ln x^2 - \ln(y^3 z^4) = 2\ln x - (\ln y^3 + \ln z^4) = 2\ln x - 3\ln y - 4\ln z$

07.

Since $a^x = e^{x \ln a}$, $4^{-\pi} = e^{-\pi \ln 4}$.

08.

Since $a^x = e^{x \ln a}$, $10^{x^2} = e^{x^2 \ln 10}$.

09.

(a) $e^{7-4x} = 6 \quad \Leftrightarrow \quad 7 - 4x = \ln 6 \quad \Leftrightarrow \quad 7 - \ln 6 = 4x \quad \Leftrightarrow \quad x = \frac{1}{4}(7 - \ln 6)$

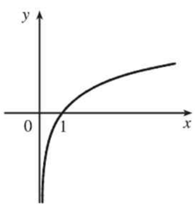
(b) $\ln(3x - 10) = 2 \quad \Leftrightarrow \quad 3x - 10 = e^2 \quad \Leftrightarrow \quad 3x = e^2 + 10 \quad \Leftrightarrow \quad x = \frac{1}{3}(e^2 + 10)$

10.

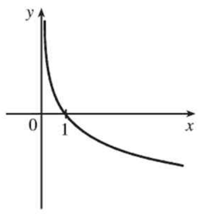
(a) $\ln x < 0 \quad \Rightarrow \quad x < e^0 \quad \Rightarrow \quad x < 1$. Since the domain of $f(x) = \ln x$ is $x > 0$, the solution of the original inequality is $0 < x < 1$.

(b) $e^x > 5 \quad \Rightarrow \quad \ln e^x > \ln 5 \quad \Rightarrow \quad x > \ln 5$

11.

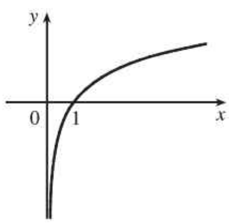


$y = \ln x$

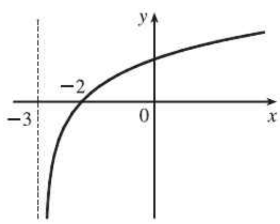


$y = -\ln x$

12.



$y = \ln x$



$y = \ln(x+3)$

13.

$$f(x) = \frac{x}{1-\ln(x-1)} \quad \Rightarrow$$

$$f'(x) = \frac{[1-\ln(x-1)]\cdot 1 - x\cdot\dfrac{-1}{x-1}}{[1-\ln(x-1)]^2} = \frac{\dfrac{(x-1)[1-\ln(x-1)]+x}{x-1}}{[1-\ln(x-1)]^2} = \frac{x-1-(x-1)\ln(x-1)+x}{(x-1)[1-\ln(x-1)]^2}$$

$$= \frac{2x-1-(x-1)\ln(x-1)}{(x-1)[1-\ln(x-1)]^2}$$

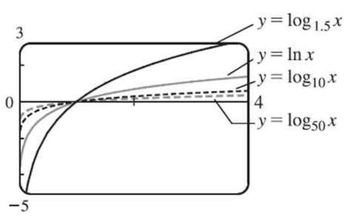
$$\mathrm{Dom}(f) = \{x \mid x-1>0 \quad \text{and} \quad 1-\ln(x-1)\neq 0\} = \{x \mid x>1 \quad \text{and} \quad \ln(x-1)\neq 1\}$$

$$= \{x \mid x>1 \quad \text{and} \quad x-1\neq e^1\} = \{x \mid x>1 \quad \text{and} \quad x\neq 1+e\} = (1, 1+e)\cup(1+e,\infty)$$

14. $y = \log_{10} x - \log_{10}(1-x)$

15. $\left(-\infty, \frac{1}{2}\ln 3\right]$, $f^{-1}(x) = \frac{1}{2}\ln(3-x^2)$, $[0, \sqrt{3}\,)$

16.



2.4 역삼각함수

01.

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b) $\cos^{-1}(-1) = \pi$ since $\cos\pi = -1$ and π is in $[0, \pi]$.

02.

(a) $\arctan(-1) = -\frac{\pi}{4}$ since $\tan\left(-\frac{\pi}{4}\right) = -1$ and $-\frac{\pi}{4}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(b) $\csc^{-1} 2 = \frac{\pi}{6}$ since $\csc\frac{\pi}{6} = 2$ and $\frac{\pi}{6}$ is in $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$.

03.

(a) $\arctan 1 = \frac{\pi}{4}$ since $\tan\frac{\pi}{4} = 1$ and $\frac{\pi}{4}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(b) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ since $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\frac{\pi}{4}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

04.

(a) $\sec^{-1}\sqrt{2} = \frac{\pi}{4}$ since $\sec\frac{\pi}{4} = \sqrt{2}$ and $\frac{\pi}{4}$ is in $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$.

(b) $\arcsin 1 = \frac{\pi}{2}$ since $\sin\frac{\pi}{2} = 1$ and $\frac{\pi}{2}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

05.

(a) In general, $\tan(\arctan x) = x$ for any real number x. Thus, $\tan(\arctan 10) = 10$.

(b) $\sin^{-1}\left(\sin\frac{7\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

[Recall that $\frac{7\pi}{3} = \frac{\pi}{3} + 2\pi$ and the sine function is periodic with period 2π.]

06.

(a) Let $\theta = \arctan 2$, so $\tan\theta = 2 \quad\Rightarrow\quad \sec^2\theta = 1 + \tan^2\theta = 1 + 4 = 5 \quad\Rightarrow\quad \sec\theta = \sqrt{5} \quad\Rightarrow$

$\sec(\arctan 2) = \sec\theta = \sqrt{5}$.

(b) Let $\theta = \sin^{-1}\left(\frac{5}{13}\right)$. Then $\sin\theta = \frac{5}{13}$, so $\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right) = \cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{5}{13}\right)^2 = \frac{119}{169}$.

07.

Let $y = \sin^{-1} x$. Then $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \quad\Rightarrow\quad \cos y \ge 0$, so $\cos(\sin^{-1} x) = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$.

08. $\tan(\sin^{-1} x) = \tan y = \dfrac{x}{\sqrt{1-x^2}}$

09. $\sin(\tan^{-1} x) = \sin y = \dfrac{x}{\sqrt{1+x^2}}$

10. $\csc(\arctan 2x) = \csc\theta = \dfrac{\sqrt{4x^2+1}}{2x}$

11. (a) 생략 (b) 생략

2.5 쌍곡선함수

01. (a) 0 (b) 1

02. (a) $\dfrac{3}{4}$ (b) ≈ 3.62686

03. (a) 1 (b) 1

04. $-\sinh x$

05. $\cosh x$

06. e^{x}

07. e^{-x}

08. $\sinh(x+y)$

09. $\cosh(x+y)$

10. $2\sinh x\cosh x$

11. e^{2x}

12. $\cosh nx + \sinh nx$

13.

$$\operatorname{sech} x = \frac{1}{\cosh x} \quad \Rightarrow \quad \operatorname{sech} x = \frac{1}{5/3} = \frac{3}{5}.$$

$$\cosh^2 x - \sinh^2 x = 1 \quad \Rightarrow \quad \sinh^2 x = \cosh^2 x - 1 = \left(\tfrac{5}{3}\right)^2 - 1 = \tfrac{16}{9} \quad \Rightarrow \quad \sinh x = \tfrac{4}{3} \quad [\text{because } x > 0].$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \quad \Rightarrow \quad \operatorname{csch} x = \frac{1}{4/3} = \frac{3}{4}.$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \Rightarrow \quad \tanh x = \frac{4/3}{5/3} = \frac{4}{5}.$$

$$\coth x = \frac{1}{\tanh x} \quad \Rightarrow \quad \coth x = \frac{1}{4/5} = \frac{5}{4}.$$

14.

(a) $\lim_{x\to\infty} \tanh x = \lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{e^{-x}}{e^{-x}} = \lim_{x\to\infty} \frac{1 - e^{-2x}}{1 + e^{-2x}} = \frac{1-0}{1+0} = 1$

(b) $\lim_{x\to-\infty} \tanh x = \lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{e^{x}}{e^{x}} = \lim_{x\to-\infty} \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{0-1}{0+1} = -1$

(c) $\lim_{x\to\infty} \sinh x = \lim_{x\to\infty} \frac{e^x - e^{-x}}{2} = \infty$

(d) $\lim_{x\to-\infty} \sinh x = \lim_{x\to-\infty} \frac{e^x - e^{-x}}{2} = -\infty$

(e) $\lim_{x\to\infty} \operatorname{sech} x = \lim_{x\to\infty} \frac{2}{e^x + e^{-x}} = 0$

(f) $\lim_{x\to\infty} \coth x = \lim_{x\to\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} \cdot \frac{e^{-x}}{e^{-x}} = \lim_{x\to\infty} \frac{1 + e^{-2x}}{1 - e^{-2x}} = \frac{1+0}{1-0} = 1$ [*Or:* Use part (a)]

(g) $\lim_{x\to0^+} \coth x = \lim_{x\to0^+} \frac{\cosh x}{\sinh x} = \infty$, since $\sinh x \to 0$ through positive values and $\cosh x \to 1$.

(h) $\lim_{x\to0^-} \coth x = \lim_{x\to0^-} \frac{\cosh x}{\sinh x} = -\infty$, since $\sinh x \to 0$ through negative values and $\cosh x \to 1$.

(i) $\lim_{x\to-\infty} \operatorname{csch} x = \lim_{x\to-\infty} \frac{2}{e^x - e^{-x}} = 0$

15. $y = \ln\left(x + \sqrt{1+x^2}\right)$

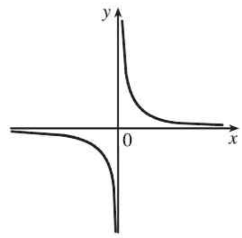
16. 생략

17. (a) 생략 (b) 생략

18.

(a) (i) $y = \operatorname{csch}^{-1} x \iff \operatorname{csch} y = x \quad (x \neq 0)$

(ii) We sketch the graph of csch^{-1} by reflecting the graph of csch (see Exercise 18) about the line $y = x$.

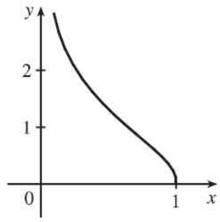


(iii) Let $y = \operatorname{csch}^{-1} x$. Then $x = \operatorname{csch} y = \frac{2}{e^y - e^{-y}} \Rightarrow xe^y - xe^{-y} = 2 \Rightarrow$

$x(e^y)^2 - 2e^y - x = 0 \Rightarrow e^y = \frac{1 \pm \sqrt{x^2+1}}{x}$. But $e^y > 0$, so for $x > 0$,

$e^y = \frac{1 + \sqrt{x^2+1}}{x}$ and for $x < 0$, $e^y = \frac{1 - \sqrt{x^2+1}}{x}$. Thus, $\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{x^2+1}}{|x|}\right)$.

(b) (i) $y = \operatorname{sech}^{-1} x \quad \Leftrightarrow \quad \operatorname{sech} y = x$ and $y > 0$.



(ii) We sketch the graph of sech^{-1} by reflecting the graph of sech (see Exercise 18) about the line $y = x$.

(iii) Let $y = \operatorname{sech}^{-1} x$, so $x = \operatorname{sech} y = \dfrac{2}{e^y + e^{-y}} \quad \Rightarrow \quad xe^y + xe^{-y} = 2 \quad \Rightarrow$

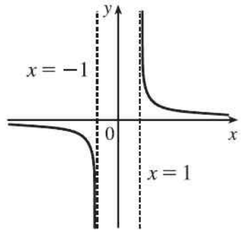
$x\,(e^y)^2 - 2e^y + x = 0 \quad \Leftrightarrow \quad e^y = \dfrac{1 \pm \sqrt{1 - x^2}}{x}$. But $y > 0 \quad \Rightarrow \quad e^y > 1$.

This rules out the minus sign because $\dfrac{1 - \sqrt{1 - x^2}}{x} > 1 \quad \Leftrightarrow \quad 1 - \sqrt{1 - x^2} > x \quad \Leftrightarrow \quad 1 - x > \sqrt{1 - x^2} \quad \Leftrightarrow$

$1 - 2x + x^2 > 1 - x^2 \quad \Leftrightarrow \quad x^2 > x \quad \Leftrightarrow \quad x > 1$, but $x = \operatorname{sech} y \le 1$.

Thus, $e^y = \dfrac{1 + \sqrt{1 - x^2}}{x} \quad \Rightarrow \quad \operatorname{sech}^{-1} x = \ln\left(\dfrac{1 + \sqrt{1 - x^2}}{x}\right)$.

(c) (i) $y = \coth^{-1} x \quad \Leftrightarrow \quad \coth y = x$



(ii) We sketch the graph of $\coth^{-1}$ by reflecting the graph of coth (see Exercise 18) about the line $y = x$.

(iii) Let $y = \coth^{-1} x$. Then $x = \coth y = \dfrac{e^y + e^{-y}}{e^y - e^{-y}} \quad \Rightarrow$

$xe^y - xe^{-y} = e^y + e^{-y} \quad \Rightarrow \quad (x - 1)e^y = (x + 1)e^{-y} \quad \Rightarrow \quad e^{2y} = \dfrac{x + 1}{x - 1} \quad \Rightarrow$

$2y = \ln \dfrac{x + 1}{x - 1} \quad \Rightarrow \quad \coth^{-1} x = \dfrac{1}{2} \ln \dfrac{x + 1}{x - 1}$

19. $\dfrac{1}{2}$

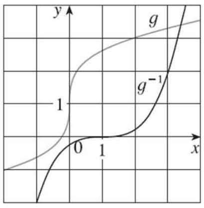
20. 생략

2장 복습문제

연습문제

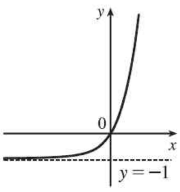
01. 아니오.

02.
(a) 수평선 판정법을 통과했기 때문에 일대일이다.
(b) ≈ 0.2
(c) $[-1, 3.5]$
(d)



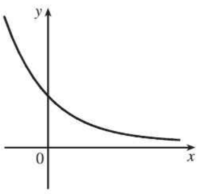
03. $y = \dfrac{1-x}{2x-1} = f^{-1}(x)$

04.

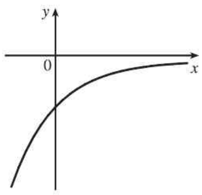


$y = 5^x - 1$

05.

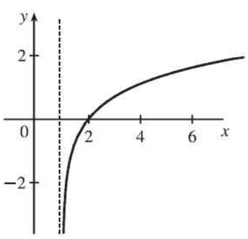


$y = e^{-x}$



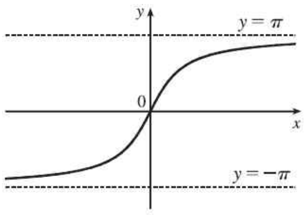
$y = -e^{-x}$

06.



$y = \ln(x-1)$

07.



$y = 2\arctan x$

08. 생략

09. (a) 9 (b) 2

10. (a) π (b) $\dfrac{1}{\sqrt{3}}$

11. (a) $x = \ln 5$ (b) $x = e^2$

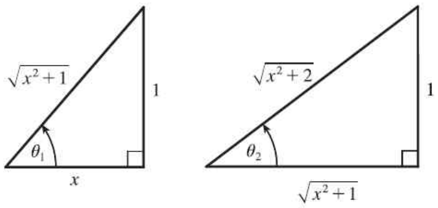
12. (a) $x = -\ln(e^3 - 1)$ (b) $x = \sin^{-1} 0.3 = \alpha$

13.

Let $\theta_1 = \operatorname{arccot} x$, so $\cot\theta_1 = x = x/1$.

So $\sin(\operatorname{arccot} x) = \sin\theta_1 = \dfrac{1}{\sqrt{x^2+1}}$.

Let $\theta_2 = \arctan\left[\dfrac{1}{\sqrt{x^2+1}}\right]$, so $\tan\theta_2 = \dfrac{1}{\sqrt{x^2+1}}$.



Hence, $\cos\{\arctan[\sin(\operatorname{arccot} x)]\} = \cos\theta_2 = \dfrac{\sqrt{x^2+1}}{\sqrt{x^2+2}} = \sqrt{\dfrac{x^2+1}{x^2+2}}$.